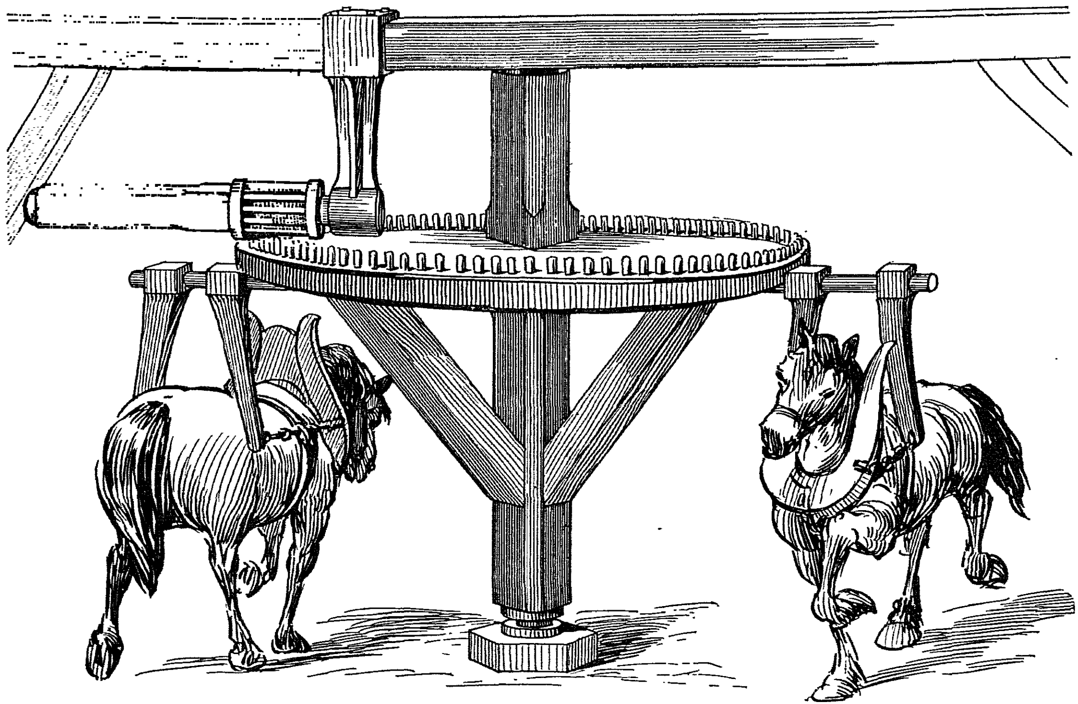
ENGINEERING KINEMATICS

McGraw-Hill Book Co. Inc.
PUBLISHERS OF BOOKS FOR
Electrical World ▿ Engineering News-Record
Power ▿ Engineering and Mining Journal-Press
Chemical and Metallurgical Engineering
Electric Railway Journal ▿ Coal Age
American Machinist ▿ Ingenieria Internacional
Electrical Merchandising ▿ Bus Transportation
Journal of Electricity and Western Industry
Industrial Engineer



Kinematics 200 years ago. After an old print. (*Frontispiece*)

ENGINEERING KINEMATICS

BY

WILLIAM GRISWOLD SMITH, M. E.

Associate Professor of Mechanical Drawing, College of Engineering, Northwestern University; Formerly Associate Professor of Kinematics and Descriptive Geometry, Armour Institute of Technology; Author of "Practical Descriptive Geometry."

FIRST EDITION

McGRAW-HILL BOOK COMPANY, INC.
NEW YORK: 370 SEVENTH AVENUE
LONDON: 6 & 8 BOUVERIE ST., E. C. 4
1923

PRINTED IN THE UNITED STATES OF AMERICA

PREFACE

The name "Engineering Kinematics" was chosen deliberately. The abstract science of Kinematics in itself does not make sufficient appeal to the engineer. He is primarily interested in *what can be done* with a science. The titles of existing text books show that this idea is present in the minds of writers on this subject; for example, "Elements of Mechanism," "Kinematics of Machinery," "Theory of Machines," and others. So in this book, while the fundamental principles of motion, its laws, its conversion, and its transfer, are given first consideration, the applications of these principles to the design of the innumerable agencies for transmission, transportation, and production are given as much attention as the size of the volume will permit.

As a natural result of this trend toward the practical use of the science, the author has used as his chief source of material, not so much the text books (valuable and authoritative as many of them are), as he has the vast treasure house of industrial literature existing in catalogues, bulletins, handbooks, and magazine articles. This industrial literature is of outstanding importance to the engineer, because it is the accumulated wisdom of specialists, each in his own field, in which he is devoting his life to the appraisement of records of many actual performances in the great laboratory of practical utility.

Handling such vital material as this, studying such practical illustrations, a student can hardly fail to desire a more thorough knowledge of the subject, realizing, as he must, that Kinematics is the basis of a most important part of our national industrial life. A thorough mastery of its principles and applications means that one has linked himself up with the great men who develop and conserve the earth's store of wealth for the comfort and happiness of our race.

A man learns a subject that interests him, but this should not be construed that the facts and theories here presented have been sugar-coated to make them palatable and easy. The close and obvious relation of the matter of this book to actual engineering problems and conditions will cause many more students to take

an active interest in it than would be the case were it only presented in the abstract. If it accomplishes this, its issuance will be justified.

The problems presented at the close of each chapter are varied, have real engineering value, and are presented in almost unlimited supply. In most cases a typical example with complete information is given first, and this is followed by one with blank spaces for data tc be supplied by the teacher. With varying data it is possible to assign different problems to every man in the class, and new ones each year.

WILLIAM GRISWOLD SMITH

URBANA, ILL
May, 1923.

CONTENTS

PAGE

PREFACE vii

CHAPTER I

DEFINITIONS AND FUNDAMENTALS 1
Laws and varieties of motion and acceleration—Measurement of work—Graphic solutions and analysis—Mechanical advantage—Automatic handling.

CHAPTER II

DETERMINATION OF RELATIVE VELOCITIES AND ACCELERATIONS OF SIMPLE LINKWORK PARTS 27
Instantaneous centers—Velocity and acceleration diagrams—Centrodes.

CHAPTER III

CHAINS OF LINKWORK 53
The quadric chain, slider-crank chain and inversions—Quick-return motions, multiple vibrating linkwork—Piston displacement—Eccentrics, rockers, straight-line mechanisms—Pantographs and applications.

CHAPTER IV

DIRECT CONTACT PAIRS, ROLLING CURVES, FRICTION TRANSMISSION . . 79
Line and surface contact—Pure rolling—Slipping—Positive drive—Rolling curves—Angular velocity ratios—Friction transmission—Plain frictions—Coefficients of friction—Facts for designers—Bevel frictions—Grooved frictions—Disc frictions.

CHAPTER V

TOOTHED GEARINGS FOR PARALLEL SHAFTS 105
Theory of gears—Tooth outlines and dimensions—Involute gears—Modern developments in gear design—Stub-tooth gears—Proportions of gear teeth—Internal gears—Twisted gears—Herringbone gears—Methods of drawing tooth outlines—Methods of gear manufacture—Elliptical gears.

PAGE

CHAPTER VI

Gears on Non-Parallel Shafting 140
Bevel gears—Non-interchangeability of bevel gears—Design and drawing of bevel gearing—Manufacture of bevel gears—Spiral bevel gears—Helical gears and their theory—Worm gears—Efficiency of worm gearing—Drawing and design of worm gears—Manufacture of worms and gears—Skew gears.

CHAPTER VII

Cams 173
Varieties and design of cams—Displacement diagrams—Adjustable cams—Involute cams—Wiper cams—Transmissible cams.

CHAPTER VIII

Belt, Rope and Chain Transmission 197
Classes and general principles of flexible transmission—Belt drives—Horsepower of belting—Pulleys—Cone pulleys—Methods of determining step diameters—Substitutes for leather belting—Rope drive and transmission systems—Wire rope drive—Chain transmission—Summary.

CHAPTER IX

Trains of Mechanism 225
Gear trains, double and triple reduction—Idlers—Lathe back gears—Screw cutting trains, automobile gear box, epicyclical gears, differential gear train, pulley blocks, differential pulleys.

APPENDIX 253

Devices and attachments to supplement standard transmission—Couplings, clutches, intermittent transmittors, compressed air, solenoids, brakes, springs, ball bearings, roller bearings, conveyors, automatic pickups and jacks.

Index 279

ENGINEERING KINEMATICS

CHAPTER I

DEFINITIONS AND FUNDAMENTALS

1. **Kinematics is the Science of Motion.**[1]—Thus, every problem in transmission and transportation is a **kinematic problem. Transmission** means "**sending** over, across, or beyond," and **transportation** means "**carrying** over, across, or beyond." Wherever we may be, in a manufacturing district, in a mercantile locality, in farming, shipping, or residential places, we see countless evidences of the application of Kinematics to the solution of the problems of everyday life. Every article of sustenance, protection, comfort, and luxury that enters into our lives, is either produced or transported by kinematic mechanism. All the machines in the shop, the steam or gas engine, the giant crane, the tiny wrist watch, the rolling mill, the farm implement, the automobile, the child's velocipede, in fact, **all things that go,** have been discovered and developed by the understanding of the principles of Kinematics.

All work is motion. Hence, power without motion is useless. If you heat a boiler, the steam will do no useful work unless it is led into some kinematic device, like the engine, which transforms its potential energy into work that can be utilized.

It should be evident, therefore, that the applications of Kinematics are so universal that a clear understanding of its principles, and the chief agencies in their distribution will be needed by every engineer. Many of these devices have become commonplaces and standardized, and their principles so familiar and established, that a study of them seems unnecessary. Nevertheless, new developments and refinements are being introduced in an unending stream, and these demand a higher understanding than ever before of the underlying principles of this subject, and the application of them into useful mechanisms that will help to make happier and more livable the lives of all mankind.

[1]Rankine defines it as the "Geometry of Machinery."

MOTION

2. **Motion is a Change of Position.**—All motion is relative, since we have no knowledge of any body that is absolutely at rest. The building you are in seems to be a stationary object; yet it, with the earth to which it is bound, is flying rapidly through space, whirling as it goes, about a sun, which is also moving about some more remote center, and no man can determine the ultimate center. Therefore, a body is at rest or in motion only as it is related to some other body, and absolute motion is inconceivable, and does not exist.

Standards.—In practical kinematics there are two reference bodies, the **earth,** and the **frame** of the machine. In stationary machines the frame has no motion relative to the earth, hence they are kinematically the same. In moving machines (locomotives, automobiles, traveling cranes) the motions of various members are most intimately related to the frame of the machine. It is sometimes necessary to relate the motions of two moving parts to one another.

Note.—To avoid further explanation, let it be understood that the frame is the so-called **stationary** member, and that parts moving relatively to the frame are called moving parts.

VARIETIES OF MOTION

3. **Motion may be constant or intermittent.**—Either of these may be **regular** or **irregular.**

Examples.—Constant regular.—A steam engine flywheel.
Constant irregular.—An automobile under way.
Intermittent regular.—A gas engine valve.
Intermittent irregular.—Any humanly controlled effort.

Free Motion.—A body so connected to other bodies (or not connected), that it need not follow a definite path, is a free body, and has **free motion,** to greater or less extent. Every outside force may change the path or position of the body.

Example.—A weight carried by a traveling crane, or a ball rolling down hill, where every irregularity will change its path. In the case of the crane, this freedom is an advantage in case of a collision. The swaying of the weight is "lost motion."

Constrained Motion.—A body so connected to other bodies, that its motion must be over a certain path, whatever outside forces may be acting, is **constrained,** and it has **constrained motion.** All machinery is dependent on constrainment for its proper action.

Example.—The crank of an engine **must** rotate on its axis, because the circular bearings of the crankshaft will permit no motion but rotation. Although it is impelled by forces acting in other directions than the one in which it is moving, no outside force can alter this direction unless it is strong enough to break the machine, or part.

PLANE, HELICAL AND SPHERICAL MOTION

4. **Plane Motion.**—A body in which every particle moves in the same or parallel planes is said to have **plane motion.** When the particles move in parallel lines, the motion is called **translation.** When all points move in concentric circles, the motion

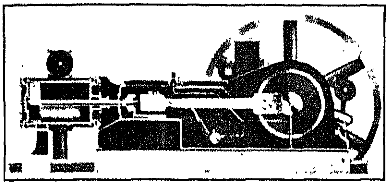


FIG. 1.—Sectional view of a steam engine. The Houston.

is called **rotary.** When all points move in varied paths, it is called **complex.**

Note.—These distinctions are only convenient descriptions. In the broadest analysis they are identical. The only difference between parallelism and concentricity is that in the case of a straight path the **center is at infinity.** All plane motion can be reduced to **rotation.**

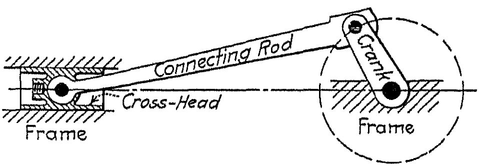


FIG. 2.—Four examples of plane motion: Frame—stationary; cross-head—rectilinear, reciprocating; crank—rotary; connecting rod—complex.

Example.—Figure 1 shows the essential parts of a steam engine. The **frame** is **stationary.** The **piston, piston rod** and **crosshead** have **translation** (rectilinear and reciprocating). The **crank** has **rotary motion,** and the **connecting-rod** has **complex motion,** since one end of the rod moves in a straight line and the other in a circle.

Note.—In a steam engine the piston, piston rod, and crosshead are kinematically one piece, and in the single acting gas engine they are actually one piece.

5. Helical Motion.—The helix is the path of a point that moves about an axis at a constant distance, and in the direction of the axis at a constant rate. A body whose particles move in helices has **helical motion.**

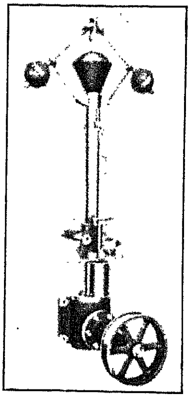


FIG. 3.—Fly ball governor. The Hooven, Owens, Rentschler Co., Hamilton, Ohio.

Example.—A screw as it moves in the recess provided for it. This motion in machinery is less common than would appear, for most helically cut members (lead screws in lathes, worms) have plane rotary motion and **transmit** plane motion.

6. Spherical Motion.—A sphere is the surface generated by point moving at a constant distance from another point. The only conspicuous instance of spherical motion is the fly-ball governor, and its spherical motion is of very slight kinematical importance.

Note.—The instances of helical or spherical motion are so infrequent that they need not be studied in this work. Hence, all mention of motion will be understood to be **plane motion,** unless otherwise stated. It is true that the relative motion between two moving parts is frequently helical and, very rarely, spherical, yet they impose no such conditions on the problem.

VELOCITY

7. Velocity is the **rate of motion.**

Equation.—$V = \frac{S}{T}$.

Example.—A train travels 60 miles in 2 hours Its velocity is therefore 30 miles an hour. Since this motion includes starting and stopping, this velocity is the **average** velocity. The instant velocity at any position (**phase**) is variable, and it is possible that the **instant velocity** is 30 miles per hour only twice in the trip.

LINEAR AND ANGULAR VELOCITIES

8. When motion is measured in **linear units** (inches, feet, meters), the velocity is called **linear velocity.** If **angular units** are employed, it is called **angular velocity.**

Example.—The point A on the rim of the pulley, Figure 4, as well as all other points in the body, has both linear and angular velocity as it rotates about O. If the distance moved along the circumference is the

standard, the velocity is **linear.** If the motion is measured by the angle traversed, the velocity is **angular.** Therefore

$$V \text{ (linear velocity)} = \frac{\text{length of arc } \theta}{T}, \text{ and}$$

$$\omega \text{ (angular velocity)} = \frac{\text{angle } \theta}{T}.$$

Note.—All particles of a rigid body have the same angular velocity.

9. Units of Velocity.—The usual units of linear velocity are miles per hour, feet per minute, and inches per second. In

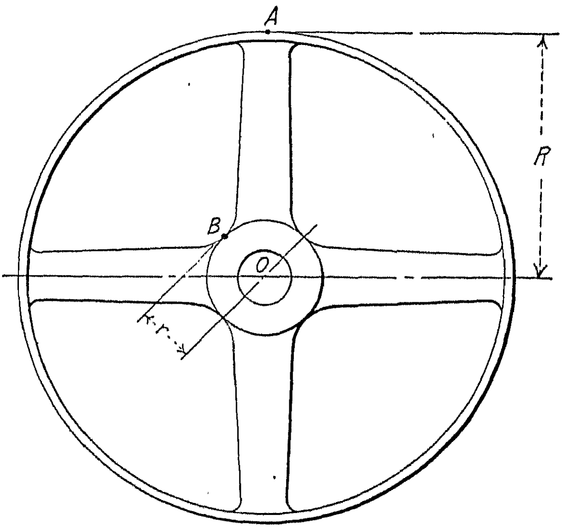


Fig. 4.

Machine Design the standard unit for belt speeds, cutting speeds, piston speeds, etc., is **feet per minute,** and the symbol is *f/m.*

Example.—A steam engine having a stroke of 16 in., runs at 225 strokes per minute. Required the **average** piston speed.

$$V_P = \frac{225 \times 16}{12} = 300 \text{ ft. per min.}$$

10. The units of **angular velocity** are radians per second, radians per minute (symbol ω), or revolutions per minute (abbreviation **r.p.m.,** symbol N). The radian is the angle whose arc is equal in length to the radius; value in degrees $= \dfrac{360}{2\pi} = 57.3$

degrees approximately. Theoretically it is the ideal unit, for by using it, the ratio of linear and angular velocities are in direct proportion to the radius. **Equation:**

$$\omega = \frac{V}{R} \therefore V = \omega R.$$

In practice, however, it is seldom used, because **r.p.m.** is the unit best understood by shop workers, engineers, and designers.

Example.—A belt runs over a 36-in. pulley at a linear velocity of 3,000 ft. per minute. Required the angular velocity in revolutions and radians.

$$\text{Circumference} = \pi D \therefore N = \frac{V}{\pi D} = \frac{3{,}000}{\pi \cdot \frac{36}{12}} = 318 \text{ r.p.m.}$$

Note.—Where values in an equation are in ft. per minute and inches, the distances in inches must be reduced to feet.

For **radians:**

$$\omega = \frac{V}{R} = \frac{3{,}000}{\frac{3}{2}} = 2{,}000 \text{ radians per min.}$$

Note.—In all machinery and measurements of material the **radius is almost unknown.** Dimensions of pulleys, gears, rods, shafts, pins, rollers, balls, screws, tubes, drills, holes, taps, dies, etc. are **invariably the diameter** of the piece. A 24-in. pulley means **24-in. diameter.**

ACCELERATION

11. Acceleration is the rate of change of velocity. **Positive acceleration,** called simply **acceleration,** is the speeding up, and **negative acceleration,** called **retardation,** is its slowing down. Acceleration may be **uniform** or **irregular,** but it is taken as **uniform,** unless otherwise specified. When velocity is uniform, acceleration is zero. Gravity is the most conspicuous example of uniform acceleration, because the force applied is constant and omnipresent. **Uniform** acceleration means that each successive unit of time shows a constant increase (or decrease, if retardation) of velocity over the preceding unit.

Example.—A stone falling from a height (in a vacuum) attains a velocity of 161 ft. per sec. at the end of 5 sec. Required the acceleration due to gravity.

$$A = \frac{V}{T} = \frac{161}{5} = 32.2 \text{ ft. per. sec. per sec., or } 32.2 \text{ ft. per sec.}^2$$

NEWTON'S LAWS OF FORCE AND MOTION

12. 1. Any material point acted upon by no force, or by a system of balanced forces, maintains its condition of rest or motion; if at rest, it remains so; if in motion, it moves uniformly in a straight line.

2. Any material point acted upon by one force, or by a system of unbalanced forces, moves with an accelerated motion proportional to, and in the direction of the force, or the resultant of the system.

3. Action and reaction are equal, opposite, and simultaneous.

RESOLUTION AND COMPOSITION OF FORCES AND VELOCITIES

13. From the foregoing laws, which have been abundantly proven by physical experiments, it is evident that two or more forces acting on a point will produce motion in the direction of their resultant, with an acceleration proportional to this resultant. The resultant of two or more forces is obtained graphically by the **parallelogram of forces.** Similarly, **velocities** may compose a resultant, or a velocity may be resolved into components. For, while it is true that a point can have but one velocity or motion at any instant, when related to a single body, yet it may be acted upon by various forces, each tending to produce velocity in its own direction. **Components may be taken to show velocity in other directions than that of the actual motion.** Refer to Fig. 5. A man is walking diagonally across a field in a northeasterly direction. As he goes with a certain velocity toward his objective, he is making definite progress, both in a direction north and a direction east, directly proportional to the sides of the parallelogram.

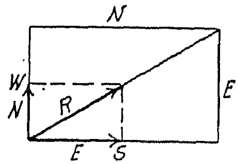


Fig. 5.

Example.—Let the velocity of $A = 300$ ft. per minute, and let the direction be N 60° E. Required his velocity north and his velocity east. Solving the parallelogram mathematically gives a northward velocity of 150 ft. per minute, and an eastward velocity of 276 ft. per minute.

MEASUREMENT OF WORK

14. The unit of work is the foot-pound, and the larger unit, which takes time into consideration is the **horsepower** (hp.).

This has been universally adopted as 33,000 ft.-lb. per minute. When linear velocity and force are known, the equation becomes

$$\text{hp.} = \frac{VP}{33,000}.$$

The velocity must be given in ft. per min. to agree with the denominator, and the symbol P represents the **pounds** of pressure or pull (*i. e.*, force) exerted at the given velocity. It is assumed to be constant, and if not, as in the case of steam or gas pressure in engines, it is averaged through the cycle.

When the velocity is given in r.p.m., the equation becomes

$$\text{hp.} = \frac{2\pi RNP}{33,000}.$$

In this equation, care must be taken to express the radius in feet, since the diameters of pulleys, gears, crank circles, etc., are expressed usually in inches.

Example 1.—A steam engine of 16-in. stroke runs 180 r.p.m., with an average piston pressure of 10,000 lb. Required the work of the steam inside the cylinder, called the **Indicated Horsepower** (i.hp.)

$$\text{hp.} = \frac{2 \times 16 \times 180 \times 10,000}{33,000 \times 12} = 145.4 +$$

Why is 2 in the numerator, and 12 in the denominator?

Example 2.—The driving tension on a belt, over a 24-in. pulley, **running** 400 r.p.m., is 250 lb. Required the delivered horsepower.

$$\text{hp.} = \frac{2 \times \pi \times 12 \times 400 \times 250}{33,000 \times 12} = 19 +.$$

Note.—The writer uses a value for $\pi = \frac{22}{7}$ as sufficiently exact for problems in kinematics. It simplifies expressions like the foregoing, and reduces errors to a minimum.

GRAPHIC SOLUTIONS

15. Kinematical problems can be solved both by graphic and analytic methods. In some cases, chiefly simple problems, the mathematical solution is easier and quicker, but for the majority of problems the **graphic** method is more rapid, and the result is more clearly shown. With good drawing instruments the result can be obtained sufficiently exact for all practical purposes. Graphic solutions are more convincing than tables of figures, as is evidenced by their increasing use by business executives, advertisers, and lecturers, because the eye responds more quickly

to the comparisons on a chart than does the imagination to the same values when set down in numbers. It is advisable frequently to check graphic results by numerical calculations.

SKELETON LAYOUTS

16. Kinematical problems are best solved when the drawing is stripped of all details that do not influence the relations of path or speed. Consequently, detailed drawings, or even sketches, are superfluous for this part of their design. Instead a skeleton layout serves better than a drawing showing all the parts. From a layout like Fig. 6, the various paths, velocities, accelerations, turning effort, and the related positions of all parts at any phase of the cycle, can be determined.

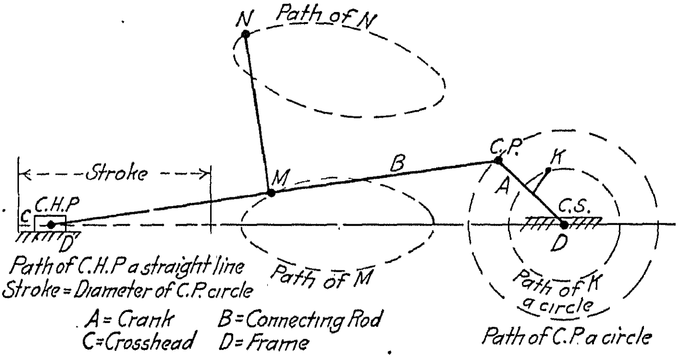


FIG. 6.—Kinematic layout of the steam engine, showing paths of various points in the mechanism, relative to the frame.

In the skeleton layout shown will be found the essential members of the steam engine mechanism. This might represent any machine having a sliding reciprocating member; such as a planer, shaper, bull dozer, punch press, gas engine, pump, and many another machine. The technical name is the slider-crank four-link chain, but in this work it will be referred to as the steam engine, its great example.

Analyzing Fig. 6, we find four members, *A*, *B*, *C*, and *D*. *A* is the rotating **crank,** its rotation being designated by the circle. *B* is the **connecting-rod,** *C* the **slider** (piston, crosshead, plunger, block, sleeve, ram, tool carrier, or whatever form it may assume), and *D* the **frame.**

In all skeletons the fixed link is designated by the hatched

lines, making no attempt to outline the part. The constrainment of the moving parts is shown by dots representing pins, or parallel lines representing sliding guides. Thus A is pinned to D and must revolve or oscillate, and C must slide horizontally on (or inside) D. The connecting-rod B is not connected directly to the frame, but is pinned to A and to C.

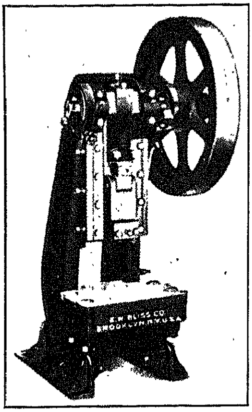


FIG. 7.—Punch press. The E. W. Bliss Co., Brooklyn, N. Y.

In all measurements and mention of pins, the reference is made to the **exact center** of the pin. The pin connecting A and D is called the **crank-shaft** (c.s.), that connecting A and B the **crank-pin** (c.p.), and that connecting B and C the **cross-head-pin** (c.h.p.). The constrainment is complete, and every member has a definite and invariable position for every phase of the mechanism.

Note.—The arms N and K in this sketch are not essential to this mechanism, but represent points in A and B not on their lines of centers.

Considering this layout as a steam engine, the motive power is exerted to move C back and forth (usually horizontally as

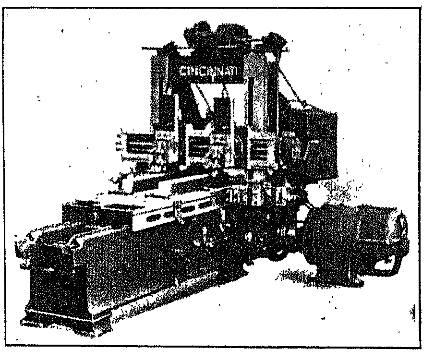


FIG. 8.—Planer. Cincinnati Planer Co., Cincinnati.

illustrated). By its constrainment (the cylinder walls and cross-head guides) the piston can only travel in a straight path. This rectilinear motion is converted into the rotary motion of the crank through the connecting-rod.

The length of the **stroke** is equal to the diameter of the crank-pin circle (center); *i. e.*,

$$\text{Stroke} = D$$
$$\therefore \text{ Piston travel for one crank revolution} = 2D.$$

The ratio of the **average** piston velocity to that of the crank-pin is

$$\frac{V^p}{V^{cp}} = \frac{2D}{\pi D} = \frac{2}{\pi}.$$

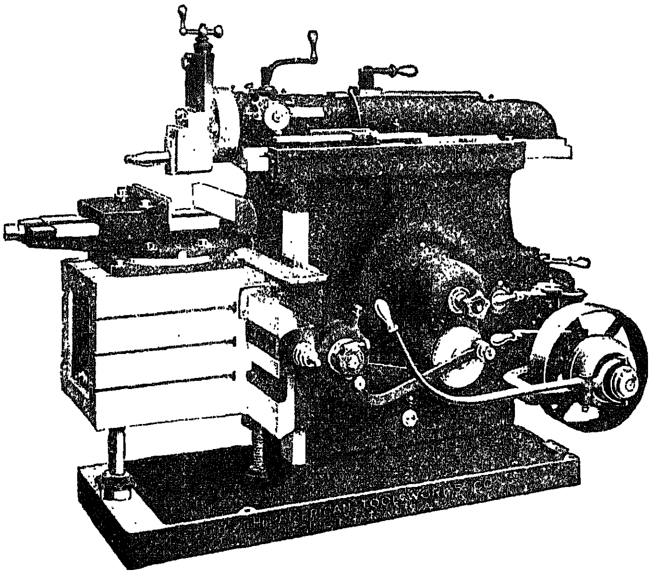


FIG. 9.—Shaper. American Tool Works Co., Cincinnati.

Example.—The crank-pin velocity of an engine is 300 ft. per minute. Required the average piston velocity.

$$V^p = \frac{2}{\pi} \times 300 = 191 \text{ ft. per min. (nearly).}$$

Note.—In the case of the steam engine the motive power is applied to move the slider, but in nearly all machines of this type, the power is applied to revolve A, and to convert rotary motion to sliding.

THE FUNCTION OF THE FLYWHEEL

17. If the revolution of the crank depended entirely on the impulse of the steam within the cylinder, acting unregulated

through the piston and connecting-rod, only a very irregular and almost useless angular velocity would be imparted to the crank. In a single-acting gas engine of less than six cylinders it is doubtful if it could complete the compression stroke. The **fly-wheel** regulates this defect, by storing up the surplus energy where the turning effort is greatest, and giving it out where the turning effort is diminished, giving the crank-pin an even and continuous pressure throughout the revolution.

How the steam force is deflected, as it is transmitted to the crankshaft, is shown in Fig. 10. The steam P produces a horizontal movement in the piston, and acting on the crosshead-pin is deflected to the center-line of the connecting-rod. This deflection could not take place without the reaction of the guides, hence, P is resolved into the components T (thrust) and R (reaction of guides). The thrust is then resolved at the crank-

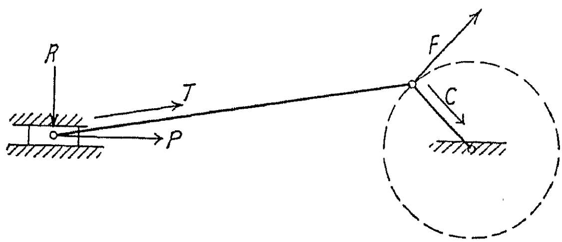


Fig. 10.—Deflection of the forces in a steam engine.

pin into two components, F and C, the **turning effort** and the **crank bearing pressure,** respectively tangent and normal to the crank-pin circle.

The average amount of the accumulated turning effort, multiplied by the crank-pin travel, determines the work actually developed by the engine (d.hp.). The thrust on the forward stroke becomes a pull on the return, the bearing pressure is grinding the crankshaft into bearings part of each stroke, and acting to wrench it from them the remainder of each stroke. When the connecting-rod and crank are in line, the thrust is entirely in the direction of the bearings and there is **no turning effort.** When the connecting-rod and crank are at right angles, the **thrust** and **turning effort** are **equal,** and there is no bearing pressure. It will be evident then that the turning effort is zero at the beginning of each stroke, and equal to the thrust at the point where crank and connecting-rod are perpendicular. This

may not be the maximum turning effort, as the piston pressure is reduced after steam is cut off.

Figure 11 shows how the turning effort at a certain phase of an engine stroke is determined. A steam pressure diagram (either the actual card of the engine, or an imaginary one from estimates) is placed over the stroke space. The ordinates of this diagram show the piston pressure at any phase, to a certain scale. Using the ordinate for the particular phase, the turning effort for that phase can be worked out graphically, as shown in the figure. If a number of representative phases are thus taken, a diagram of turning effort can be drawn. The area of the enclosed space is the total amount of work delivered to the

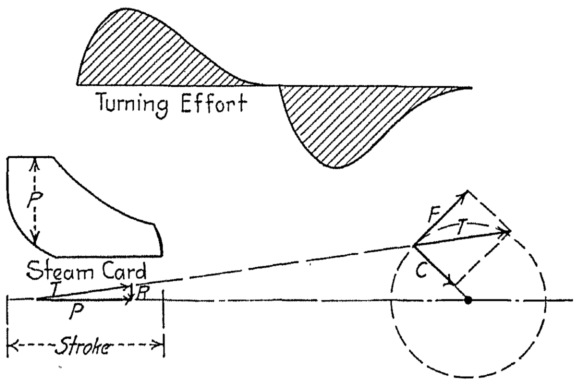


FIG. 11.—The steam indicator card and the diagram of turning effort.

crankshaft. This is the theoretical amount, and will be more than the actual work on account of frictional and other losses.

A study of the crank effort diagram will show a jerky force imparted, varying from zero at the dead points to a maximum near the center of the stroke. The flywheel is designed to correct this. It is a heavy wheel, its weight concentrated at the rim as much as is possible. To start this great mass requires heavy power, but once under way, it distributes the high turning effort over the low points, and thus gives a nearly uniform torque to the crankshaft throughout the revolution.

Cranks are sometimes equipped with balance weights to offset the inertia of the moving parts. Locomotives and many automobiles have been equipped with balances, and much study has been devoted to the correction of these disturbing irregularities.

Certain machines, punch presses, bull dozers, bolt headers, and their like, have their power supplied at constant pressure, but the resistance of the work is intermittent, and flywheels are applied to them to take up the shock, as it is called.

MACHINERY

18. Mechanisms designed for useful work are called **machines,** and are given different consideration from a mechanism which merely transforms motion. This, then, is their distinction: a machine carries a load, and a mechanism need not. The study of **pure Kinematics** does not consider the effects of loading the mechanism, whereas the science of **Machine Design** considers both the mechanism and its burden.

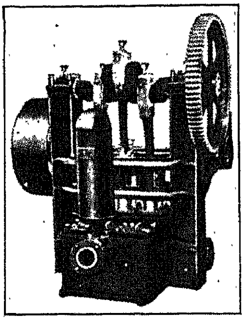


FIG. 12.—Triplex pump. Goulds Manufacturing Co., Seneca Falls, N. Y.

The imposition of a load on a mechanism makes imperative a selection of materials, arrangement of parts, appropriate dimensions, varying thicknesses, sectional design in the distribution of the material to withstand stresses and shocks, wearing parts, lubrication devices, shock absorbers, etc., that have no place in pure kinematics.

Machine Design is a subject built on kinematics, which involves a larger use of mathematics, mechanics, strength of

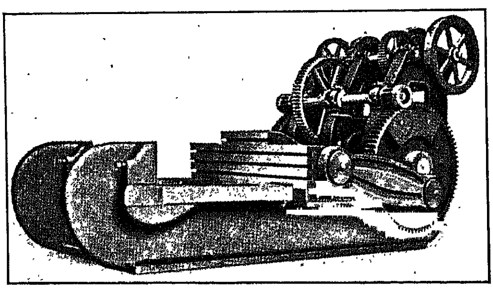


FIG. 13.—Bulldozer. Williams, White & Co., Moline, Ill.

materials, and laboratory information. A pump and a bulldozer are **kinematically** the same, but actually they do not look alike, and they have quite opposing problems to consider in

their design. Even two machines of the same type, a 10-hp. and a 1,000-hp. engine, for example, present different problems, and require different mechanical treatment, and even different materials in some of the parts. Two bolts of different diameters have different proportions, and there are different shapes of screw threads for different purposes.

A machine is made up of constrained members, so related that the same operation is repeated at regular intervals. The complete set of positions, or **phases,** thus taken is called a **cycle.**

The members of a machine may be **rigid** or **flexible.** A **rigid body** is one in which all points remain in a fixed relation at all times and in all positions. Absolute rigidity is not possible in any material under all loads, but **comparative** rigidity obtains where the deformation is negligible. By using safe loads practical rigidity is attained, and that is what the term implies.

A **flexible body** is one which can be stretched, compressed, bent, or twisted to perform the required work, and whose power of resistance to such strains can be used to give the machine approximately constrained motion.

Belts, chains, springs, liquids, gases, rubber parts, all come in this class, and rigid bodies are too common and too well understood to require examples. All wooden or metal parts, except chains and springs, are rigid.

MECHANICAL ADVANTAGE

19. In machine design it is usually desirable to keep the size of prime movers, *i. e.*, engines, motors, etc., as small as possible, even when operating heavy tools and structures. Thus a very heavy bridge may be turned or lifted by a comparatively small motor; or a man using one hand can raise with a jack a heavy automobile; and rock crushers exerting enormous pressures are geared down, so that motors of small size and high speed will drive them easily. This difference between the applied force and the resistance overcome is called **mechanical advantage.** This is accomplished through the inverse proportion of the respective velocities.

The amount of work put in (hp.) and the amount taken out (useful work + losses by friction and otherwise) must be the same. No machine can gain power, as the expression is often ignorantly used. Efficiency of 100 per cent is unknown. The

efficiency of a machine $= \frac{\text{output}}{\text{input}}$. This means (neglecting losses) that the **product** of pressure and velocity is **constant** throughout the various members of the train which constitutes the machine.

Example.—An automobile weighs 4,000 lb., and one end (2,000 lb.) is lifted 3 in. in 2 min. by a man with a jack. His hand travels at the rate of 4 ft. per sec. What pressure does he exert?

$$P = \frac{3 \times 2{,}000}{120 \times 4 \times 12} = 1\ 1/24 \text{ lb.}$$

The mechanical advantage is therefore $\frac{2,000}{1\ 1/24} = 1{,}920$; that is, each pound exerted overcomes a resistance of 1,920 lb.

One of the main functions of kinematics is to increase man power through **speed reducing devices.** Many agencies are available for speed reduction—levers, link work, gears, pulleys, and combinations of these. The problem is to determine velocity ratios which can be made to produce the desired result.

AUTOMATIC HANDLING

20. Labor shortage and labor demands have recently (1923) put a serious curtailment on all production, resulting in abnormal prices for articles, without the handicap of any scarcity of raw materials, or the difficulty of obtaining them. The necessity is apparent therefore of a greatly increased substitution of automatic machinery, and automatic control, for human labor. This is already in a more advanced state than most people realize. The **steam engine** was the first great labor saver, and marked the commencement of our present industrial era. Shortly following that invention came, in steady and accumulating succession, the power-driven machines, lathes, drills, planers, milling machines, saws, grinders, etc., of the shop, mine, and mill. Then textile machinery, the steam locomotive and boat, quickly followed by agricultural implements, cheaper steel, moulding machinery, electricity, and all manner of new devices for handling commodities, and new aids to production far too numerous to mention. Perhaps the latest development is the complete automatization of the great machines, wherein mechanical operation and mechanical control completely take the place of brain and brawn. Familiar now are the automatic

screw machines, gear cutters, multiple drills, and many single-purpose machines, but there is still much to be done to reduce man handling to its lowest terms.

The devices at hand for securing such control, for relieving man of the labor of lifting, carrying, discharging, and shaping material, and of pulling levers, turning handles, and pushing buttons to regulate such operations, are numerous indeed. A mere catalogue of these devices would fill a book much larger than this, so a detailed study of them is impossible here. The remaining chapters will consider general principles, and the application of these principles to the task in hand.

In this work the prime mover is taken for granted. This means that the power is generated, ready to use, because the study of the steam engine, electric motor, gasoline engine, water

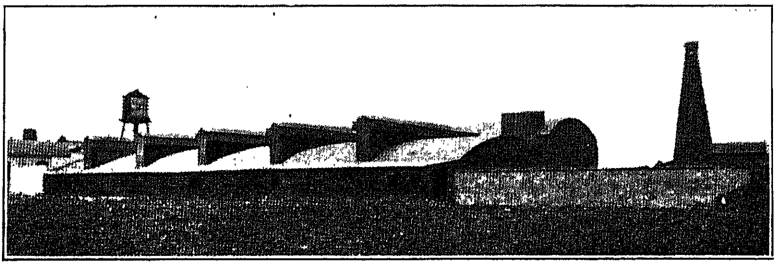


FIG. 14.—Salt mill, Willcox Engineering Co., Saginaw, Mich. This is an example of plant automatization. Two men, a day man and a night man, are on the payroll and 250 barrels a day constitutes the normal output.

turbine, and other energizers are, and should be, taken separately and individually. Kinematics takes the work of transmission from that point, and studies the adaptability of the various media for transference of motion, by linkwork, direct contact pairs (rolling pairs, frictions, gears, cams), and flexible bands (belts, ropes, chains), leaving out of consideration such agencies as electricity, gravity, compressed air, and others which accomplish transference of motion through no material contact, or are sometimes unconstrained.

Since the same conversions and speed ratios can be transmitted from shaft to shaft by different agencies of the aforesaid group, the study of the subject resolves itself, (1) in the mastery of the principles underlying the various transmissions, and (2) a study of their suitability to the necessities of the job in hand. A thorough understanding of these principles, and the charac-

teristics of the various available units, is the best equipment possible for the future engineer and inventor.

PROBLEMS

In the following problems, a typical problem with complete data is given, followed by the same problem with blank data to be supplied by the instructor.

1. An automobile goes 30 miles per hour. Express its velocity in ft. per minute and inches per second.

2. Same problem. —— miles per hour.

3. A flywheel, 66-in. in diameter, rotates 105 r.p.m. Required: (a) the velocity of a point on the rim. (b) the velocity of a point on the 8-in. hub.

4. Same problem. Diameter —— in., r.p.m. ——. Hub —— in.

5. A belt runs 3,000 ft. per min. over pulleys of 40-in. and 18-in. diameter. Required the r.p.m. of each pulley and the speed ratio of their respective shafts.

6. Same problem. Belt speed —— ft. per min., ——in. and —— in. diameters.

7. An engine flywheel, 72-in. in diameter, drives a generator direct through a belt over a 10-in. pulley. The engine speed is 180 strokes per min., and the slip of the belt is 3 per cent. What is the angular velocity of the generator?

8. Same problem. Flywheel —— in., generator pulley —— in., Engine speed —— strokes.

9. Two friction wheels have a linear velocity of 1,800 ft. per min. at the point of contact. Their diameters are 8 and 28 in. What are their angular velocities.

10. Same problem. Linear velocity —— ft. per min., diameters —— in. and —— in.

11. Two friction wheels have a linear velocity of 2,200 ft. per min., and angular velocities of 150 and 400 r.p.m. Required their diameters.

12. Same problem. Linear velocity —— ft. per min., r.p.m. —— and ——.

13. A stationary engine, 24-in. stroke, runs at 125 r.p.m.. The flywheel is 80-in. in diameter.

Required: (a) Linear velocity of crank-pin.
(b) Average piston velocity.
(c) Velocity of a point on the rim of the flywheel.

14. Same problem. Stroke —— in., r.p.m. ——, flywheel —— in.

15. A train goes 213 miles in 6 hr. and 35 min. The locomotive drivers are 72-in., and the piston stroke 26-in.

Required: (a) Train speed in miles per hr., ft. per min., and ft. per sec.
(b) Average crank pin velocity relative to frame.
(c) Average piston velocity relative to frame.
(d) Distance traveled by the crosshead on its guides on the trip.
(e) Plot the path of the crank-pin for one revolution of the driver.

16. Same problem. Miles ——, hr. ——, min. ——, drivers —— in., piston stroke —— in.

17. An automobile travels 37 miles in 1 hr. and 5 min. It has 32-in. wheels, and the crankshaft rotates 3½ times as fast as the rear axle. The stroke is 5½ in. Required : the average r.p.m., both of wheels and crank, and the average piston velocity relative to the frame.

18. Same problem. Distance —— miles, hr. ——, min. ——, ratio of rear axle to crankshaft ——, stroke —— in.

19. If high-speed steel drills will cut cast iron with a peripheral velocity of 130 ft. per min., mild steel 100 ft. per min., and tool steel (annealed) at 70 ft. per min., at what speed should a 1¼-in. drill be run for the three materials?

20. Same problem. Cast iron —— ft. per min., mild steel —— ft. per min., tool steel —— ft. per min., drill —— in. (⅛ to 5 in.).

21. A stationary engine, 19½-in. stroke, 560 ft. per min. piston velocity, with a 64-in. flywheel, drives a generator direct by belt from flywheel to 11-in. pulley on the armature shaft. Required: the belt speed and the generator speed.

22. Same problem. Stroke —— in., piston velocity —— ft. per min., flywheel —— in., generator pulley —— in.

23. How fast is a body falling (neglecting air resistance) at the end of 7 sec.? How far has it fallen?

24. Same problem. —— sec.

25. A train attains a speed of 35 miles per hr. in 44 sec. after starting. What is its acceleration, and how far does it go in that time?

26. Same problem. —— miles per hr., —— sec.

27. An automobile is capable of an acceleration of 5 ft. per sec.² How long and how far must it go to attain a speed of 50 miles per hour?

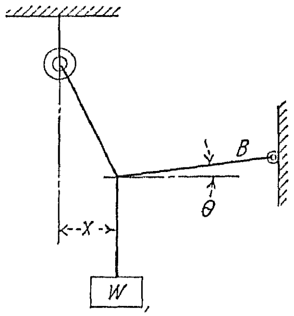


FIG. 15.—Problems 28–29.

28. A weight of 250 lb. hangs by a 72-in. rope, as shown in Fig. 15, and is pulled 18 in. (x) out of line by a rope attached to its middle, running to the wall at an angle of 10 degrees, θ. How much tension is there in the upper half of the supporting rope and in the rope B? Graphic solution.

29. Same problem. Rope —— in. long, $x =$ —— in., θ —— degree.

30. The piston pressure of an engine, of 21-in. stroke, and 56-in. connecting-rod, is 6,000 lb. throughout the stroke. Required the thrust in the connecting-rod, the guide reaction, the turning effort of the crank, and the pressure on the crankshaft bearings at 45 degree position of the crank.

31. Same problem. Pressure —— lb. stroke —— in., connecting-rod —— in. Phase of crank —— degree.

32. Plot the turning effort of the engine in Prob. 30 through a complete revolution of the crank, making 12 or 16 readings at the instructor's option. Calculate the i.hp. and the d.hp.

33. Same problem. Pressure —— lb. stroke —— in., connecting-rod —— in.

34. A Corliss engine, 28-in. stroke, 72-in. connecting-rod, 14-in. cylinder diameter, runs under steam at 125 lb. boiler pressure. During the first quarter stroke (to cutoff) the piston pressure averages 95/100 *P*, the second quarter ⅔ *P*, the third quarter ½ *P*, and the final quarter ⅜ *P*. Using these values, plot a pressure diagram. What will be the connecting-rod thrust, guide reaction, turning effort, and bearing pressure at the 60 degree crank position?

35. Same problem. Stroke —— in., cylinder —— in., boiler pressure —— lb., crank position —— degree.

36. Same problem. Stroke —— in., cylinder —— in., boiler pressure —— lb. Make a crank effort diagram.

37. Engine in Prob. 34. R.p.m. = 90, mechanical efficiency of the engine = 90 per cent. Required the i.hp. and the d.hp. Use graphic methods to obtain average pressures and turning effort.

38. Engine in Prob. 34. Stroke —— in., cylinder —— in., boiler pressure —— lb., r.p.m. = —— in., mechanical efficiency —— per cent. Required the i.hp. and d.hp.

39. An engine flywheel 76 in. D., delivers 250 hp. at 90 r.p.m. through a belt. What is the continuous pull on the belt?

40. Same problem. Flywheel —— in. D, hp. ——, r.p.m. ——.

41. A stationary engine, running 320 r.p.m., drives a 24-in hoisting drum at 1/10 its own velocity. The drum can hoist a weight of 5 tons. How quickly can it raise the weight 50 ft? Assuming the efficiency of the train at 75 per cent, what hp. is required of the engine?

42. Same problem. R.p.m. of engine ——, speed reduction ——, weight to be raised ——, distance ——, efficiency —— per cent.

43. A lift bridge over a canal weighs 25 tons, and has two counterweights of 10 tons each. The height of the lift is 65 ft., and the friction loss is 40 per cent. Required the capacity of a motor to lift it full height in 45 sec.

44. Same problem. Bridge —— tons, counterweights —— tons, height of lift —— ft., friction loss —— per cent, time —— sec.

45. A hand hoist for two men has a 16-in. drum, and a rated capacity of one ton. The turning levers have a throw of 15 in., and the double-reduction gear gives a 30 to 1 ratio between handle shaft and drum shaft. It operates at 80 per cent efficiency. How fast must the men turn to raise the weight 40 ft. in 8½ min.? What handle pressure must each exert continuously during the lift? What is the mechanical advantage? Make a sketch of the layout.

46. Same problem. Drum —— in., capacity —— lb., crank throw —— in., gear reduction ——, efficiency —— per cent. How fast must the men turn the cranks to raise the weight —— ft. in —— min.? Handle pressure? Mechanical advantage?

47. A hand hoist is designed to lift 3,000 lb. Two men can exert 35 lb. each on the turning levers, which are 16-in. throw, and the drum is 12-in. diameter. The efficiency is 75 per cent. What gear reduction is

necessary? What is the mechanical advantage? How fast can they raise the weight 56 ft., by turning the handles 20 r.p.m.?

DRAFTING PROBLEMS

For outside work, drawing board, or blackboard work. Plot the path of the point K in these problems. The link A is always the driving crank, and the various phases for a full cycle should be made, taking twelve to sixteen readings to obtain the path. There are fifteen sets of proportions to each layout.

For blackboard work, use a scale three times the dimensions given.

For paper work, use the same scale or twice the scale given.

Note—In the following figures, the links A, B, C and D are rigid and their design is not affected by pinning such links as E and F to them, as these are pinned to B and C in Problems 3, 4 and 10, to A and B in Problem 6, to B in Problems 8, 9 and 11 and to C and D in Problem 12.

1.	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
$A=$	1	1	1	1¼	1¼	1¼	1½	1½
$B=$	2	2¼	2½	2½	2¾	3⅛	3	3½
$C=$	1¼	1½	1⅝	1½	1⅝	1¾	1¾	2
$D=$	2	2	2¼	2½	2¾	3	3	3¼
$E=$	1½	1	2	1½	2	2	2	2
$F=$	1½	2	1¼	2	1	2	2	2½

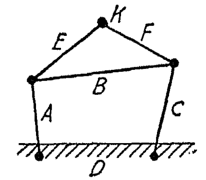


FIG. 16.—Problem 1.

	(j)	(k)	(l)	(m)	(n)	(o)	(p)
$A=$	1½	1¾	1¾	1¾	2	2	2
$B=$	4	3	3½	4	4	5	6
$C=$	2¼	2	2¼	2½	2¼	2½	3
$D=$	3½	3	3¼	3¾	4	5¼	6
$E=$	3	3¼	2½	4	1½	2	5
$F=$	2	1	2	2	5	4	3

Also: Required the angle of oscillation of C, and the time ratio of each oscillation.

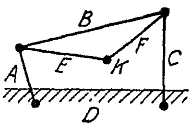


FIG. 17.—Problem 2.

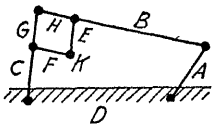


FIG. 18.—Problem 3.

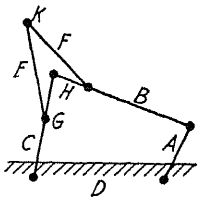


FIG. 19.—Problem 4.

2. Use the same proportions and same requirements as in Prob. **1.**

3. Use the same proportions and requirements as Prob. **1.** Make $E = F = G = H$, and choose ½, ¾, 1, 1½, 2 in., etc. for the value.

4. Use the same proportions for A, B, C, and D as in Prob. **1**, and the same requirements. Make E, F, G, and H any reasonable values.

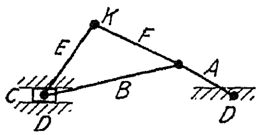


FIG. 20.—Problem 5.

5.	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
$A =$	1	1	1	1½	1½	1½	2	2
$B =$	3	4	5	3½	5	7	6	7½
$E =$	2	3	3	4	2	5	6	8
$F =$	2	4	4	1	5	4	3	2

	(j)	(k)	(l)	(m)	(n)	(o)	(p)
$A =$	2	2½	2½	2½	3	3	3
$B =$	9	10	12	14	10	12	15
$E =$	5	9	7	6	5	8	10
$F =$	7	6	8	10	7	7	7

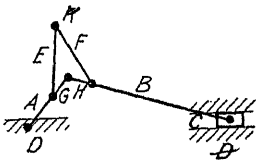


FIG. 21.—Problem 6.

6. Same proportions and requirements as Prob. **5.** Make $G = H =$ ½, ¾, 1 in., and $E = F =$ 1 to 3 in.

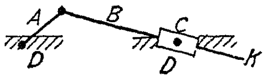


FIG. 22.—Problem 7.

7.	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
$A =$	1	1	1	1½	1½	1½	1¾	1¾
$B =$	4	5	6	4	5	6	4	5
$D =$	2	2½	3	2½	3	3½	2¾	3½

	(j)	(k)	(l)	(m)	(n)	(o)	(p)
$A =$	1¾	2	2	2	2½	2½	2½
$B =$	6	5	6	7	5	6	7
$D =$	4	3	4	4½	3½	3¾	4½

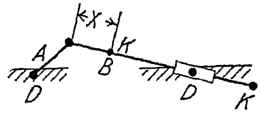


FIG. 23.—Problem 8 (Link B is not jointed at K.)

8.	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
$A =$	1	1	1	1½	1½	1½	1¾	1¾
$X =$	½	¾	1¼	¾	1	2	¾	1½
$D =$	2	3	4	2½	3½	4½	3	4

	(j)	(k)	(l)	(m)	(n)	(o)	(p)
$A =$	1¾	2	2	2	2½	2½	2½
$X =$	1	1	2	1½	1	1½	3
$D =$	6	4	6	7	4	5	7

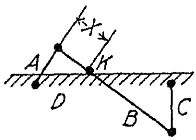


FIG. 24.—Problem 9 (Link B is not jointed at K).

9.	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
$A = C =$	1	1	1	1½	1½	1½	1¾	1¾
$B = D =$	2	3	4	3	4	5	4	5
$X =$	½	2	1	1½	3	1	1	1

	(j)	(k)	(l)	(m)	(n)	(o)	(p)
$A = C =$	1¾	2	2	2	2½	2½	2½
$B = D =$	6	5	6	7	6	7	8
$X =$	2	1½	1	2	2	2	3

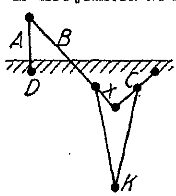


FIG. 25.—Problem 10.

10. A, B, C, D the same as in Prob. **9.** $E = F =$ 2 to 6 in. $X = Y =$ ½ to 1 in.

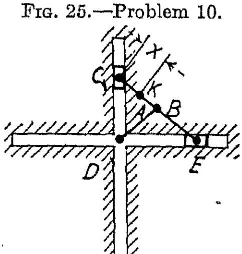


FIG. 26.—Problem 11.

11.	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
$A =$	1	1	1	1¼	1¼	1¼	1½	1½
$B = 2A.$								
$X =$	½	⅝	¾	½	⅝	¾	½	¾

	(j)	(k)	(l)	(m)	(n)	(o)	(p)
$A =$	1½	1¾	1¾	1¾	2	2	2
$X =$	1	1	1¼	½	1	1½	¾

12. A, B, and C the same as in Prob. **1**:

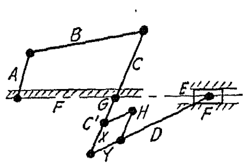


FIG. 27.—Problem 12.

	(a)	(b)	(c)	(d)	(e)
$C' =$	1	2	1	2	1
$Z = D$ in Prob. **1**.					
$D =$	3	3	3	3	3
$X = Y =$	½	¾	1	½	¾
$G = H =$	1½	2	2½	1½	2

	(f)	(g)	(h)	(j)	(k)
$C' =$	1¼	2	1	1½	3
$D =$	3	4	4	4	4
$X = Y =$	1	½	¾	1	¾
$G = H =$	2½	2	2½	3	2

	(l)	(m)	(n)	(o)	(p)
$C' =$	1	2	1	1½	2
$D =$	5	5	5	5	5
$X = Y =$	1	1½	1	1½	1½
$G = H =$	2½	3	2	3	4

Required: (a) Path of K
(b) Angle of oscillation of C
(c) Length of stroke of E
(d) Time ratio of forward and return strokes E.

Note: If instructor desires, he may change elevation of E, and he may make CC' a bell crank, making the angle anything between 90 and 225 degrees.

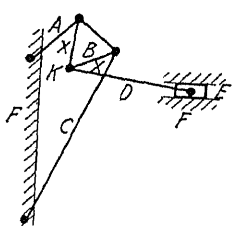


FIG. 28.—Problem 13.

13.	(a)	(b)	(c)	(d)	(e)
$A =$	1	1	1	1¼	1¼
$B =$	1½	1¾	1¾	1½	1¾
$X = B$					
$C =$	2¼	2½	3	2½	3
$Y =$	2	3	2¾	2½	3¼
$D =$	3	3	3	4	4

	(f)	(g)	(h)	(j)	(k)
$A =$	1¼	1½	1½	1½	1¾
$B =$	2	1¾	2	2¼	2
$C =$	3½	3	3½	4	3½
$Y =$	3	3	3¼	4¼	3½
$D =$	4	4	5	5	5

	(l)	(m)	(n)	(o)	(p)
$A =$	1¾	1¾	2	2	2
$B =$	2½	2¾	2½	2¾	3
$C =$	4	5	4	5	6
$Y =$	4½	4½	3¾	5½	5½
$D =$	5	5	6	6	6

Same requirements as Prob. **12.**

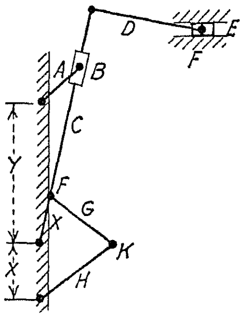


FIG. 29.—Problem 14.

14.

	(a)	(b)	(c)	(d)	(e)
$A =$	1	1	1	$1\frac{1}{4}$	$1\frac{1}{4}$
$C =$	4	5	6	4	$5\frac{1}{2}$
$D =$	2	3	4	3	4
$X =$	1	1	1	$\frac{3}{4}$	1
$Y =$	2	3	4	$2\frac{1}{2}$	$3\frac{1}{2}$
$G = H =$	1	2	3	2	$2\frac{1}{2}$

	(f)	(g)	(h)	(j)	(k)
$A =$	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{3}{4}$
$C =$	$6\frac{1}{2}$	5	7	8	6
$D =$	5	3	4	5	4
$X =$	$1\frac{1}{2}$	1	1	2	$1\frac{1}{2}$
$Y =$	$4\frac{1}{2}$	3	4	5	4
$G = H =$	3	2	2	3	3

	(l)	(m)	(n)	(o)	(p)
$A =$	$1\frac{3}{4}$	$1\frac{3}{4}$	2	2	2
$C =$	$7\frac{1}{2}$	8	8	9	10
$D =$	5	6	5	6	7
$X =$	1	1	1	2	$1\frac{1}{2}$
$Y =$	5	6	5	6	7
$G = H =$	$1\frac{1}{2}$	2	2	4	3

Same requirements as Prob. **12.**

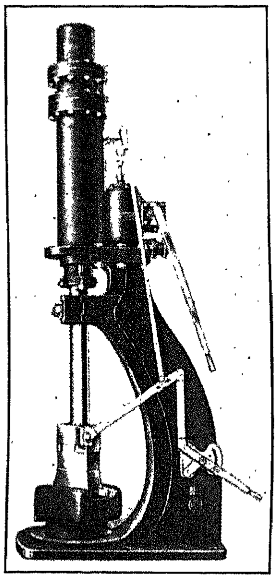


Fig. 30.—Steam hammer. Buffalo Foundry & Machine Co., Buffalo.

CHAPTER II

DETERMINATION OF RELATIVE VELOCITIES AND ACCELERATIONS OF SIMPLE LINKWORK PARTS

21. The foregoing problems have shown that every point in a link train has a definite path, and that each phase of any constrained mechanism brings every part into a definite position relative to every other part. It is evident therefore that there is a possibility of computing the velocity of **any point** in the mechanism, and also its acceleration, in every phase of a cycle. This chapter will deal entirely with the methods of obtaining the velocity and acceleration in links having **variable** velocities. Graphic methods only are used.

TURNING AND SLIDING PAIRS

22. Two members of a mechanism, that are so constrained as to admit only of pure rotation relative to each other, constitute a **turning pair.**

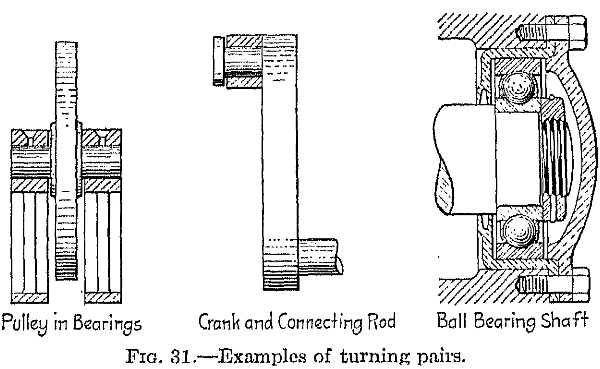


FIG. 31.—Examples of turning pairs.

Examples.—A pulley running loose on a shaft constitutes a turning pair with the shaft; a shaft turning in its bearing, a crank pinned to a connecting-rod, and all similar instances. Either member of a turning pair may be considered the rotating member; *i.e.*, **each member revolves about the other.** The flywheel rotates relative to the frame and the frame rotates relative to the flywheel.

If the interposition of a pin, ball bearing, sleeve or roller does not change this relation, **it is not considered kinematically separate.**

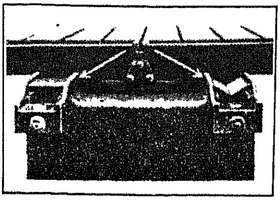


FIG. 32.—Example of a sliding pair. Planer table and bed. Cincinnati Planer Co.

23. Two members of a mechanism, that are so constrained that one slides in parallel guides relative to the other, make what is termed a **sliding pair.**

Examples.—The piston of an engine and its cylinder, the ram of a shaper, the table of a planer, and any parts that are constrained by blocks, sleeves, pins, or rollers to slide in grooves, are examples of sliding pairs. These grooves are usually straight, but they are often curved. The guides, called templates, which are used to feed the tool correctly over the surface of a bevel gear, are examples of sliding mechanism.

CENTERS OF ROTATION

24. In every turning pair the **center of the pin is the center of rotation of both bodies.** It is a **common point.** It is the only point in both bodies that always remains the same distance from all other points in **both bodies.** It is **inseparable** in both bodies, and therefore can have but one **motion**, one **velocity,**

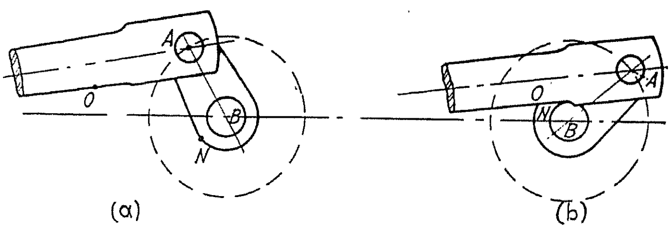


FIG. 33.

or one acceleration, regarded as a point in either the **one** or the **other** of the bodies.

Proof.—In Fig. 33 are shown two typical positions of a crank and connecting-rod. A is the crank pin center, and B the crank-shaft center. A travels in a circle about B, and is therefore at a constant distance from it. Whether A is regarded as a point in the crank or the connecting-rod, it **must** follow that path.

Take two random points O and N, one on each member, at equal distance from A. Their distances from B are obviously not

the same, and they are at some distance from each other. In position *b*, *O* and *N* coincide, and are still the same distance from *A* as before. They are not the same distance from *B* or from each other. Since *O* and *N* are random points, they are typical, and *A* is therefore the **only point in both members whose distance from all points in both members is constant for all phases.**

FIG. 34.

It is called the **virtual center** of the two bodies, and since they are permanently connected at that point, it is a **permanent center.** All pinned links have this permanent center.

25. Instantaneous Centers.—When two links are not directly connected, they have no permanent centers, and the relative velocities are not as simple in consequence. The problems given in Chapter I show conclusively that even the remote members of a mechanism have definite motion relative to each other, and that there must be some means of determining their relative velocities, where the motions of each are fully constrained by the others.

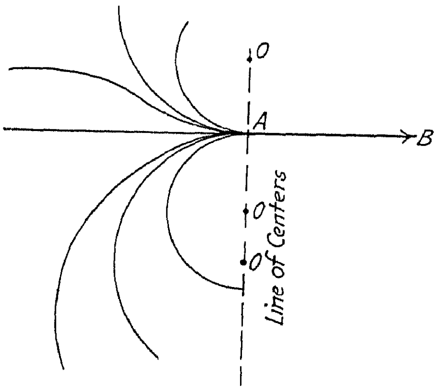


FIG. 35.

All Motion at any instant is Rotation. In what direction is a point of a rotating body moving at any given instant? The natural reply would be around the circle. Where does a whirling weight go when the string is released? How do the sparks from a grinder fly? Where does a flywheel go when it bursts?

The instant direction is the direction of the tangent. Given the direction of the point *A* (Fig. 34) as *AB*. What does this

tell of its past or future? It may have come from straight behind, and it may be continuing straight ahead. However, it is possible for it to have come from an infinite number of curved paths, as shown in Fig. 35. Since the tangent is perpendicular to the radius, all the centers of curvature of all the curves to which AB is tangent, including the **center of rotation of A at this instant** are on the Line of Centers (Fig. 35); *i.e.*, **somewhere on a perpendicular to AB through A.**

The rule may be stated thus:

Rule.—The motion of a point at any instant, relative to some body, is that of rotation about a center on the perpendicular through itself, to its direction of motion at that instant.

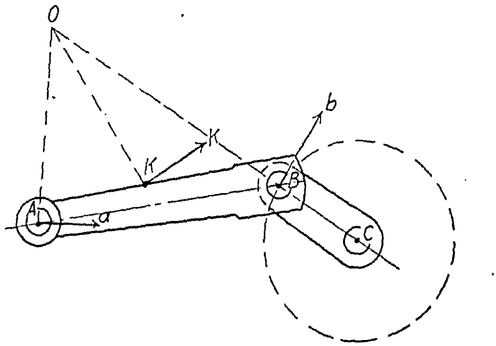


FIG. 36.

This center may be any distance from the point from zero to infinity (∞), and is called the **instantaneous center,** or for brevity **instant center,** or **centro.**

26. To find the Centro of a Body.—If the direction of two points in a body are known, the line of centers of these points will be known (relative to any other body). The intersection of these lines will be the center of rotation for these two points (at this instant), and therefore of the entire body, if it is a rigid body.

Example (a).—Figure 36 shows a steam engine mechanism. By the constrainment of the linkage, we know that the motion of A (c.h.p.) is in the direction Aa, relative to the frame D. We also know that B is moving in a circle about C, relative to D, and that its motion at this instant is in the direction of Bb. The lines of centers of these two points must then be OA and OB, centering at O. This means that at this instant AB is rotating about D with O as a center.

Example (b).—Figure 37 shows a crosshead between its guides on an engine frame. A and B are two points in the crosshead, and their motions at this instant are Aa and Bb, respectively parallel to the guides on the frame. Their lines of center are perpendicular respectively to Aa and Bb, and are therefore parallel, and intersect at infinity. The crosshead therefore revolves relative to the frame about a center at infinity.

All sliding pairs having rectilinear relative motion have their centros at infinity.

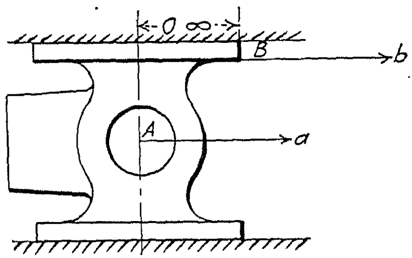


FIG. 37.

27. Instant Radii.—In Fig. 36, the instant radii of A and B are OA and OB. If a point K is taken anywhere on the same body, its instant radius is OK, and its direction is therefore Kk.

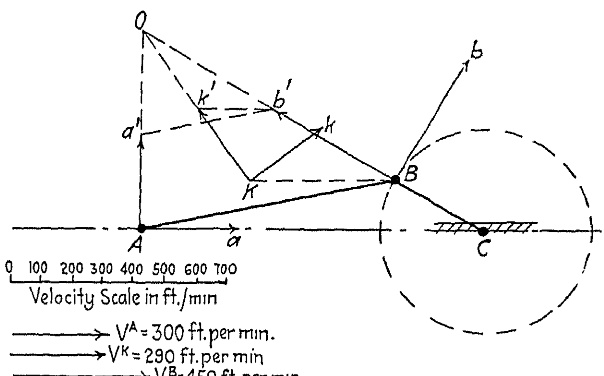


FIG. 38.—Linear velocities of A, K, and B of the connecting rod shown in Fig. 36.

Instant Velocities.—It has been shown that, in any rotating body, the linear velocities of two points in the body are proportional to the radii.

$$\frac{V^A}{V^B} = \frac{2\pi R^A N}{2\pi R^B N} = \frac{R^A}{R^B}.$$

Example.—Figure 38 is a skeleton layout of the connecting-rod in Fig. 36,

in the phase given, and laid off to the same scale. The instant radii of A, B, and K are OA, OB, and OK. Their velocities at this instant are proportional to their instant radii, OA, OB, and OK.

The point B in this example is the center of the crank-pin. Assume its velocity to be known ($= 2\pi RN$), and that it = 450 ft. per min. By laying out a scale, like the one shown in the sketch, and making $Bb = 450$ ft. per min. from the scale, a segment of OA will be found in the same proportion to Bb, as $OA:OB$, by similar triangles. Then by scaling this segment Aa, the piston velocity is determined, and in this figure $Aa = 300$ ft. per min. $= V$. By the same process, the velocity of K is found to be 290 ft. per min.

Note.—This process of determining the velocities of any point in any mechanism can always be reduced to definite figures, because the r.p.m.

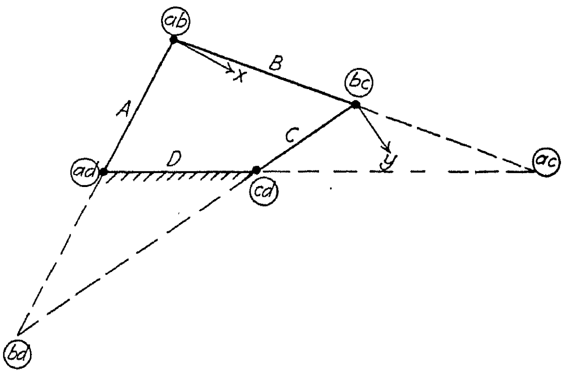


FIG. 39.

of the driving crank or pulley is usually known, and with its known diameter or radius, the linear velocity of this uniformly rotating member can thus be used as a starting point for these calculations.

Note.—The radial segments, Aa, Bb, and Kk, show the relative **amounts** of the velocities of these points, not the direction. The direction is, of course, always perpendicular to the radius.

28. To locate all the Centros of a Four-link Mechanism at any phase.

(1) Determine the number of centros by the formula $\frac{N(N-1)}{2}$, where N = number of links. This would yield 6 centros for 4 links, 15 for 6, 28 for 8, etc.

(2) Select the **permanent centros**; *i.e.*, the centros of the links that are directly connected, either by pins or guides. In Figs. 39 and 40 these are ab, ad, bc, and cd. In Fig. 40, note that cd is at infinity. Why?

(3) Determine the centros *ac* and *bd* by the motion of two points, as in Art. 26.

Example (a).—To find *bd* in the mechanism in Fig. 39.

The motion of *ab* relative to *D* is known to be in the direction of the arrow *X* since it is a point in *A*, as well as *B*. Similarly, *bc* is moving in the direction *y*. Both points are in *B*, and have known motions, hence the centro *bd* lies at the intersection of the perpendiculars to their motion.

Example (b).—To find *ac* in the mechanism in Fig. 40. The motion of *ab*, as a point in *B* relative to *C*, is in the direction of the arrow *X*, and that of *ad* in the same manner is that of *y*. Therefore the relative motion of *A* or *C* with respect to the other is about the point *ac* as a center (for this instant at this phase).

Note.—The centros *bd* and *ac* in these mechanisms, as well as all non-permanent centros in any mechanism, are changing position with every change in the arrangement of the members.

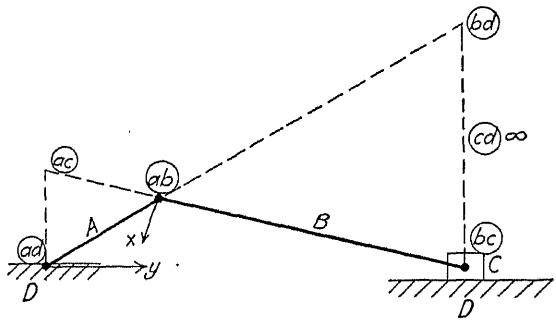


Fig. 40.

KENNEDY'S THEOREM

29. If any three members of a mechanism have plane motion, their centros are three points in the same line.

In corroboration of this theorem, note that, in both linkages just considered, *ab*, *ac*, and *bc* are in a straight line, also *ab*, *ad*, and *bd*, also *ac*, *ad*, and *cd*, and also *bc*, *bd*, and *cd*. This shows that the theorem holds for these cases, but does not prove it.

30. Proof.—Let *A*, *B*, and *C*, Fig. 41, be any three bodies in motion. Let *ab* be the centro (common point) of *A* and *B*, and *bc* the common point of *B* and *C*. Assume *ac* anywhere as the centro of *A* and *C*.

As a point in *A*, *ac* must move in the direction *x*, and as a point in *C*, it must move in the direction *y*, relatively to *B*, Since a point can have but one motion relatively to any body,

x and y must coincide. To coincide, x and y must be perpendicular to a straight line through ab and bc. Therefore ac must lie on the line of these points.

Note.—It is evident that this theorem does not locate ac in this case, as the links shown are not constrained, and there is

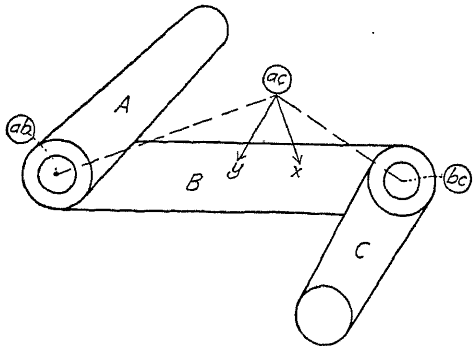


FIG. 41.

no compelled motion of A and C relatively to each other. In a constrained linkage their motions are governed, and the centro desired can be located through the influence of the constraining members.

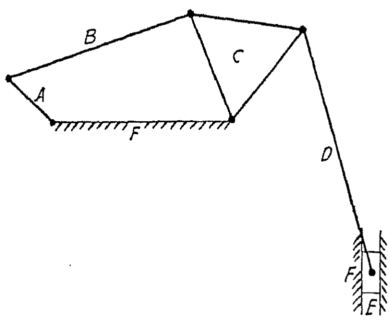


FIG. 42.—A 6-link chain.

31. To find the Centros of any Linkage.

Example.—Find the centros of the six-link chain, Fig. 42, similar to certain valve motions, in the phase shown. This is a typical chain, and the method pursued in this case will locate the centros in any mechanism, of greater or less complexity.

(1) Lay out all the possible combinations of link pairs of the mechanism

in the form of the Score Card, Fig. 43. This card shows the fifteen centros possible in a six-link mechanism.

Note.—The Hexagon Score Card is preferred by some teachers. In this, any pair of letters joined by a line indicates that the centro of that line has been located.

(2) Locate all the permanent centros, Fig. 45. Since all the directly connected links are pinned, except *E* and *F*, which constitute a sliding pair

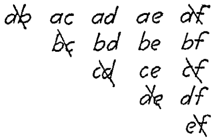


Fig. 43.—"The Score Card."

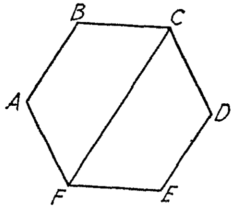


Fig. 44.—The hexagon score card.

there is a permanent centro at each of the six pins, and one, *ef*, at infinity. Having located these, cross them off the score card; see Fig. 43. In the hexagon score card, the bounding sides and the diagonal *CF* tell the same story. This gives centros enough to locate all the remainder by Kennedy's Theorem.

(3) Begin by locating the centros not yet checked on the score card, or joined on the hexagon, *using any that have already been found.* As

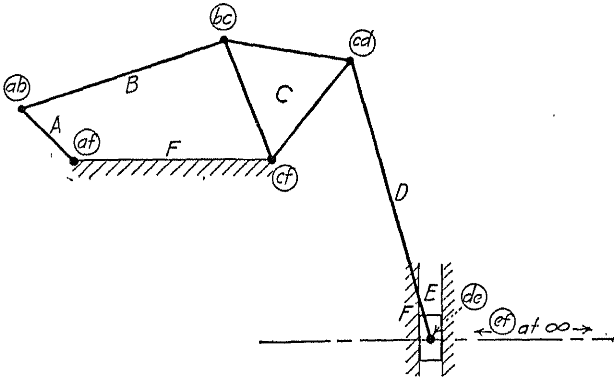


Fig. 45.—All the permanent centros.

each new one is located, check it on the score card, and use the new one for locating the more remote ones. Kennedy's Theorem is the most useful tool for locating centros of links not directly connected.

Example.—The seven primary centers are located (Fig. 45) and are checked on the score cards (Figs. 43–44). Note that *ab–bc* and *af–cf* are checked, and that *A* and *C* are common to both pairs. In the hexagon,

note that a line from A to C will make two triangles ABC and AFC with AC in both triangles. The significance of this is that the centro ac will lie on both straight lines through these pairs of centros, and therefore at their intersection, as in Fig. 46, which is a part of the linkage of Fig. 45. Having

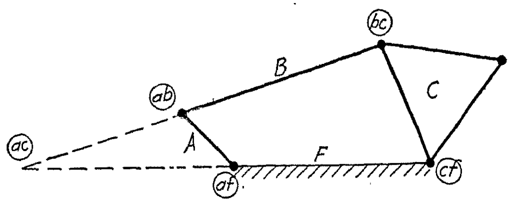


FIG. 46.—Locating ac by Kennedy's theorem.

found ac, the eighth centro, it can be checked on the score card, and used in locating some of the remainder. Figure 47 shows how the various centros are located by using the least remote of the floating centros, and checking them until all are located. A little practice will enable the student to work these mechanically.

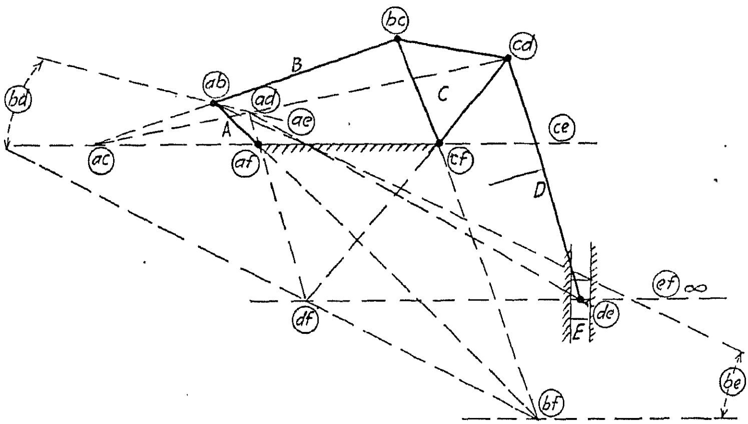


FIG. 47.—All the centros.

VELOCITIES IN COMPLEX LINKWORK

32. To obtain the velocity of some point remote from the driver-link, whose velocity is known, the method by centros will prove easiest in some instances, and the method by transference of velocities in others. The link B has complex motion, since one end is in complete rotation about af, and the other end is oscillating about cf. The link E has rectilinear motion (reciprocating) relatively to F; B and E are not directly con-

nected, yet at this instant they have a relative motion of pure rotation about the centro *be*, and therefore their velocities, either with reference to *F*, the stationary link, or any moving link can be determined.

Example.—In the linkage shown in Fig. 42, let the driving crank $A = 12$ in., and be revolving uniformly at 100 r.p.m., and let it be required to find the linear velocities of *bc* and the slider *E* in the phase illustrated.

$$V^{AB} = 2\pi \text{RN} = 2 \times \frac{22}{7} \times \frac{12}{12} \times 100 = 628 \text{ ft. per min.}$$

Since *ab* is a point in *B*, as well as *A*, and the centro *bf* is known, the velocities of all points in the link *B*, as it moves relatively to *F*, can be determined.

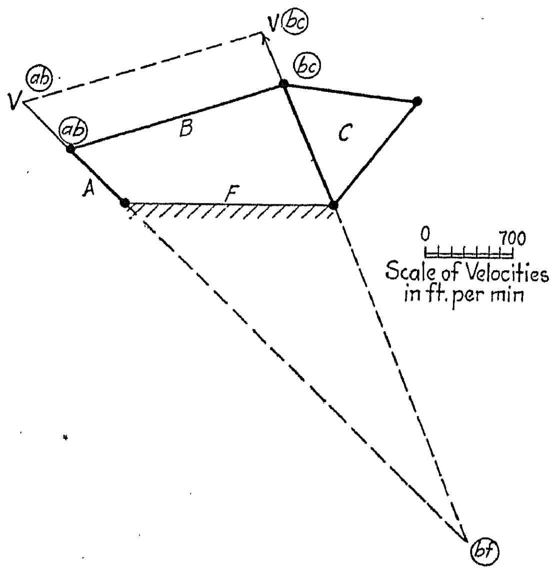


Fig. 48.—Determing the linear velocity of a point.

In any rigid body the velocities are directly proportional to the radii of the points. Figure 48 shows the centro of *B* and *F*, and the instant radii, at this phase, of *ab* (velocity 628 ft. per min.) and *bc*, whose velocity is required.

(1) Lay out a scale of linear velocities; see Fig. 48.

(2) Set off 628 ft. per min. from this scale on the instant radius of *ab*.

(3) Draw through this point a line parallel to the link *B*.

(4) The segment of the instant radius of *bc* included between the parallel lines will be, by similar triangles, the velocity of *bc*, or 514 ft. per min. measured on the scale.

33. To Find the Velocity of the Sliding Link.

The common point of *A* and *E* is *ae*. Knowing the velocity of *ab* as 628 ft. per min., it is possible to find that of *ae* by comparing the instant radii of *ab* and *ae* about *af*. See Fig. 47.

The velocity of *ae* is found to be 495 ft. per min. Since the block *E* rotates relatively to *F* about a centro at infinity, the radii of all points in *E* are infinite in length, and therefore their velocities are all equal. The conclusion is that the velocity of the block *E* at this phase is 495 ft. per min.

Note.—The same result should be obtained, if the velocity of *ab* should be compared with that of *be*, both rotating about *bf*. In obtaining this result, the writer used this latter point in *E*, rather than *ae*. Why?

THE "METHOD OF TRANSFERENCE"

34. Many cases occur where it is easier to transfer the reference velocity from one point to another, instead of locating all the instant centers.

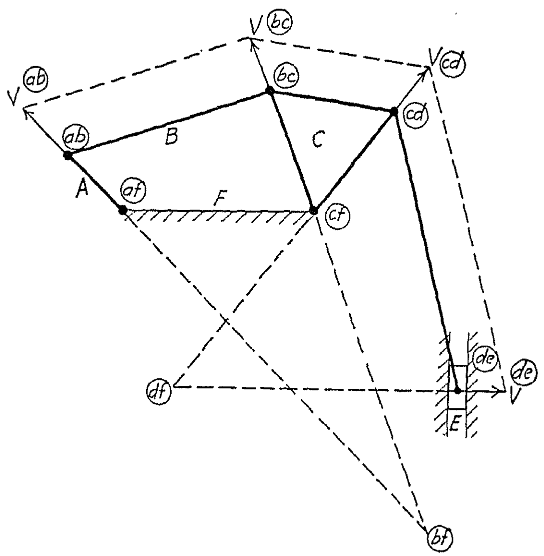


Fig. 49.—Velocity determination by transfer.

This should give 495 ft. per min., the same as obtained by the direct method.

Example.—To find the velocity of the slider *E* by transference, if that of *ab* is known to be 628 ft. per min.

(1) Obtain V^{BC} as shown in paragraph 32.
(2) Using V^{BC} as reference, obtain V^{CD}, with *cf* as the center.
(3) Using V^{CD} as reference, obtain V^{E} with *df* as the center.
This should give 495 ft. per min., the same as obtained by the direct method.

35. To obtain the Velocity of a Remote Member, when there is an Intermediate Slider.

One instance of this arrangement is shown in Fig. 50. *A* is the driver, and its velocity is known. The block *B* is pinned to *A*, and slides in a slot in the beam *C*. The linear velocity of *ab* is known, and this point is also a point in *C*, though not directly connected to *A*. The velocity of *ab* is laid off, $ab - x$ n its correct direction at this phase. Since *C* is centered about *cf*, the motion of this point in *C* must be in the direction of the arrow *y*, and since the block *B* is sliding on *C* in the direction of *cf*, the velocities in these directions must be components of *x*. Resolving *x*, it is apparent that this point in *C* has a velocity $=y$. The point *cd* is on the same link, so its velocity is proportional to the radii of the points about their center *cf*, or V^{cd}. Knowing the velocity of *cd*, it is possible to find that of *de*, and therefore all points in *E*, since *ef* is at infinity, by referring both points to *df* as the center of all points in *D* relatively to the frame.

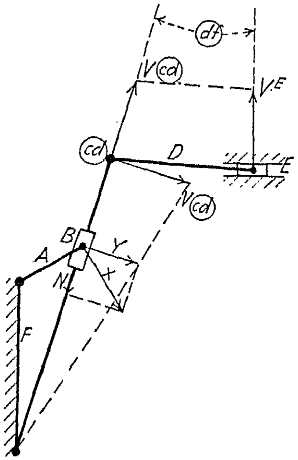


FIG. 50.—The oscillating beam quick return linkage.

VELOCITY DIAGRAMS

36. It is frequently desirable to show graphically the speed fluctuations of certain parts throughout an entire stroke, revolution, or cycle of a mechanism. This is accomplished by obtaining the velocities of the part in a number of its phases, laying them out in a certain order, and plotting a curve through the ends of the lines. If these curves are worked out in a definite arrangement, the velocities of intermediate points may be read from the curve. Three such arrangements, **Space, Time** and **Polar Velocity Diagrams,** are commonly used.

A **Space Velocity Diagram** is one in which the velocity values are laid off on the space traversed by the point whose velocities are required.

Figure 51*a* shows the velocity diagram of the piston of a steam engine. It is laid off over the path of the crosshead-pin, and shows the piston with zero velocity at the beginning and end of each stroke, with its maximum near the center, slightly faster than the uniform speed of the crank-pin.

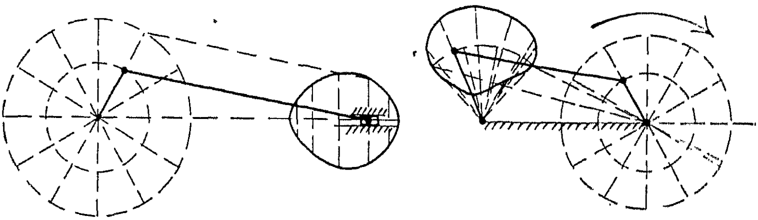


FIG. 51.—Space velocity diagrams.

Figure 51*b* shows that of an oscillating point. The crank, as before, travels a complete circle at a uniform speed, and imparts an oscillation to the follower, a form of linkage commonly used in machine feeds and Corliss engines. The velocities of *bc* are laid off radially, with one stroke outside the circle, and the other stroke inside, denoting the travel in opposite direction, or **sense,** as it is often called.

38. A **Time Velocity Diagram** is one where the time units are laid off in equal spaces. Figure 52*a* gives the same diagram

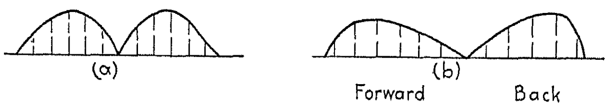


FIG. 52.—Time velocity diagrams.

as that in Fig. 51*a*, except that the time spaces in Fig. 52 are equal, and those in Fig. 51 are not. The ordinates, denoting velocity values, are the same in both figures. Figure 52*b* does the same thing for Fig. 51*b*, and in both cases no definite distances need be assigned for the equal time units.

39. A **Polar Velocity Diagram** is one where the velocity values of the variable link are laid off on the corresponding radii of the uniformly traveling link. This gives a direct comparison

of their values at all points in the cycle. A favorite device is to scale the velocity of the crank-pin equal to the radius of the crank-pin, as is done in these sketches. This gives a direct comparison of their velocities.

There are certain conditions where each of these diagrams is best suited for the work in hand, the circumstances of the problem determining the one to use.

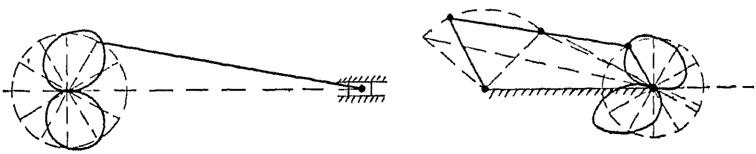


FIG. 53.—Polar velocity diagrams.

ACCELERATION AND ACCELERATION DIAGRAMS

40. In recent years, the introduction of very high speed and heavy loads in steam engines, gas engines, and machinery, has compelled engineers to give attention to the serious condition brought about by quick and often violent reactions due to excessive acceleration, particularly in the reciprocating parts. These reactions find expression in heavy vibration, shocks, wear, lost motion, noise, and fatigue of the metal, and cause much loss of power and destruction of parts, through inattention to the effects of acceleration. So-called "shaking forces" have been studied extensively, and much ingenuity has been displayed in counteracting them. Proper balancing and heavy flywheels will do much to overcome the vibratory and uneven action of the parts due to acceleration produced by the motive power and the kinematic results of the particular constrainment and proportions of the machine. One automobile concern announces that it has increased the power of its engine 70 per cent by a system of balancing, so effectively designed that the vibratory action of the reciprocators is virtually nullified. Without disputing or accepting this claim, it is safe to say that all automobiles would be benefited, and would show more smoothness of action, less wear on vital parts, and develop more horsepower, if the shaking forces were more effectively studied and corrected. Besides balancing to reduce these ill effects, much reduction in weight of the reciprocatory parts is possible through the use of better materials and better design to resist stresses.

41. Acceleration is expressed:

$$A = \frac{V}{T} = \frac{S}{T^2}$$

The unit of acceleration in linear measure is feet per square second.

The unit of acceleration in angular measure is radians per square second.

ACCELERATION DIAGRAMS

42. The acceleration is proportional to the subnormal to the velocity curve at any point.

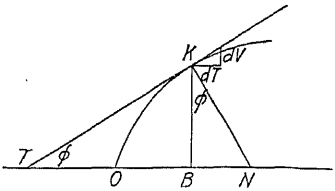


FIG. 54.

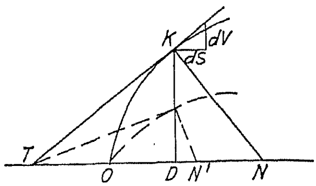


FIG. 55.

Proof.—Let K be any point on a **time velocity curve.**

The ordinate $BK =$ the velocity at that time.

TK and NK are the tangent and normal at K.

$\frac{dv}{dt} =$ acceleration A at the phase $K = \tan \phi$.

$\frac{BN}{BK} = \tan \phi = A$ at the phase K.

$\therefore$, when BK represents the velocity, BN may represent the acceleration, and may be laid off as an ordinate at B, and an acceleration curve may be drawn.

Important.—Note that this acceleration curve only shows the **relative** acceleration. The acceleration in actual units can only be shown by this curve, when the correct scale has been applied to it.

43. Numerical Value of the Acceleration Ordinates.—For the correct reading of these accelerations, the ordinates and abscissæ must have a certain relation in laying off the velocity diagram. Figure 55 shows a **space velocity curve,** in which DK represents the velocity at the phase D over the space OD.

If **similar units** are used for the linear values of these ordinates and abscissæ, the acceleration will be given in **similar units.**

Example.—If 1-in. on the scale = 1 ft. in space value, and 1 in. on the scale = 1 ft. per sec. in velocity value, then the acceleration ordinates will scale 1 in. = 1 ft. per sec.^{2}.

If these values are scaled so that 1 in. = 10 ft., and 1 in. = 10 ft. per sec., the acceleration ordinates will scale 1 in. = 10 ft. per sec.^{2}.

This is true, because

$$\tan \phi = \frac{DN}{DK} = \frac{dv}{ds} = \frac{\text{Acceleration}}{\text{Velocity}} = \frac{\text{Subnormal}}{\text{Velocity Ordinate.}}$$

This could only obtain when dv and ds were given **similar values.**

As this is seldom convenient, never in fast moving parts, a rule must be devised for scaling the acceleration, when dissimilar units are employed.

THE SCALE OF THE ACCELERATION

44. Figure 55 shows two curves of the same velocities, the **solid line being laid out to similar scales,** and the **dotted to dissimilar scales.** The equation for this curve is $y = fx$, $\therefore \frac{dy}{dx} = \tan \phi = f'x.$

If a unit length of ordinate = n times as many ft. per sec. as the linear unit of the abscissæ, the ordinates will be $\frac{1}{n}$ as long as they would be if the scales were similar. The equation becomes, therefore,

$$y = \frac{1}{n} f'x = \frac{1}{n} \tan \phi.$$

Example (1).—The subnormal depends for its length on the inclination of TK.

If the dotted curve in Fig. 55 represents ordinates laid off on ½ the scale of the space units, then $n = 2$. OD remains the same, $DK' = \frac{1}{2}DK$.

If $\phi = \phi'$, DN' would = $\frac{1}{2}DN$, and the acceleration units would be similar to the velocity units.

However, ϕ and ϕ' are not equal, and $\tan \phi$ does not equal $\tan \phi'$, therefore there is a **double reduction** in the length of the subnormal ($\tan \phi' = \frac{1}{2} \tan \phi$, and $DK' = \frac{1}{2}DK$), consequently $DN' = \frac{1}{n^2} DN$, or $\frac{1}{4}DN$.

Since K is a random point on the curve, all subnormals are reduced in the same proportion. In laying out any acceleration curve, then, the acceleration scale will be n^2 times the velocity scale.

Example (2).—Let the velocity ordinates be laid off to the scale of 20 ft. per sec. = 1 in., and the space abscissæ be laid off 10 ft. = 1 in., then $n = \frac{1}{2}$, and $\frac{1}{n^2} = 4$. $\therefore$ the acceleration scale will be 40 ft. per sec.^{2} = 1

in.; *i.e.*, the subnormal at any point will give the acceleration if measured to that scale.

Example (3).—A steam engine of 12-in stroke, runs 100 r.p.m. To determine its velocity and acceleration curves, and the scale for the latter. V^{CP} = 314 ft. per min. = 5.23 ft. per sec. The layout in Fig. 56 is made

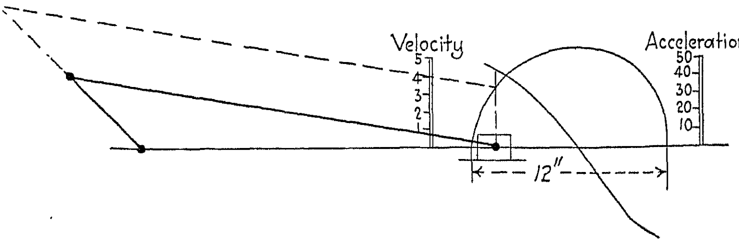


FIG. 56.—Velocity and acceleration diagrams of engine, with scale for each.

to a scale, 2 in. = 1 ft., or ½ ft. = 1 in. on the paper (reduced in the illustration). The piston velocity curve is laid off to the scale of 5 ft. per sec. = 1 in., which makes $n = 10$.

The acceleration is laid off on the velocity ordinates, making the length = the subnormal at each point. Since $n^2 = 100$ and the space unit is ½ ft. = 1 in.; the acceleration unit is therefore 50 ft. per sec.² = 1 in. This is laid off on the right of the figure, and reading the values of velocity and acceleration of the piston, when in the 45-degree crank phase shown, they are seen to be about 3 ft. per sec. and 40 ft. per sec.², respectively. These results check up quite closely to the mathematical calculations.

CENTRODES

45. The path of a centro is a *centrode*. Where the centro is fixed, as *ad* in Figs. 57 and 58, the

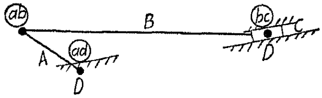


FIG. 57.

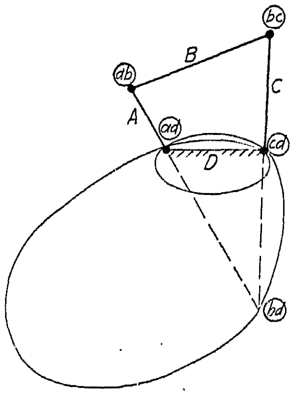


FIG. 58.—Centrode of *bd*.

centrode is a point *on the stationary member*. And when it is a point on some rotary member, as *ab*, the centrode is a circle *on the stationary* member. The centrode of *bc*, Fig. 57, is a straight line on the stationary link. If the centro be floating, as *bd*, its centrode may be one of an unlimited variety of curves. In the

drag link mechanism shown in Fig. 58, the path of *bd* takes the outline of the complex curve drawn. By inverting the same linkage, making B the stationary link, a different curve would result, tangent to the first curve, the point of tangency depending on the phase in which it was inverted. Curves generated by inverting a link chain will roll together, and certain substitutes for link chains have been worked out to give the same results through direct contact mechanisms.

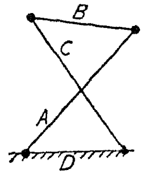


FIG. 59.

The crossed link chain, shown in Fig. 59, wherein $A = C$, and $B = D$, will produce centrodes, about D and B respectively, which are equal and tangent ellipses. The angular velocity ratios of such curves will be treated in Chapter IV.

PROBLEMS

The following problems are all intended for graphic solution, excepting the calculation of the velocity of the crank-pin center. In all the problems, the crank A is considered to have uniform velocity and complete rotation. Its velocity, therefore, is known, as the r.p.m. will be supplied by the instructor. All velocity results are required in ft. per min., and accelerations in ft. per sec.^{2}. The entire list of requirements is not applicable to all problems, and only those which the instructor chooses need be assigned to any of them.

REQUIREMENTS

(a) All centros at phase ——— degrees (30 degrees, 45 degrees, etc.) of A.

(b) Linear velocity of *bc* (if four-link), or E (if six-link), at phase ——— degree of A.

(c) Linear velocity of centro ——— (not *ab*) at phase ——— degree of A.

(d) Linear velocity of some point in the mechanism, *not* one of the centros, at phase ——— degree of A.

(e) Space velocity diagram of ———.

(f) Time velocity diagram of ———.

(g) Polar velocity diagram of ———.

(k) Acceleration diagram of ———.

(l) Acceleration diagram of ———. (Different points.)

(m) The centrode of link ——— relative to link ———.

(n) Angle of oscillation of arm ———.

(o) Length of stroke of slider ———.

(p) Time ratio of advance stroke to return stroke of slider ———, or arm ———.

1. A horizontal steam engine having a ——— in. stroke, ——— in. connecting-rod, and ——— r.p.m.

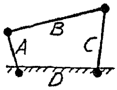


FIG. 60.—Problem 2.

2.	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
A	1	1	1	1¼	1¼	1¼	1½	1½
B	2	2¼	2½	2½	2¾	3⅛	3	3½
C	1¼	1½	1⅝	1½	1⅝	1¾	1¾	2
D	2	2	2¼	2½	2¾	3	3	3¼

	(j)	(k)	(l)	(m)	(n)	(o)	(p)
A	1½	1¾	1¾	1¾	2	2	2
B	4	3	3½	4	4	5	6
C	2¼	2	2¼	2½	2¼	2½	3
D	3½	3	3¼	3¾	4	5¼	6

3.	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
A	1	1	1⅛	1	1	1	⅞	1⅛
B	1¾	2	2¼	1½	1¾	2	1¾	2
C	1⅝	2	2	1½	1¾	1⅞	1⅝	1⅞
D	¾	¾	¾	⅞	⅞	¾	⅝	⅞

	(j)	(k)	(l)	(m)	(n)	(o)	(p)
A	1⅛	1¼	1¼	1¼	1½	1½	2
B	2	2	2¼	3	2	2½	3
C	1⅞	2	2⅛	2¾	1⅞	2⅜	3
D	⅞	1⅛	⅞	⅞	1	1¼	1¾

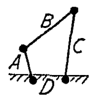


FIG. 61.—Problem 3.

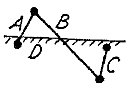


FIG. 62.—Problem 4.

4.	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
$A = C =$	1	1	1	1¼	1¼	1¼	1½	1½
$B = D =$	1½	2	3	2	3	4	2	2½

	(j)	(k)	(l)	(m)	(n)	(o)	(p)
$A = C =$	1½	1¾	1¾	1¾	2	2	2
$B = D =$	4½	3	4	5	4	5	6

5.	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
$A = C$	1¾	2½	3	2	3	4	2	3
$B = D$	1	1	1	1¼	1¼	1¼	1½	1½

	(j)	(k)	(l)	(m)	(n)	(o)	(p)
$A = C$	4	3	4	5	4	5	6
$B = D$	1½	1¾	1¾	1¾	2	2	2

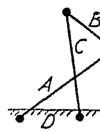


FIG. 63.—Problem 5.

6.	(a)	(b)	(c)	(d)	(e)
A	1	1	1	1¼	1¼
B	2	3	4	3	4
X	½	1	0	½	1

	(f)	(g)	(h)	(j)	(k)
A	1¼	1½	1½	1½	1¾
B	5	4	5	6	5
X	2	½	0	1	2

	(l)	(m)	(n)	(o)	(p)
A	1¾	1¾	2	2	2
B	6	7	6	7	8
X	1	0	3	1	2

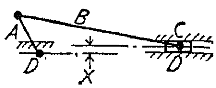


FIG. 64.—Problem 6.

7.

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
A	1	1	1	1¼	1¼	1¼	1½	1½
C	3½	4½	6	3	4	5	3	5
D	2	3	4	1	2	3	1	3

	(j)	(k)	(l)	(m)	(n)	(o)	(p)
A	1½	1¾	1¾	1¾	2	2	2
C	5	6	4	5	6	5	8
D	4	4	1½	3	3	1½	5

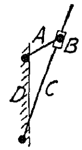


Fig. 65.—Problem 7.

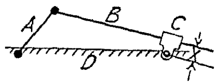


Fig. 66.—Problem 8.

8.

	(a)	(b)	(c)	(d)	(e)
A	1	1	1	1¼	1¼
B	4	5	6	5	6
X	½	½	¾	½	¾
D	2	2½	3	3	3½

	(f)	(g)	(h)	(j)	(k)
A	1¼	1½	1½	1½	1¾
B	7	5	6	8	6
X	1	½	1	1½	1
D	4	3	4	5	4

	(l)	(m)	(n)	(o)	(p)
A	1¾	1¾	2	2	2
B	7	9	8	10	12
X	1½	2	1	½	2
D	5	6	5	6	8

9.

	(a)	(b)	(c)	(d)	(e)
A	1	1	1	1¼	1¼
B	4	5	6	5	6
D	2	2½	3	3	3½

	(f)	(g)	(h)	(j)	(k)
A	1¼	1½	1½	1½	1¾
B	7	5	6	8	6
D	4	3	4	5	4

	(l)	(m)	(n)	(o)	(p)
A	1¾	1¾	2	2	2
B	7	9	8	10	12
D	5	6	5	6	8

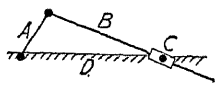


Fig. 67.—Problem 9.

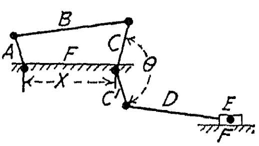


Fig. 68.—Problem 10.

10.

	(a)	(b)	(c)	(d)	(e)
A	1	1	1	1¼	1¼
B	2	2¼	2½	2½	3¼
C	1¼	1½	1⅝	1½	1⅝
X	2	2	2¼	2½	2¾
C'	1	¾	1½	1	1½
Y	1	½	0	0	1½
D	2	2	2	3	3
θ	180	150	135	120	180

	(f)	(g)	(h)	(j)	(k)	(l)	(m)	(n)	(o)	(p)
A	1¼	1½	1½	1½	1¾	1¾	1¾	2	2	2
B	3⅛	3	3½	4	3	3½	4	4	5	6
C	1¾	1¾	2	2¼	2	2¼	2½	2¼	2⅓	3
X	3	3	3¼	3½	3	3¼	3¾	4	5¼	6
C'	2	1	1	2	1	1	1½	2	1	1½
Y	1	1½	2	1	0	1½	1	1	0	1
D	3	3	4	4	3	3	3	4	5	6
Θ	120	150	180	120	90	135	180	150	120	180

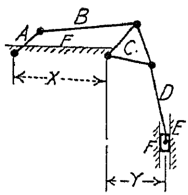


FIG. 69.—Problem 11.

11.

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
A	1	1	1	1¼	1¼	1¼	1½	1½
$D = B$	2	2¼	2½	2½	3¼	3⅛	3	3½
X	2	2	2¼	2½	2¾	3	3	3¼
C	1¼	1½	1⅝	1½	1⅝	1¾	1¾	2
(Equilateral Triangle)								
Y	1½	1½	2	2	2	2	2½	2

	(j)	(k)	(l)	(m)	(n)	(o)	(p)
A	1½	1¾	1¾	1¾	2	2	2
$D = B$	4	3	3½	4	4	5	6
X	3½	3	3½	3¾	4	5¼	6
C	2¼	2	2¼	2½	2¼	2½	3
Y	2½	2	3	3	3	2	4

12.

	(a)	(b)	(c)	(d)	(e)
A	1	1	1	1¼	1¼
B	1½	1¾	1¾	1½	1¾
(Equilateral Triangle)					
$C = D$	2¼	2½	3	2½	3
X	1	1½	1⅜	1¼	1⅝
Y	1¾	2¾	2½	2¼	3

	(f)	(g)	(h)	(j)	(k)
A	1¼	1½	1½	1½	1¾
B	2	1¾	2	2¼	2
$C = D$	3½	3	3½	4	3½
X	1½	1½	1⅝	2⅛	1¾
Y	2¾	2¾	3	3¾	2¼

	(l)	(m)	(n)	(o)	(p)
A	1¾	1¾	2	2	2
B	2½	2¾	2½	2¾	3
$C = D$	4	5	4	5	6
X	2¼	2¼	2	2¾	2¾
Y	4	4	3¾	5	5

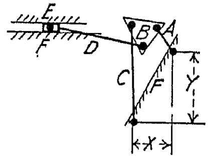


FIG. 70.—Problem 12.

13.

	(a)	(b)	(c)	(d)	(e)
A	1	1	1	1¼	1¼
X	¾	⅝	⅞	¾	⅞
C'	¾	1	1¼	¾	1
$C = D$	3	3	3	4	3
	(f)	(g)	(h)	(j)	(k)
A	1¼	1½	1½	1½	1¾
X	1	¾	1	1¼	1
C'	1¼	1	1¼	1½	1
$C = D$	4	3	4	5	4
	(l)	(m)	(n)	(o)	(p)
A	1¾	1¾	2	2	2
X	1¼	1½	1¼	1⅜	1½
C'	1¼	1½	1	1¼	1½
$C = D$	4	4	4	5	5

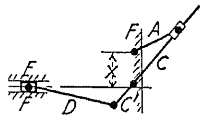


FIG. 71.—Problem 13.

14.

	(a)	(b)	(c)	(d)	(e)
A	1	1	1	1¼	1¼
B	2	2½	3	2½	3
X	1	1	1½	1	1
$C' = A + X$					
C	3	4	5	3	4
D	3	4	5	3	4
	(f)	(g)	(h)	(j)	(k)
A	1¼	1½	1½	1½	1¾
B	4	3	4	5	3
X	1½	¾	1½	2	1
C	5	3	4	5	4
D	5	3	4	5	4
	(l)	(m)	(n)	(o)	(p)
A	1¾	1¾	2	2	2
B	4	5	4	5	6
X	2	3	1	2	1
C	5	6	4	5	6
D	5	6	4	5	6

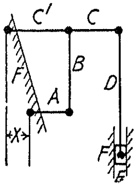


FIG. 72.—Problem 14.

15.

	(a)	(b)	(c)	(d)	(e)
A	1	1	1	1¼	1¼
X	1½	2	2½	1¾	2
$C = D$	3	4	5	3½	4
	(f)	(g)	(h)	(j)	(k)
A	1¼	1½	1½	1½	1¾
X	2½	3	2	4	3
$C = D$	5	5	4½	6	6
	(l)	(m)	(n)	(o)	(p)
A	1¾	1¾	2	2	2
X	4	5	3	4	5
$C = D$	7	8	6	7	9

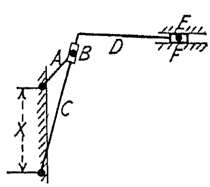


FIG. 73.—Problem 15.

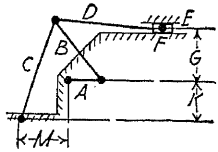


FIG. 74.—Problem 16.

16.

	(a)	(b)	(c)	(d)	(e)
A	1	1	1	1¼	1¼
B	2¾	3⅛	3⅛	3	3¼
$C = D$	3	3½	4	3½	4
G	2	2	2	2¼	2½
K	1	1	1½	1	1
M	1½	1½	1	1¾	1¾

	(f)	(g)	(h)	(j)	(k)
A	1¼	1½	1½	1½	1¾
B	3¼	4	4½	4½	4½
$C = D$	4½	4½	5	6	5
G	2½	3	3	3	3½
K	1½	1½	1½	2¼	1¾
M	1¼	2¼	2¼	1½	2½

	(l)	(m)	(n)	(o)	(p)
A	1¾	1¾	2	2	2
B	5	5	5½	6¼	6½
$C = D$	6	7	6	7	8
G	3½	3½	4	4½	4½
K	1¾	2½	2	2	3
M	2½	1¾	3	3	2

17.

	(a)	(b)	(c)	(d)	(e)
A	1	1	1	1¼	1¼
B	1⅜	1½	1¾	1⅝	1¾
C	1½	1½	1¾	1½	1¾
D	4	4	4	5	5
X	1½	1½	1¾	1½	1¾
Y	¾	¾	⅞	1	1
Z	¼	¼	⅜	⅜	⅜

	(f)	(g)	(h)	(j)	(k)
A	1¼	1½	1½	1½	1¾
B	2	2	2	2¼	2½
C	2	1⅞	2	2¼	2¼
D	5	6	6	6	7
X	2	2	2	2½	2½
Y	1⅛	1	1¼	1¼	1¼
Z	½	½	½	⅝	⅝

	(l)	(m)	(n)	(o)	(p)
A	1¾	1¾	2	2	2
B	2¾	3	3¼	3¼	3½
C	2½	3	3	3¼	3½
D	7	7	8	8	8
X	3	3	3½	3½	4
Y	1½	1½	1½	1¾	1¾
Z	⅝	¾	¾	¾	⅞

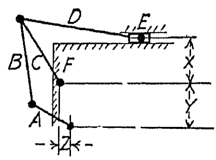


FIG. 75.—Problem 17.

18.

	(a)	(b)	(c)	(d)	(e)
$A = C$	1	1	1	$1\frac{1}{4}$	$1\frac{1}{4}$
$B = X$	2	$2\frac{1}{2}$	3	$2\frac{1}{4}$	3
D	3	3	3	4	4

	(f)	(g)	(h)	(j)	(k)
$A = C$	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{3}{4}$
$B = X$	$3\frac{1}{2}$	3	4	5	4
D	4	4	4	4	5

	(l)	(m)	(n)	(o)	(p)
$A = C$	$1\frac{3}{4}$	$1\frac{3}{4}$	2	2	2
$B = X$	5	6	$4\frac{1}{2}$	6	7
D	5	5	6	6	8

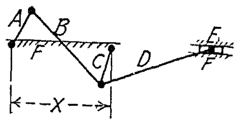
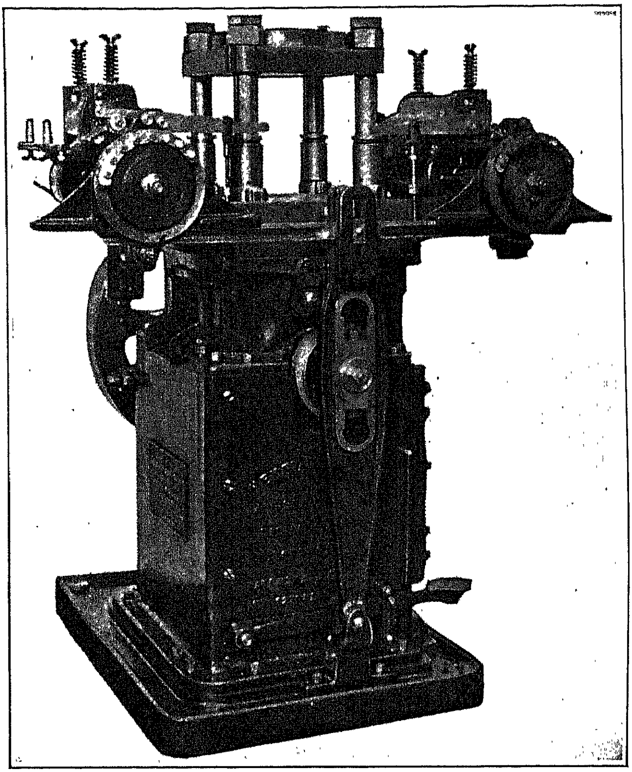


FIG. 76.—Problem 18.



Henry and Wright Dieing Machine showing six-link oscillating beam quick return Mechanism.

CHAPTER III

CHAINS OF LINK WORK

46. Link work offers a most attractive field to the designer of machinery. Positive in action and economical to make (since most of the machine work is plain drilling and facing), an unlimited variety of combinations is provided. The designer is fascinated with the marvelous range of its usefulness, for through link work **rotary motion** may be converted into **complete** or **partial rotation,** with either uniform or variable velocity, or into **translation**; and points may be made to run in an infinite variety of paths, **straight, circular, elliptical, looped,** and many odd-shaped curved lines, with no other constrainment than pins or straight guides. Moreover, any of these motions may

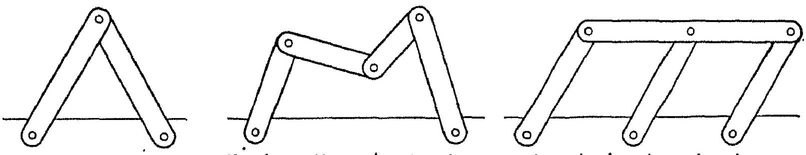


FIG. 77.

be converted into any of the others, to take advantage of whatever motive power is available, and to accomplish any desired result.

Quick-return mechanisms, straight-line motions, sudden blows, parallel motions, powerful pressures, rapid velocity changes, reversals of drive or transmission, intermittent feeds, shifts of material from one conveyor to another; these and many other results, often spectacular, are achieved easily and simply through this medium.

47. Link work must consist of four or more rigid links to function properly.

A chain of three links cannot change the relative posistion of its members, since only one triangle can be made up of three definite sides. A chain of five links, pinned consecutively, would also be no mechanism, since there is no positive constrainment.

Five links, however, can be so joined, as in the example of the locomotive side rods, that they will operate.

THE QUADRIC CHAIN

48. Simplest of all link chains is the one called the quadric chain, four links pinned consecutively. By proper proportioning of parts, the following results can be secured.

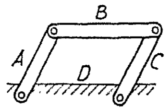


FIG. 78.—Parallel motion. A—C. B—D.

Complete rotation of both cranks A **and** C. **Parallel Motion.** Angular velocities of A and C always equal. Rotation not positively transmitted at the dead points.

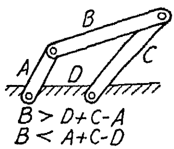


FIG. 79.—"Drag link."

Complete rotation of A **and** C. The angular velocity of C is variable, if A is uniform. It is called the drag link chain.

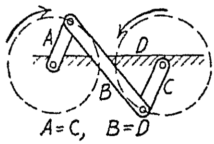


FIG. 80.—Contrary motion.

Complete rotation of A **and** C. Called the crossed link chain. With A rotating uniformly, C will have a very variable velocity, and its rotation will be in the opposite sense to A. Can be made into parallel motion.

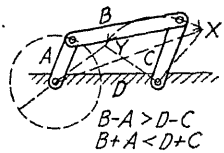


FIG. 81.

Complete rotation of A, **oscillation of** C. Has inner and outer dead points. The times of oscillation of the forward and return strokes of C are different.

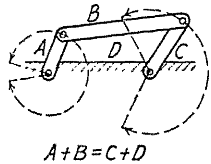


FIG. 82.

Oscillation of both cranks. Figure 82 shows an example of symmetrical oscillation of both A and C, with C working on the outside.

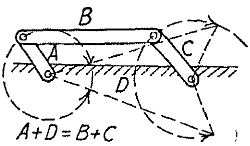


FIG. 83.

Oscillation of both cranks. This case shows symmetrical oscillation of both A and C, with C working inside.

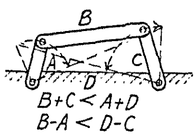


FIG. 84.

Oscillation of both cranks. Non-symmetrical.

Note.—For a more complete list of link combinations, see Reuleaux.

DEAD POINTS

49. When two links coincide in direction, either outstretched or overlapping, there can be no further motion by the connecting link in the direction of its centers, therefore there is no positive transmission of motion from one member to another. Such positions are called **dead points,** or **dead centers.** Where they exist in the mechanisms just reviewed, they can be found by striking arcs from the center of A, with a radius equal to $A + B$, and with a radius $B - A$, intersecting the circle of C. Using this process in the chain in Fig. 81, the dead points, the limits of oscillation of C, will be found at x and y. When the links are outstretched (position x), the dead point is called outer, and when they overlap (position y), the point is called the inner dead point. A steam engine on dead point (0 and 180 degrees) will not start, because the force of the steam is exerted against piston, connecting-rod, and crank all in one line, and it therefore applies only a tensile or compressive force on the crank, but no turning effort. In a single cylinder engine the starting is accomplished by pulling the fly-wheel past the dead point, in small engines by hand, in large ones by a gear. Locomotives are protected against this by having two engines, one 90 degrees in advance of the other. In chains in which both members oscillate (Figs. 82, 83, 84), the action near the dead points is not smooth, hence it is advisable to limit the operation to a smaller range than the maximum.

50. The Slider-Crank Linkage.—This familiar mechanism, the steam engine mechanism, is a four-link chain with one sliding member. The slider may, and sometimes does, slide in a curved guide, but its use in such cases involves no special treatment. In the case of the steam engine, the sliding link does the driving, but in nearly all machines of this type the sliding link is the driven.

51. Inversions of the Slider-Crank Linkage.

Any member of any chain may be made the stationary member. This results in four so-called inversions, each of which in the slider-crank chain develops characteristics of motion and velocity peculiar to itself. The **first** inversion is the most familiar arrangement, in which D is the fixed link.

In the **second** inversion, A is made the fixed link and B becomes the driving crank. There are two possibilities, which

result in two distinct mechanisms with characteristics of their own; *a* where *B* is longer than *A*; and *b* where *A* is longer than *B*. In the first of these both *B* and *D* have complete rotation, and the chain is the basis of the Whitworth Quick Return Motion. In the second case, *B* has complete rotation, and *D* oscillates only. This chain forms the basis of the Oscillating Beam Quick Return Motion. Both of these are used in shapers to furnish the transmission for the tool carrier, or ram, as it is called. The purpose of these mechanisms is to give the tool a cutting stroke of long duration, and a relatively short time on the return idle stroke. The usefulness of this in such machines as planers, shapers, slotters is shown by an example. If a man, working on a shaper with the cutting and idle strokes equal, turns out \$100 worth of work in a week, he will turn out

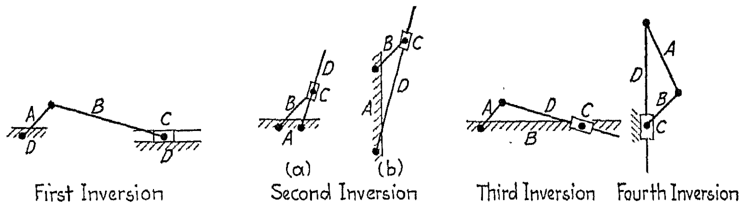


FIG. 85.—The slider-crank inversions.

approximately \$150 worth, if the time ratio is 3:1. In the first case the useful work is 50 per cent of the total, and in the second it is 75 per cent.

In the third inversion, *B* is the fixed link and *A* the driving crank, *C* is an oscillating sleeve or slotted block, and *D* is a link with complex motion. The linkage is not common, but has been used in oscillating engines, such as the Gnome airplane motor, the Case engine (no longer made), and a good many others.

In the **fourth inversion,** *C* is the fixed link, *D* a slider, *B* an oscillator and *A* a rotating crank. If *A* were made long enough, *B* could be given complete rotation. The possibilities of this inversion have not made much of an appeal to the designer, but it has application in the case of wind mills and pump jacks.

THE ELLIPSOGRAPH

52. When a four-link chain has two sliding members there is no rotary motion imparted to any of the links although

the mid-point of D between B and C moves in a circle. The link D has complex motion, two of its points, bd and cd, moving in straight lines. All the other points, except the mid-point mentioned, move in elliptical paths. This is in accordance with that property of the ellipse: to wit, if the focal distance becomes zero, the ellipse becomes a circle, and if the focal distance equals the major axis, the ellipse becomes a straight line.

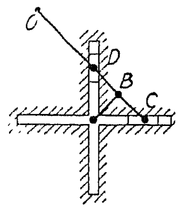
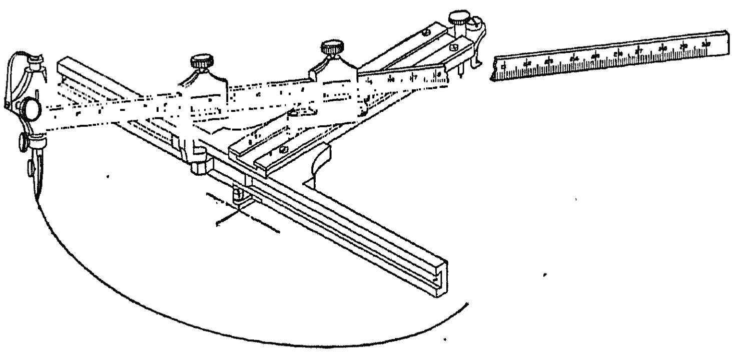


FIG. 86.

If the link D is extended beyond the slider pin, the path of any point, as O, will also be an ellipse. This fact is made use of in the ellipsograph, one of the forms of instruments designed to draw ellipses, sometimes called the trammel. If the arm B is fitted with close-fitting rollers, with the smallest possible play (lost motion), an ellipse of any degree of flatness or roundness, that will be called for in practical work, can be drawn as perfectly as the slight play in the grooves will permit.

The **Elliptical Chuck** is another interesting application of the crossed double slider. This device consists of a stationary base



(Courtesy, Professor Thomas E. French).
FIG. 87.—Ellipsograph, from "French Engineering Drawing."

plate, grooved as is the skeleton in Fig. 86. The chuck, having rollers to run in these grooves, is revolved so that the motion is the same as that of link B. Castings, plates, and elliptical gears to be turned in that outline are fastened in the chuck, and the tool, held at a point on the center line of the horizontal groove, will cut the work in the desired outline. By varying

the distance between the rollers, shortening or lengthening the crank throw to correspond, and changing the tool distance, ellipses of all practical proportions and sizes can be cut. The danger here, as in the ellipsograph, is from loose fitting parts having lost motion, which will result in defective outlines.

Another interesting application of this chain is the Oldham coupling, which will be found illustrated and described in the Appendix.

CHAINS OF MORE THAN FOUR LINKS

53. Quick Return Mechanisms.—The number of combinations of the numerous quadric chains that can be made by adding more links to them is so large that a few of the most important only can be reviewed here. In the preceding chapter a detailed study of the chief characteristics of these mechanisms was given in problem form, so that the intention here is to confine the examination of their applications to a few representatives out of the hundreds of possibilities.

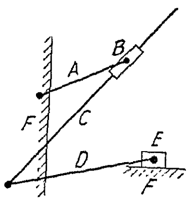


FIG. 88.—Whitworth quick return linkage.

The Whitworth Quick Return.—The skeleton layout of this chain is given in Fig. 88. A is the crank, B a slider, C a completely rotating member, and F the frame. Thus far it is the second inversion of the slider-crank chain, with B sliding on C to compensate for the varying distance necessary between

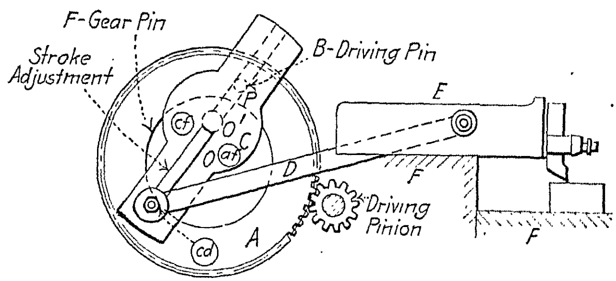


FIG. 88a.—Whitworth quick return linkage as designed for shaper driving.

ab and cf. This distance is the maximum when A is vertical and above af, and the minimum when A is in the opposite phase. The link connecting the ram E with the variably rotating C, is D and is usually attached to an extension of C beyond cf. This extension is made adjustable, so that the stroke can

be changed without affecting the ratio of advance to return. The ordinary mechanical design is shown in Fig. 88*a*. In this, the crank A is only kinematically present, as the drive is made through a pinion engaging a large gear, which rotates on a large pin, really a disc d. This disc is stationary and holds the pins O and O' (*af* and *cf*). A pin P, on the gear, with a roller, is the link B, and is carried through the revolution by the gear. The link C has two slots, the inside one for B to slide in, and the outside one, a T-slot, to allow for the adjustment of the arm *cf-cd*. This is the oldest of the quick returns.

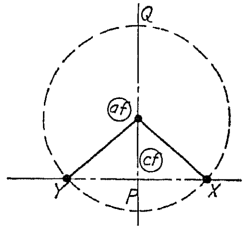


FIG. 89.

The time ratio is shown in the diagram, Fig. 89. The radius of the circle is the length of the crank arm A. The horizontal XY, which passes through *cf*, is the line taken by the link C at the extremes of its strokes. This line divides the circle (path of *ab*) into two arcs, YQX and XPY. The arc YQX is the slow stroke and XPY is the quick stroke, since the velocity of *ab* is uniform. Hence, the time ratio of advance to return is

$$\frac{XQY}{XPY} = \frac{\text{angle } \beta.}{\text{angle } \alpha.}$$

To complete the arc for any desired ratio,

$$\text{angle } \alpha = \frac{\text{return}}{\text{return} + \text{advance}} \times 180 \text{ deg.}$$

The distance for this ratio between *af* and *cf* is,

$$OO' = A \cos \alpha = A \cos \frac{\text{return}}{\text{return} + \text{advance}} 180 \text{ deg.}$$

Example.—To obtain the distance between centers, for a Whitworth chain having a 6-in. crank, for a time ratio of 3:1.

$$OO' = A \times \cos \frac{\text{return}}{\text{return} + \text{advance}} \times 180 \text{ deg.} = 6 \times \cos \frac{1}{1+3} \times 180 \text{ deg.}$$
$$= 6 \times \cos 45 \text{ deg.} = 6 \times .707 = 4.242 \text{ in.} = 4\tfrac{1}{4} \text{ in.}$$

The Oscillating Beam Quick Return.—This useful linkage is probably employed more frequently than the Whitworth, because it is simpler. The driving crank A has complete rotation, B is a slider, pinned to A, sliding in the slot in C. This slotted

link oscillates with its extremes at the points where its center line is tangent to the circle of ab. D is the connecting-rod, and E the ram. By changing the length of A, the stroke of E is adjusted, and the time ratio of the strokes is altered. The length of the beam C may also be altered to change the stroke.

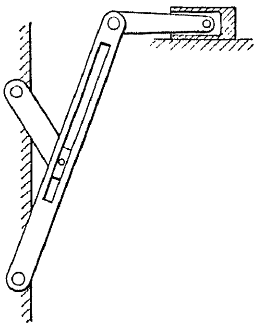


FIG. 90.—The oscillating beam linkage.

The time ratio depends on the points of tangency, X and Y, in the same manner as the Whitworth.

$$\text{The time ratio} = \frac{\text{angle } \beta}{\text{angle } \alpha}.$$

The angle α is determined by the distance OO' (from af to cf) and the length of the crank A, *i.e.*,

$$\cos \alpha = \frac{A}{OO'}.$$

Example.—If the shaper crank is 6-in. throw, and the ratio of strokes is to be $2\frac{1}{2}:1$, what must be the distance from center to center of crank and beam?

$$\alpha = \frac{2}{7} \times 180 \text{ deg.} = 51 \text{ deg. } 25 \text{ min. } 43 \text{ sec.}$$

$$OO' = \frac{A}{\cos 51 \text{ deg. } 25 \text{ min. } 43 \text{ sec.}} = \frac{6}{.624} = 9.61 \text{ in.} = 9\tfrac{5}{8} \text{ in.}$$

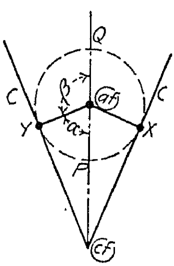


FIG. 91.

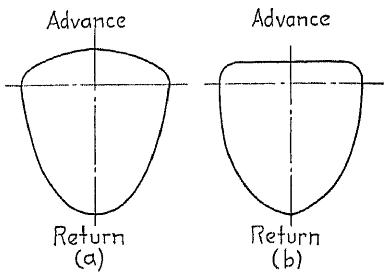


FIG. 92.

Note.—As the length of A decreases in proportion to OO', the time ratio approaches unity. Greater than a 3 : 1 ratio is impracticable, because of the extreme dip of the pin cd, and the length of the stroke compared to the throw of the crank.

Many efforts to improve this chain have been put forth by machine designers, chiefly in the endeavor to give a uniform speed to the cutting stroke. Figure 92a shows the velocity

diagram of this mechanism. Note that, on the advance stroke, the velocity increases toward the middle, and then falls off. The ideal stroke is shown in the right-hand diagram, where the velocity attains a certain amount proper for cutting and maintains that until near the end of the stroke. One way of approxi-

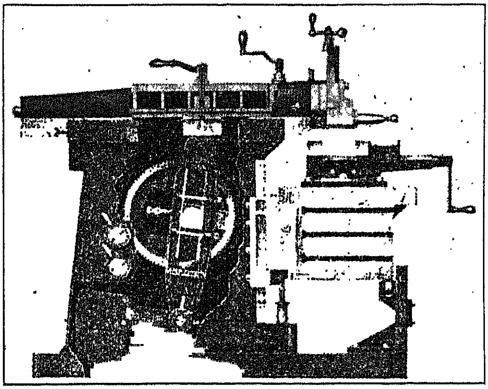


FIG. 93.—The Averbeck shaper. The Steel Products Co., Springfield, Ohio.

mating this ideal has been to insert a cam or eccentric, which changes the length of the crank *A*, so as to maintain a nearly uniform velocity. Another effort in that direction is clearly shown in the picture of the Averbeck Shaper, Fig. 93. This is an entirely different, though similar, linkage and it is also employed on other makes of shapers and key-seating machines.

Single-Slide Swinging Quick Return.—This chain has great merit theoretically, but it has never found any recognition from machine designers. By proper proportioning of parts a ratio of advance to return as high as 3 : 1 can be obtained, and the length of stroke is easily regulated by moving the pin joining *C* to *D*, on the swinging beam *C*. This would in nowise affect the stroke ratio, and any reasonable length of stroke could be attained.

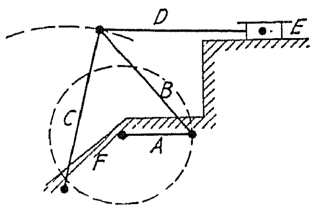


FIG. 94.—Single-slide swinging quick return linkage.

Professor MacCord shows in his book "Velocity Diagrams"* that the linkage, which bears his name, gives a better velocity

* MACCORD, C. W., "Velocity Diagrams," J. Wiley & Sons, New York.

curve than that produced by the Whitworth. It consists essentially of the drag link chain augmented by a connecting-rod D and slider E. The stroke adjustment is made by connecting D to C in such a way as to bring cd at varying distances from cf.

Either of these quick return motions will give good velocity diagrams, but they have not been given much attention as yet by designers of machinery.

The **rolling ellipse** pair, or elliptical gears, with link connections makes a remarkable quick return motion, which has not been given the exploitation it deserves. Its equivalent in link motion is the crossed-link, with opposite members of equal length, sometimes called **Reverse Anti-parallel Cranks.** This chain gives the same remarkable results as the equal ellipse pair, which will be studied in Chapter IV.

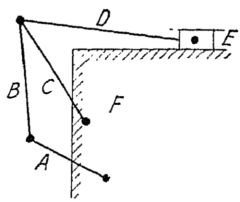


Fig. 95.—Double crank quick return linkage (MacCord).

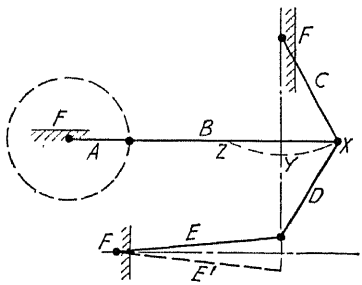


Fig. 96.—Multiple vibrating linkwork.

The few examples cited here, of practically undeveloped mechanisms of great possibilities, should demonstrate to the student the immense opportunity that remains to the future inventor and designer, to employ his own skill and the accumulated research of his predecessors in the creation of new resources in automatic machines.

54. Multiple Vibrating Linkwork.—Where very rapid vibrations of small amplitude are wanted, the linkwork shown in Fig. 96 will operate. As the link C is oscillated by the driver A, it dips from the level of X to that of Y twice in each revolution of A. By adding a connecting-rod D and another oscillator E, the latter is given two full oscillations, four strokes, for every revolution of A. This arrangement can be further expanded to eight or sixteen strokes per revolution by continuing the same idea, but at considerable loss by friction, and diminution of the

amplitude. Unless there is good reason for this expansion, it would be better to use a higher speed motor than to use the more complex linkage.

55. Rapid Change in Follower Velocity.—Numerous occasions arise when sudden rapidity is needed, as, for example, in opening the steam ports of engines to give full and unimpeded entrance to the steam, and free exit for the exhaust. This is

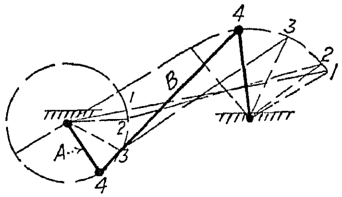


FIG. 97.—Rapid velocity change in follower. From 2 to 4, the follower moves very rapidly.

accomplished through linkwork, notably in the case of the wrist-plate of the Corliss valve mechanism. In Fig. 97, A is the driver oscillating at a reasonably uniform velocity, but imparting, at a portion of the stroke, a rapid increase in the follower. By taking advantage of this rapidity at the start, the valve is given a swift turn at the opening, and is slowed down during the expansion and compression periods. This

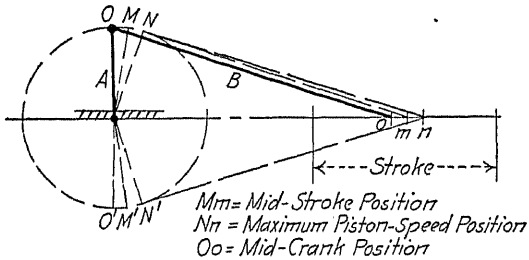


FIG. 98.—The piston displacement.

device is taken advantage of in the automatic feeding of various machines which use rod or bar stock; for example, rivet and bolt-making machines. A certain type of slotting machine utilizes a modification of this chain for its quick return.

56. The Problem of Piston Displacement.—The motion of the crosshead of a steam engine, and therefore of the piston, is reciprocating translation, each stroke being twice the length of

the crank throw. The velocity of the piston is not harmonic, but its maximum is attained when the crank positions are at N and N^1. When the crank is at its middle point, 90 deg. the piston is off center, at o, a distance om from its center point. Thus, on the forward stroke **more** than half is traveled in a quarter-revolution of the crank, and on the return stroke, **less** than half is traveled in the first quarter-revolution. In steam engines this means that different valve settings are required for the two strokes to equalize the steam cards; *i.e.*, to give uniform power on both sides of the piston. This **displacement of the piston** is lessened by increasing the length of the connecting-rod in proportion to the crank throw. To attain a value of $om = 0$, the length of the connecting-rod must be infinite.

There are valid objections to a large disparity in the lengths of connecting-rod and crank. First, the compactness of design

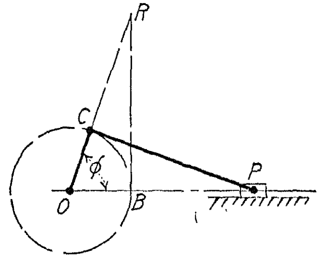


FIG. 99.—To locate the maximum piston velocity.

must be considered, and more than that, as the length of the connecting-rod increases, its weight must be more than proportionally increased. This is true, since its cross-section must be increased with the length increase, to prevent buckling. This weight increase is of very serious detriment in reciprocating members, because the inertia of these members robs the engine of considerable power with each reversal. This reciprocation, moreover, is in two directions, longitudinally for the piston, crosshead, and connecting-rod, and transversely for the connecting-rod as it oscillates, and this causes the chief shaking force in the engine. Reasonable limits for the connecting-rod have been found to be four to six times the crank throw. In single-acting gas engines, automobile engines for example, the power strokes are all forward (toward the crank), and this displacement is of no consequence, so that the connecting rods

may be made relatively short. Figure 99 shows a graphical method of obtaining the position of maximum piston velocity. Since this position is at the point where the crank and connecting-rod are perpendicular, an equal triangle to OCP can be drawn by laying out a tangent $BR = CP$, the connecting-rod, and completing the triangle OBR. The point C on the crank-pin circle will be the crank position of maximum piston velocity. The mathematical expression is, $\tan \phi = \frac{\text{conn. rod}}{\text{crank}}$.

At this point acceleration is zero, crank effort = thrust, and bearing pressure = zero.

57. The Scotch Yoke.—In machines where harmonic motion is a requisite, the **connecting-rod of infinite length,** or **Scotch Yoke,** is used. Observation of Fig. 100 will show that its velocity

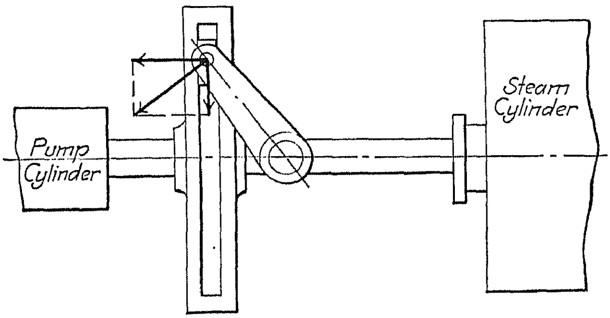


FIG. 100.—The Scotch yoke used as a pump connecting rod.

is harmonic, symmetrical about a vertical axis. While it has this advantage, both strokes alike, and also the advantage of compactness, it has never found favor in steam or gas engine design, because it is not adapted to rapid strokes, and is inefficient and subject to wear. It has, however, found a useful place in fire engine and donkey steam pump design, and in various machines, as the harmonic analyzer, where its particular motion is necessary. As a link chain, it is a variant on the **crossed-guide double slider.**

58. The Eccentric.—In order to produce **small travel** in the slider, the device called the **eccentric** is usually adopted. See Fig. 101. If a circular disc A, be mounted on a shaft at O, away from its geometrical center O', the result on the slider is the same as that of a crank whose throw is OO'; that is, the slider

travel is twice OO'. It is usually more convenient to employ the eccentric than to place a crank of small throw on the crank shaft. Moreover, by making use of the swinging eccentric (see Art. 60), the slider travel can be changed to suit conditions, an impossibility with a rigid crank.

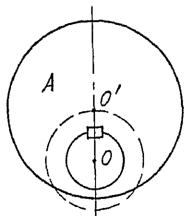


FIG. 101.—The eccentric.

In effect the eccentric is an enlarged crank-pin with a radius exceeding the crank throw.

The most important application of the eccentric is in the steam engine, where it is often the driving crank of the steam valves. The usual design is shown in Fig. 102. The eccentric is keyed to the crank shaft, and is surrounded by a rigid band (called a sheave), which allows the eccentric to turn freely inside. The sheave is connected rigidly to a connecting-rod (called the valve rod), which imparts to the

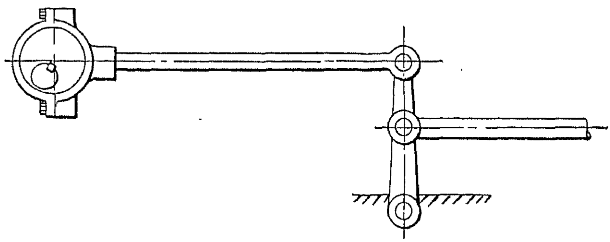


FIG. 102.—Eccentric, sheave, rod, and reducing rocker.

valve, either directly, or through a rocker, a stroke much shorter than that of the piston.

The eccentric is also much use as da cam, but its operation in that function is quite different from its operation as a crank.

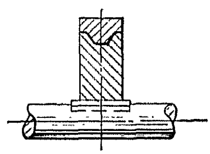


FIG. 103.—Sectional view of eccentric and sheave.

59. Reversing the Eccentric Stroke.—In operating locomotives, it is necessary to reverse the wheel drive by changing the inlet of steam into the valves. Since both crank-pins can never be on dead center (Art. 49), this is possible, and a reversing link like the sketch Fig. 104 is employed. Two eccentrics, A and B, are keyed to the same shaft in opposing positions, and the sheave rods are pinned to a slotted link C, which carries a sliding block attached to the valve rod. When the block is in mid-position, as in the

sketch, there is not enough motion given the valve to open the parts to steam, and the engine is throttled. A lever in the locomotive cab operates the block, and, as the block is moved toward 1, the valve is operated by the eccentric *A*. As it is moved toward 2, the valve is worked by the eccentric *B*, and the wheels are turned in the opposite direction. The farther from the mid-point the block is moved, the more steam will be admitted. Thus both speed and direction are controlled by the throttle lever, acting to move the block in its slot.

60. Swinging Eccentrics.—In stationary engines, a reversal is not necessary, as all machinery is designed to reverse by gear or belt shifters, but some sort of an automatic governor is required. There are two varieties of governors in general use,

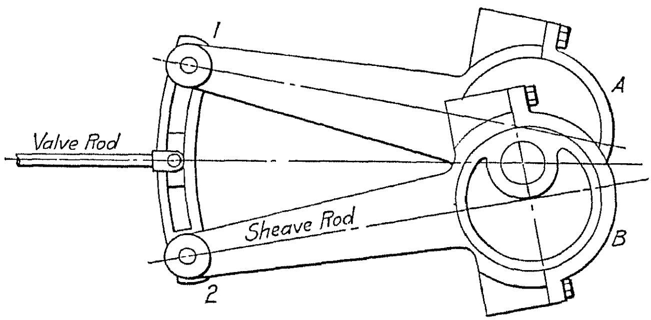


FIG. 104.—Reversing eccentric link.

the flyball governor, and the **flywheel governor.** Both are used to throttle the steam when the engine travels too rapidly. In some engines both governors are used, one to regulate the steam flow from the boiler, and the other to regulate it through the valve action. Both operate because of centrifugal force.

There are many designs of flywheel governors, but the operating principle is always the same. Since the length of the valve travel controls the steam admission, the way to control the steam admission is to control the length of the valve travel. The valve travel is equal to **twice** the throw of the eccentric (*i.e.*, the eccentricity), so by altering the eccentricity, the valve travel and steam admission are altered.

The problem, then, is to regulate the eccentricity so as to give maximum port opening to start the engine, and then to shut off the steam supply gradually, as the engine gathers

speed and momentum, and finally to cut down the steam supply to the point of throttling, if the engine speed increases beyond the point of safety or efficiency.

To do this, the eccentric is mounted on a swinging arm, instead of being keyed to the shaft. By the motion of this arm, the center of the eccentric is brought nearer the center of the flywheel shaft. For starting purposes, the eccentricity is the maximum, and for throttling it is smaller (not necessarily zero). Heavy weights, held back by springs, are attached to the swinging arm, and the entire mechanism is attached to the flywheel.

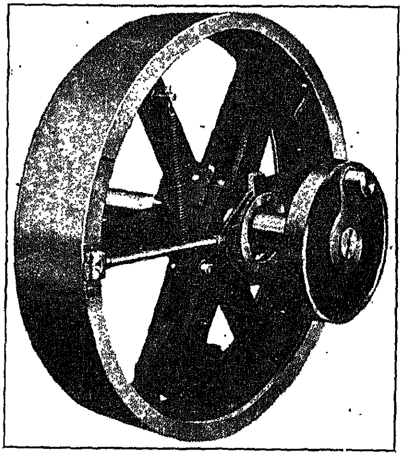


Fig. 105.—Flywheel governor with swinging eccentric. The Brownell Co., Dayton, Ohio.

When the engine is at rest, the weights are held against shoulders by the springs, and the eccentricity is such that steam can be admitted throughout about three-quarters of the stroke. As the flywheel is revolved, the weights are forced toward the rim, and the eccentric is brought nearer the center of the shaft. There are a great many designs, some with coiled springs and some with leaf springs, but the specimen shown in Fig. 105 is a good representative, and illustrates the operation. If the problem is carefully worked out, the result is a smooth-running and safe engine.

ROCKER ARMS AND BELL CRANKS

61. The rocker arm is often utilized in effecting changes in direction and length of stroke, as well as in increasing the lever-

age, on reciprocating parts. Figure 106 shows a steam engine employing rocker arms for this purpose. The skeleton sketches in Figs. 107 and 108 show two typical and simple means of

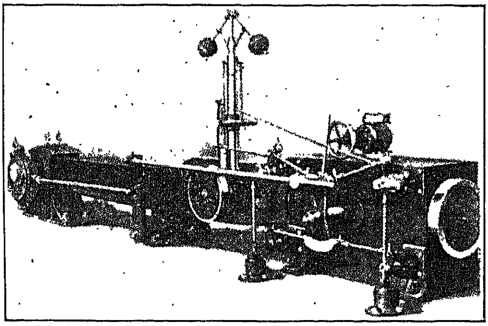


FIG. 106.—Corliss steam engine showing rocker arm between eccentric and valve rod. Notice the quick velocity change linkage in the valve motion. Hooven, Owens, Rentschler Co., Hamilton, Ohio.

accomplishing this result. These devices act also to transmit the motion from eccentric to valve, when the travel of the latter is out of line with the center of the crankshaft. The difference between the two arrangements is that (*a*) reverses

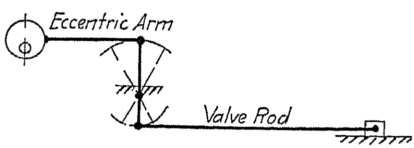


FIG. 107.—Rocker arm between eccentric arm and valve rod.

the direction of valve and eccentric. Both arrangements may be made reducing or enlarging.

A bent rocker is called in text books a **Bell Crank,** but the distinction is seldom made in ordinary practice, and it is called

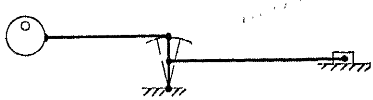


FIG. 108.—Rocker arm between eccentric arm and valve rod.

a rocker, whether straight or bent. To lay out such a link, to transmit reciprocation along **intersecting** lines, instead of parallel, is shown in Fig. 110.

Example.—Lay out a bell crank to transmit motion from a given line (direction AB) to a direction 30 deg. from the given line, at a relative velocity of 2 : 3.

(1) Draw a line AC making 30 deg. with AB.

(2) Make $AB = 3$ and $AC = 2$, the inverse ratio of their velocities.

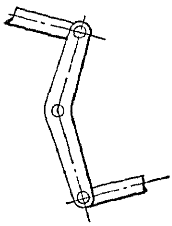


FIG. 109.—The bell crank.

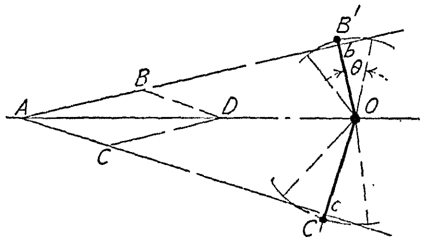


FIG. 110.—The layout of the bell crank.

(3) Complete the parallelogram $ABCD$.

(4) Draw the diagonal AD, and locate O. the center of the bell crank, wherever desired on AD.

(5) Draw OB' perpendicular to AB, and OC' perpendicular to AC.

(6) Make OB' greater than Ob, and OC' greater than Oc, each by an amount equal to the product of half the versed sine of the angle on either side of the central position of the crank and Ob and Oc, respectively.

$$(OB' - Ob = \tfrac{1}{2} \text{ vers. sin } \theta \times Ob)$$

This will reduce to a minimum the angular deviation of the connecting-rods.

STRAIGHT LINE MOTIONS

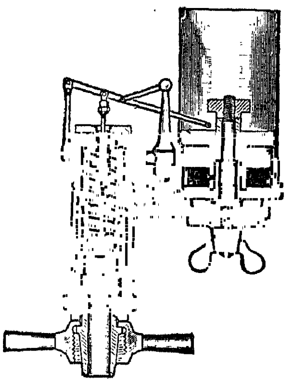


FIG. 111.—Thompson indicator. American Steam Gauge & Valve Mfg. Co., Boston.

62. A straight line can be **ruled,** or it can be **generated.** Many link chains have been devised to compel a point to travel a straight path without the assistance of guides. The chief employment for **straight-line motions** is in steam indicators, but in former years they were employed in the steam engine link trains.

The **Steam Indicator** is an indispensable adjunct to every power plant and steam engine laboratory. Its function is to give an accurate record of the steam pressure in the cylinder at every phase. An examination of Fig. 111 will show the operation of the straight line motion as it is applied to this particular indicator.

The instrument is screwed into a tapped hole in the cylinder, leading into the clearance space. A spring in one of the indicator cylinders carries a piston, and this resists the pressure of the steam. The piston and spring are compressed upward in the cylinder in proportion to the steam pressure. For example: if 40 lb. steam pressure shortens the spring $\frac{1}{2}$ in., 80 lb. will shorten it 1 in. This motion of the piston actuates the straight line link work in such a way that the point of a pencil attached to it is given a straight line travel.

A second cylinder, carrying a sheet of paper, in touch with the pencil point, is partially rotated by a cord attached to a hook on the crosshead. The spool carrying the cord is geared to the cylinder, so that the length of the arc traveled by the paper is a definite fraction of the engine stroke. A spring returns the paper (called the **Indicator Card**) to its original position. The pencil traveling as it does in a vertical straight line, records on the card its displacement due to the steam pressure, as the card moves past the pencil through a distance representing to scale the piston stroke.

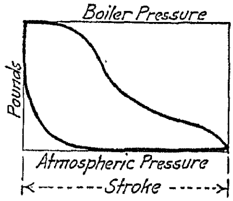


FIG. 112.—Typical indicator card.

Figure 112 shows a typical diagram. These diagrams are indispensable in analyzing the work done inside the cylinder, and in studying the setting of the valves, to insure best results from the steam. The area of the card represents the amount of work (mean steam pressure × length of stroke) done inside the cylinder. By dividing this area by the stroke, the **mean ordinate** is obtained. The pressure expressed by this mean ordinate is called **Mean Effective Pressure** (m.e.p.), distinguishing it from the boiler pressure. By means of the m.e.p. the i.hp. of the engine is calculated. The d.hp. is usually measured by a Prony Brake, and it is often termed Brake Horsepower.

It is of utmost importance that the straight line motion operating a steam indicator shall not only work in a straight line at the pencil point, but also that the distances of pencil travel shall be in proportion to the compression of the spring.

Figure 113 shows a few of the numerous link chains for this purpose. More or less elaborate treatment of this form of link work, together with proof of their accuracy, where such exists, is found in many books on Kinematics; for example, in Barr

and Wood's "Kinematics of Machinery," Keown's "Mechanism," and McKay's "Theory of Machines."*

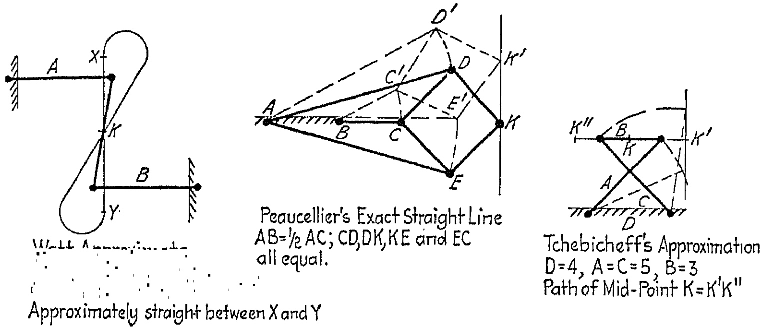


FIG. 113.—Straight line linkage.

THE PANTOGRAPH

63. The diagram in Fig. 114 shows that the lines AP and AK in the parallelograms $ABEF$ and $ABCD$ will always lie in a straight line, and will always be in the same proportion, whatever the angles of the parallelograms may be. **Why?** This

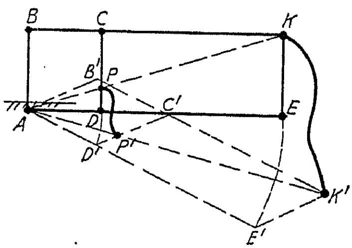


FIG. 114.—Pantograph.

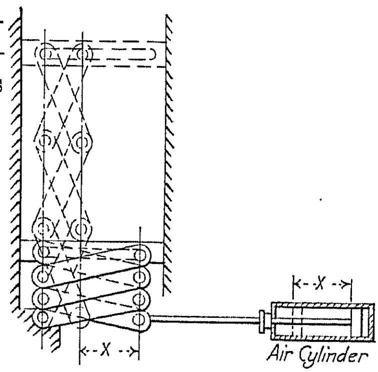


FIG. 115.—Pantograph elevator.

means that the path PP' will be an enlargement of the path of KK'; that is, it will be correctly magnified.

This device is principally used as an instrument for enlarging drawings and pictures. The pencil point if placed at P, the

* KEOWN, ROBERT MCA., "Mechanism," McGraw-Hill Book Co., New York.

BARR, J. H., and WOOD, E. H., "Kinematics of Machinery," J. Wiley & Sons, New York; MCKAY, R. S., "Theory of Machines," Longmans, Green & Co., New York.

tracing stylus at K, and the lines of the drawing traced by the stylus will be reproduced in enlargement by the pencil P.

If the pantograph were limited to the function of enlarging pictures, it would scarcely merit consideration here. A device is shown in Fig. 115, where the same mechanism is used to lift material from one level to another. One end of the base is anchored to the frame on a fixed pivot, and the companion bar is moved by the piston rod of a pneumatic cylinder. The platform slides in guides, and its vertical travel may be consider-

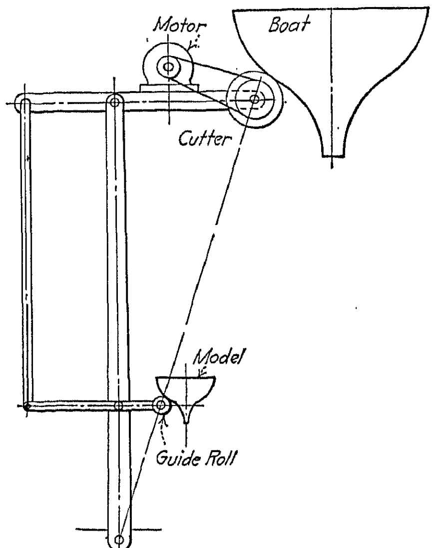


FIG. 116.—Boat planer.

ably more than the horizontal travel of the piston. By making the top of the platform tilting, the load can be dumped when it reaches the top.

Another application in a recent invention is a boat planer, operated as in the sketch, Fig. 116. To finish the outer surface of boats to conform to a certain model, a circular cutter is mounted on an arm of a pantograph, and driven by an electric motor. On an arm nearer the pivot a guide roll proportionate in diameter to the cutter is placed. As this roller is guided over the surface of a smaller model of a boat hull, the cutter finishes a surface identical in its proportions to the model. As each

segment is finished on the boat and the model, the cutter and roller are moved along their axes a distance equal to their respective widths.

Duplicate wood carvings for furniture are manufactured by a similar device, although in this case the hand-carved model is made the same size as the manufactured product.

The pantograph is used as a telephone extender, as tongs, and has been tried as a straight line chain for steam indicators, but was discarded because the joints received too much wear, and lost motion (fatal to indicators) resulted.

The same principle, parallelism, not enlarging, is used in the Universal Drafting Machine, which has received great approval

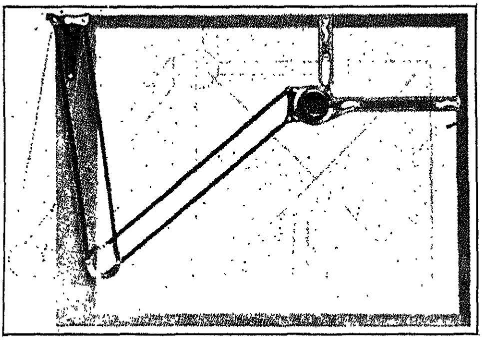


FIG. 117.—Universal drafting machine. C. F. Pease Co., Chicago.

from draftsmen, both in schools and industrial plants. It replaces the ordinary T-square and triangle. Undoubtedly the designer of the future will discover many new uses for the pantograph.

PROBLEMS

Note.—Some of these exercises are given with definite data, and are followed by the same problem with blank data to be supplied by the instructor.

1. Design an oscillating beam quick-return chain to give a 12-in. stroke, with time ratio of 3 : 1 for maximum travel. What is the maximum length of crank throw? What is the time ratio for a 6-in. stroke? Draw the velocity diagram for full length stroke.

2. Same Problem. Quick return mechanism ——— (selected from Art. **53**). Stroke ——— in., time ratio ——— (not exceeding 4 : 1).

3. Design a quick-return six-link mechanism, using a different combina-

tion of links from any examples illustrated in Art. **53**. Find its time ratio of strokes and draw the velocity diagram.

4. Design a linkage like the one in Fig. 118, with a rocker or bell crank C of such proportions that E will travel a forward stroke of 12 in., a short stroke of 3 in. (forward and back) and a full return stroke, for one revolution of A. Give full dimensions. Draw the velocity diagram of E.

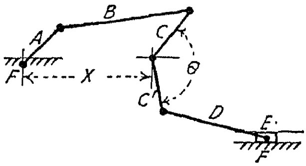


FIG. 118.—Problem 4.

5. Same problem. Forward stroke ——— in., short stroke ——— in.

6. Same problem with no short stroke. Stroke to be ——— in. If A rotates ——— r.p.m., give maximum velocity of E, angle of oscillation of C, and draw velocity diagram of E. What is time ratio of strokes?

7. Show with diagram why the centros *af* and *cf* in Fig. 95 should not be in a vertical line. What angle with the vertical will give the best results?

8. Take the data for one of the layouts for Problem ———, Chapter II, assigned by the instructor, and determine whether it is an effective arrangement of the chain. Make a layout that will be more effective, with dimensions.

9. Design an elliptical chuck. Give distance between centers of rolls, and distance from chuck center to tool point to turn an ellipse of 3-in. minor axis and 5-in. major axis.

10. Same problem. Minor axis ——— in., major axis ——— in.

11. How much is the piston displacement in a steam engine of 14-in. stroke, in which the length of the connecting-rod is 2, 2½, 3, 3½, and 4 times the stroke? In each of these cases, at what angle of the crank and at what fraction of the stroke is the maximum piston velocity? Solve graphically and mathematically.

12. Same problem. Piston stroke ——— in.

13. A foot-power grindstone is shown in Fig. 119. Treadle at P makes 20 strokes per min. $R = 3$ in. Required the diameter D of the stone to give a velocity of 350 ft. per min. at the surface. Required, the average foot velocity.

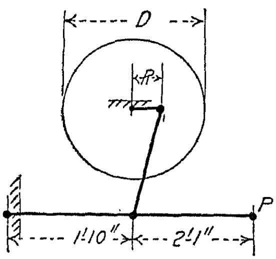


FIG. 119.—Grindstone. Problem 13.

14. Same problem. Strokes of treadle ——— per min., rim velocity of stone ——— ft. per min.

15. Same grindstone. $D = 24$ in. If the foot velocity is 40 ft. per min., what is the grinding velocity?

16. Same problem. $D =$ ——— in. Foot velocity ——— ft. per min.

17. An eccentric of 2-in. throw is placed on a crankshaft 6 ft. from the center position of the steam valve. It is desired to use a reversing rocker which will reduce the valve travel to 1¼ in. Make a sketch of the mechanism, with a design for the rocker. Compare the valve motion thus imparted with pure harmonic motion.

18. Same problem. Eccentric throw ——— in., distance ——— ft., valve travel ——— in. Make the rocker reversing or direct acting at instructor's option.

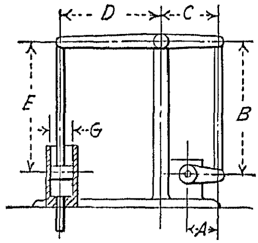


FIG. 120.—Problem 19.

19. Six link pump. Fig. 120. Data for various sizes:

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(j)	(k)	(l)	(m)	(n)	(o)	(p)
A =	2	2	2	3	3	3	4	4	4	5	5	5	6	6	6
$B = E$ =	16	18	20	22	25	28	35	40	44	35	45	50	50	55	60
C =	8	9	10	10	11	12	12	13	15	16	18	20	20	24	26
D =	16	18	20	22	25	28	32	30	34	30	32	35	40	44	48
G =	2	2	2	3	3	3	4	4	4	5	5	5	6	6	6
r.p.m. =	200	180	160	140	120	80	70	60	60	50	40	40	35	30	25

Required: (a) Stroke of Plunger.
(b) Velocity of Plunger at ——— deg. phase of A.
(c) Gallons capacity per hr.

20. Watt straight line, Fig. 121. $A = B = \frac{1}{2}C$. Use any reasonable scale. Plot path of P (mid-point of C) through a complete cycle, and give the limits of oscillation of A and B to yield an approximately straight line.

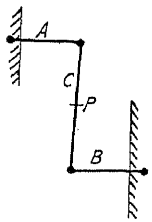


FIG. 121.—Problem 20.

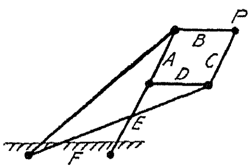


FIG. 122.—Problem 21.

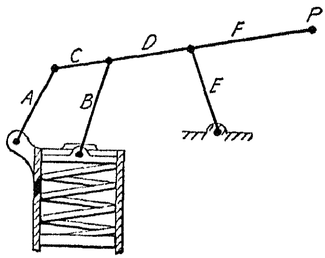


FIG. 123.—Problem 22.

21. Peaucellier's Straight Line, Fig. 122. Plot path of P through a complete cycle and give the angle through which it works. Prove geometrically that the path of P is a straight line. Use any reasonable dimensions.

22. Thompson Indicator. Let $B = C = D = E$, and $A = 1\frac{1}{4}\ B$, and $F = 2B$. Choose pivots for A and E. Plot path of P, using equal units of piston travel. Compare the units of P-travel with the corresponding units of piston travel.

23. Design a lifter similar to Fig. 115 to give the platform a lift of 6 ft., while the piston travels 18 in. Make a working drawing of the entire machine, and design a tilting arrangement for the platform.

24. Same problem. Lift = ———— ft. and piston travel = ———— in.

25. Design a pantograph for triple enlargements.

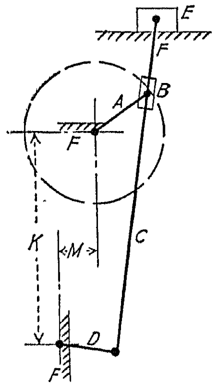


FIG. 124.—Problem 26.

26. Averbeck Shaper, Fig. 124. Data.

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(j)	(k)	(l)	(m)	(n)	(o)	(p)
$A = M =$	3	3	3	4	4	4	5	5	5	6	6	6	7	7	7
$C =$	18	20	22	20	24	26	28	36	40	36	39	42	40	45	48
$D =$	4	4	4	5	5	5	6	6	6	7	7	7	8	8	8
$K =$	12	14	16	12	15	18	16	20	23	20	24	26	24	26	28

Required: (a) Stroke of E.

(b) Ratio of advance to return of E.

(c) V^E at ———— deg. phase of A, A running ———— r.p.m.

(d) Velocity diagram of E.

BIBLIOGRAPHY

REULEAUX. "Kinematics of Machinery," an old book, but well supplied with material for the advanced student.

FESSENDEN. "Valve Gears." McGraw-Hill Book Co. Many examples of link work not treated in this chapter are given in this book.

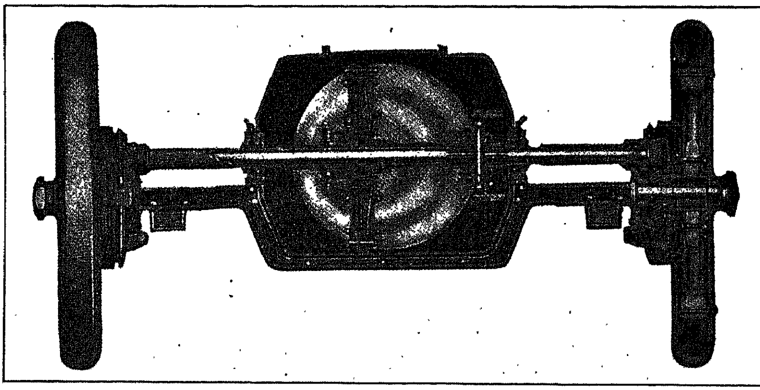


FIG. 125.—Kelsey automobile rear axle friction drive. Kelsey Motor Co. Newark, N. J.

CHAPTER IV

DIRECT CONTACT PAIRS, ROLLING CURVES, FRICTION TRANSMISSION

64. Direct contact pairs would include with propriety all transmissions working on two shafts without an intermediate link or belt. That is, rolling curves, friction wheels, cones, gears, cams, ratchets, etc., would all come under this head, but

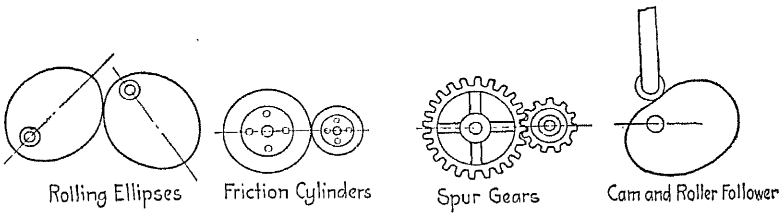


FIG. 126.—Examples of line-contact pairs.

it is advisable to treat most of these drives under separate headings, and to limit this chapter to the consideration of rolling curves and friction pairs.

65. Line and Surface Contact.—The examples shown in Figs. 126 and 127 illustrate better than any definition the two

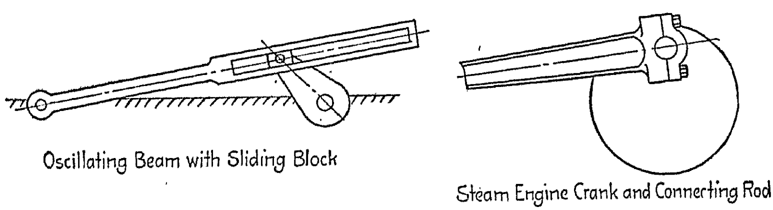


FIG. 127.—Examples of surface-contact pairs.

main classes of direct contact pairs. Ball bearings are theoretically **point contact,** but more often the point of contact is a surface of measurable area. These classes were formerly called higher pairing and lower pairing, but the terms are confusing

and pointless, and have been abandoned by most teachers in favor of the more expressive line and surface contact pairs.

Generally speaking, there is less friction, but greater wear, in line than in surface contact, but in many cases it is possible to get both ease of running and durability from a line contact design. Where a choice is necessary, it is the business fo the engineer to consider the balance of advantage of one form or the other. Where possible, ball bearings should be used, as they give almost perfect results in eliminating friction. They wear astonishingly well, due to the materials, heat treatment, and accurate finishing of balls and ball races.

66. Definition of Rolling, Slipping and Positive Drive.—**Rolling** (called also **pure** rolling) takes place when the consecutive elements of both of the contact surfaces come into contact, each with each, in regular order. To meet this condition, **the tangential velocity of any two contact points must be equal and in the same direction.** In addition to this, it is obvious that **the length of any contact arc of one member must equal the length of the corresponding contact arc of the other.**

Slipping takes place when one of the contact points **is moving tangentially faster than the other, or in the opposite direction.** Thus, the determination of the rate of slipping is the **algebraic difference** of the tangential velocities of the contact points.

Positive Drive is said to occur when the follower **is compelled to move or be broken,** to allow motion of the driver.

Note.—This distinguishes it from friction drive, since overloading on a friction drive causes the follower to remain stationary, or nearly so, while the driver continues its motion. In direct contact members there may be positive drive throughout the cycle, through part of it, or not at all. These pairs may also have rolling, slipping, or a combination of rolling and slipping.

67. The Angular Velocity Ratio in Circular Pairs.—Two circular wheels, A and B, of radii r and R, are shown in contact at P. Pure rolling between them is assumed, therefore the linear velocity of all points on both rims will be equal.

$$\therefore\ V^{PA}=V^{PB};\ \text{from which } \omega^{A}=\omega^{B}\frac{R}{r}.$$

Rule.—In circular pairs the angular velocities are inversely proportional to the radii.

This rule also applies to internal contact circular pairs. Let the student show this with a figure and prove the equation.

Example.—Two friction wheels are 8 in. and 20 in. in diameter. If the small wheel runs at 700 r.p.m. what is the speed of the larger?

Answer.—$\omega^B = \frac{8}{20} \times 700 = 280$ r.p.m.

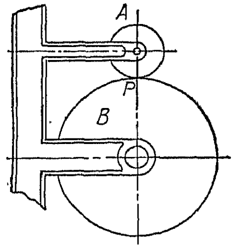


FIG. 128.

68. The General Rule for the Angular Velocity Ratio of Direct Contact Pairs.

Let A and B (Fig. 129a) be two bodies rotating respectively about O and O' as centers. At the instant they are in contact at P. It is assumed that the drive and contact are constant and continuous, which means that there shall be no **overlapping**

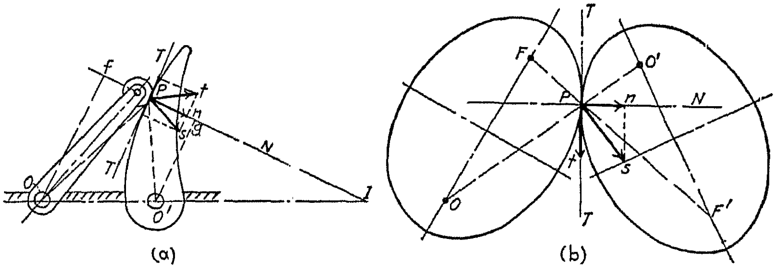


FIG. 129.

or **pulling apart,** although the point of contact may be constantly shifting.

The point P (P^A and P^B) is rotating about both centers O and O', and has, therefore, separate motions and velocities in the direction of Ps and Pt. These velocities have normal and tangential components along the common normal and tangent to the two curves at P.

The normal velocities of the two points must be equal and in the same direction. If this were not true, the contact would not be constant and continuous, which would be contrary to the hypothesis.

If a value is assigned to Pt, the value of Ps would be known, since their directions are known, and they have the same normal component, Pn. This gives the instantaneous velocity ratio between them,

$$\frac{\omega^A}{\omega^B}=\frac{\frac{Ps}{OP}}{\frac{Pt}{O'P}}$$

How is this equation derived?

To bring this complex equation into simple terms, draw perpendiculars from O and O' to the common normal, Of and $O'g$. From similar triangles (why?), we find:

$$\frac{\frac{Ps}{OP}}{\frac{Pt}{O'P}}=\frac{\frac{Pn}{Of}}{\frac{Pn}{O'g}}=\frac{O'g}{Of}$$

Rule.—The angular velocity ratio of two direct contact bodies is inversely proportional to the perpendiculars from their centers to their common normal.

A simpler equation, and usually more convenient, is derived by extending the common normal until it meets the line of centers, OO', at I.

By similar triangles (why similar?),

$$\frac{O'g}{Of}=\frac{IO'}{IO}\therefore\frac{\omega^A}{\omega^B}=\frac{IO'}{IO}.$$

Rule.—The angular velocity ratio of two direct contact bodies is inversely proportional to the segments of the line of centers made by the intersection of the common normal.

This rule applies to gear wheels, cams, frictions, and rolling surfaces.

Example.—Refer to Fig. 129*b*. Two rolling ellipses are shown, mounted on their foci, O and O', in contact at P, which is on the line of centers. P is therefore I, and the ratio of their angular velocities is $\frac{\omega^A}{\omega^B}=\frac{O'P}{OP}$; that is,

the inverse ratio of their contact radii. This is the rule, as will be shown later for all rolling surfaces.

Note.—The foregoing rule applies to link work and flexible band drive, if line of action be substituted for the words common normal. See Barr's "Kinematics of Machinery" for detailed treatment.

69. Constant Angular Velocity Ratio.—Since O and O' are fixed points, the expression $\frac{IO'}{IO}$ can only remain constant if I be a fixed point. Therefore, for constant angular velocity ratio, the normal must intersect OO' in a fixed point. This condition must be recognized to understand the theory of gear teeth.

ROLLING CURVES

70. Condition of Pure Rolling.—If the algebraic difference of the tangential components of the velocities at the point of contact be zero; that is, if both points of contact are moving

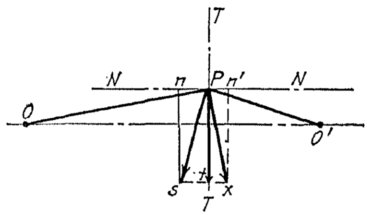


Fig. 130.

tangentially in the same direction and at the same velocity, **there will be no slipping.** By a previous hypothesis, the normal components must also be equal, in order that there shall be constant and continuous contact between the two members. In consequence of these facts, the only possible position of the point of contact is on the line of centers. Figure 130 shows this.

Assume P as the point of contact, then OP and $O'P$ are the contact radii. Let Ps and Px be the respective velocities of P^A and P^B, perpendicular to the respective radii. Since Pn and Pn' must be equal in amount and direction, and also Pt and Pt', it is necessary that OP and $O'P$ shall be one and the same straight line in order that Ps and Px shall coincide and be perpendicular to the line of centers.

This is shown to be true in the case of the rolling ellipses illustrated in Fig. 129*b*.

Conditions.—(1) The contact point must be on the line of centers.

(2) The sum of the contact radii must be equal to the dis-stance between centers.

(3) The contact arcs of both members must be equal in length.

71. **The Angular Velocity Ratio of Rolling Curves** is inversely proportional to the contact radii at any instant. This is true, because the common normal intersects the line of centers at the contact point.

POSITIVE DRIVE

72. **Positive drive,** compelling motion of the driven by the driver, may obtain in rolling curves, and it may not. It is produced by the normal pressure acting to force the follower from its position. If this pressure is in the direction of a fixed center, no turning force can be exerted, and if there is **no normal component,** or if the normal component is away from the drive, there can be no drive.

Rule.—Positive drive exists, only when the common normal does not pass through either fixed center.

Cases.—A circle mounted on its center can neither drive, nor be driven by a direct contact roller, because its normal is a radius, and passes through its center.

Other curves act positively when the normal acts toward the follower, but does not pass through its center. Rolling ellipses drive through half the revolution, but must be assisted through the other half by teeth, links, springs, or some other accessory.

If the free foci (F and F', Fig. 129*b*) were joined by a rigid link, positive drive would obtain throughout the revolution.

73. **To Lay Out a Curve to Roll on Another.**—It is possible to lay out a curve from any given center to roll on any curve or straight line rotating about some other center.

Given.—A circle mounted eccentrically at O, and the point O' outside of the circle.

Required.—To draw a rolling follower, with O' as its center. Use the conditions of pure rolling given in Art. 70.

(1) Divide the given curve into arcs, preferably equal in length.

(2) With complimentary radii to $O1$, $O2$, $O3$, etc., draw arcs from O'.

(3) With lengths equal to the chords P-1, 1-2, 2-3, etc., step off the points 1′, 2′, 3′, etc., on the arcs of O' successively.

(4) Draw a smooth curve through P, 1′, 2′, 3′, etc. This will approximate the true rolling curve.

Note.—The smaller the arcs are taken, the closer this approximation will be.

Note.—The derived curve in Fig. 131 makes one revolution to two of the given circle. It is by no means certain (the chances are much against it) that the follower curve for a center O' will

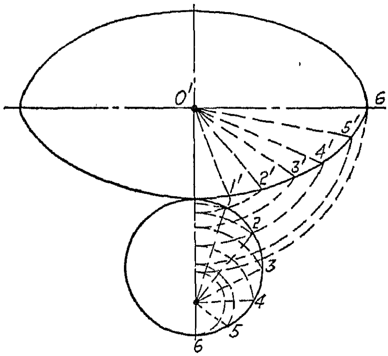


Fig. 131.

close. The approximate center position for a closed curve for the follower requires several trial centers before it can be determined.

ROLLING ELLIPSES

74. Two equal ellipses, mounted on their respective foci, at a distance equal to the major axis, possess all the conditions of pure rolling. (Let the student analyze the conditions, and their fulfilment in this case.) This pair affords an unusually interesting study in velocity ratios, quick return, high maximum speed, and the possibility of great utility, if intelligently employed. It has not received the attention it deserves from engineers and designers. The two objections urged against it are easily overcome. The first objection, that it is positive drive for only half a revolution, can be overcome by providing teeth all the way around; or on the necessary portion of the periphery, or by connecting the free foci by a rigid link, or many another device. The second objection obtains in the case of

elliptical gears, and is that elliptical gears are expensive and hard to make. This is no longer true. They can be made easily, accurately, and inexpensively with the powerful and ingenious modern machine tools. The same characteristics of motion and velocity can be obtained by using one of the quadric chains. (Which one?)

Since the peripheral lengths of the ellipses are equal, both members will make the same number of revolutions, but during each revolution there will be startling changes in the angular velocity ratio, varying from $\frac{FF' + F'B}{F'B}$ to its reciprocal. (Why?) Thus, where FF' is proportionally large, say $FF' = 2F'B$, the maximum speed of the follower will be three times that of the driver, and its minimum speed will be $\frac{1}{3}$. As FF' approaches zero (the circle) this disparity in angular velocity approaches

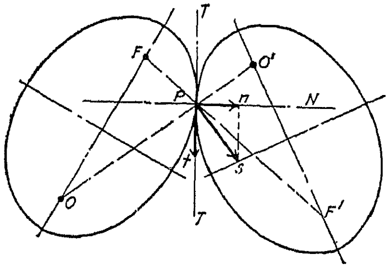


FIG. 132.

unity, and it would be unity in the case of two equal circles. Conversely, as the ellipse approaches flatness, the disparity approaches infinity.

It is evident from this that a remarkable quick-return motion can be designed to employ these characteristics. Also, on account of the extraordinary linear velocity of the point B', at the maximum follower velocity, a very powerful blow can be struck using that point as a hammer. This property has been utilized by at least one concern in the design of well-digging machines, and the quick return property has been applied to shapers, but its possibilities are capable of far more widespread application. Throughout this work there will be found a number of problems illustrating this pair, and the student will do well to examine the properties and possibilities of it, with a view to its adoption when the opportunity arises.

THE LOGARITHMIC SPIRAL

75. The Logarithmic Spiral, or curve of constant angularity, is occasionally used as a rolling member and frequently as a part of the outline of a cam. It is not a closed curve, consequently must be used as a reciprocator, unless it is made in the

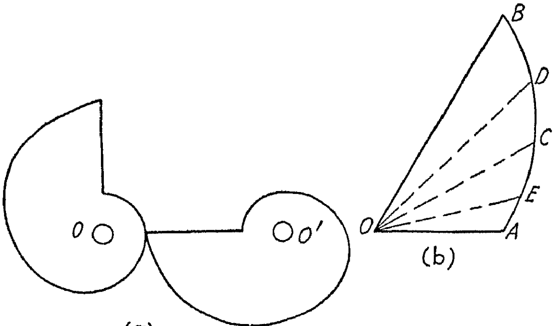


FIG. 133.—Rolling logarithmic spirals and their construction.

form of a lobed wheel, as in Fig. 134. Where such a curve is to be laid out between two radius vectors, the construction is made according to the sketch, Fig. 133*b*. Given *OB* and *OA*, to construct the spiral. Draw the bisector *OC* of the angle *AOB*, and make its length the mean proportional between *AO* and *BO*. Then draw *OE* and *OD*, bisectors of *AOC* and *BOC*,

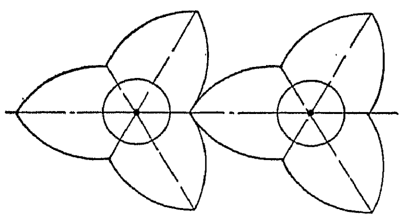


FIG. 134.—Lobed wheels with logarithmic spiral outlines.

making their lengths mean proportionals between *AO* and *OC*, and *OC* and *OB*. The chief property of this curve is that the tangent at any point always makes the same angle with its radius.

When this curve is used as the outline of a pair of lobed wheels, a rapid fluctuation of the speed of the follower will occur, as an examination of the sketch will show. On account of the

violence of this change of velocity, these wheels are seldom used. A very exhaustive study of these and similar curiosities will be found in MacCord's "Kinematics," and sufficient information for all purposes in Barr and Wood.

FRICTION WHEELS

76. Friction drives are frequently found in transmission where compactness, simplicity, high speed, gradual starting, or flexibility of speed changes (sensitiveness) are needed. Such drives are chiefly limited to **rolling cylinders, rolling cones,** and **disc wheels.** Other rolling surfaces, such as rolling elliptical cylinders, and rolling hyperboloids, are possible, and may find

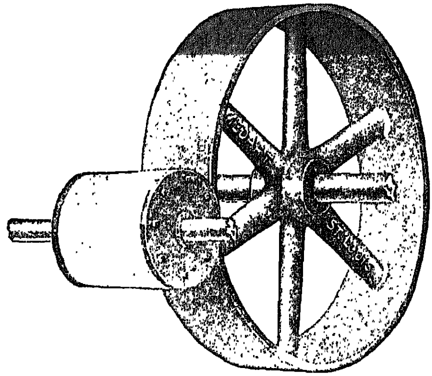


FIG. 135.—Spur friction gearing. The Medart Co., St. Louis.

application, but to this writer's knowledge they have received only slight attention from machine designers.

A distinct place for friction wheels and bevels has been found in hoisting apparatus, wood-working machines, sensitive drills, grinders, variable speed machines, intermittent motions, quick reversals, and in several makes of automobiles. The latest automobile to use friction disc drive is the Kelsey, and the transmission of this car is shown in a cut furnished by them.

Friction drives (belting, etc., included) only transmit power to the slipping point. This results in advantage and disadvantage. The main advantage is that overloading the transmission results in stopping the machine without serious injury. This is also its main limitation, and is what keeps it in the class of light machinery. Another limitation is the uncertainty

that it will guarantee a **definite speed** to the follower, since there is nearly always a small and variable amount of slippage. This loss amounts to about 2 per cent in most transmissions. Therefore, for heavy duty and for definite speeds, the friction drive is unsuited, and some form of positive drive must be employed.

The advantages of friction wheels may be summed up as quiet running, gradual starting (making it possible to apply the power with least disturbance), sensitiveness, and freedom from

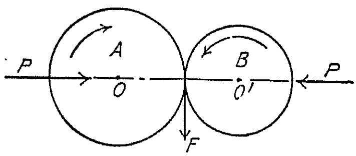


FIG. 136.

accidents through overloading. In positive drive, the machine must go, or something must break when an overload is imposed.

PRESSURE REQUIRED FOR FRICTION DRIVE

77. It has been shown that there is **no drive** in tangent circles. Hence two perfect cylinders, perfectly smooth, without pressure of surface to surface, would be uninfluenced by the motion of one another. In order to obtain a drive from one to another, there must be a deformation of the surface to deflect the common normal from the center. This is brought about by pressure (called **bearing pressure**) acting from one surface to the other, and the resulting effect is called **Friction.** In this case friction is the great agency for transmission, but in bearings it is the great enemy of efficiency.

From this it is evident that some outside agency must be employed to force the follower to take the desired motion. Gear teeth will do it, and links can be made to do it, or the centers can be moved and a crossed belt will suffice. Very acceptable drives, however, can be produced by applying pressure, by take-ups such as are illustrated here, or by other means. Referring to Fig. 136, the contact line between the cylinders A and B becomes a contact surface, and the normal is diverted from the center, and a transmission results, called **friction transmission.**

THE COEFFICIENT OF FRICTION

78. If a body of weight W, resting on a surface, is acted upon by a force F, the tendency of motion of the body is in the direction of their resultant. In a static condition the weight W is opposed by an equal reaction R, and the force F is opposed by a reaction resulting from certain resisting qualities of the supporting surface. This resistance depends upon the character of both surfaces, their smoothness, hardness, and their mutual attractiveness. On account of the impenetrability of the supporting surface, the motion of the weight must be in the direction of the tangent to the surfaces in contact, horizontally in the case illustrated. Since the resistance to be overcome is equal to the force necessary to overcome it, the ratio between this weight and the force necessary to move it is taken as the measure, or coefficient, of this particular pair of surfaces. This ratio is called the **coefficient of friction** (μ), and $\mu = \frac{F}{W}$. Thus, if W is increased or decreased, F will also increase or decrease in proportion. Until motion is accomplished, the resistance is called **static friction.** When W is in motion, the resistance is less, and it is called **kinetic friction.** The value of μ for the various surfaces is based on the static friction.

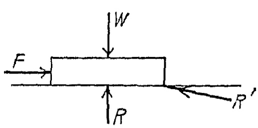


FIG. 137.

Generally speaking, the coefficient of friction is greater, if one of the bodies is softer than the other, than if they are of equal hardness. In friction drives it is usual to make one wheel of softer material, but it must possess sufficient elasticity to regain its normal shape after being deformed by pressure.

If the body rests on an inclined plane, it will remain at rest unless the angle of inclination is greater than the angle of repose, in which case the weight will slide down the inclination. The angle of repose is that in which $\tan \theta = \mu$.

FRICTION CYLINDERS

79. There is a large and increasing employment of friction cylinders in many transmissions where a light load is to be moved by a high speed driver, and where the necessity for exact speed ratios is not present. The speed ratio is inversely proportional to the diameters. The amount of horsepower trans-

mitted depends (1) on the materials used (their coefficient of friction), (2) the bearing pressure, and (3) the peripheral speed. In Fig. 138, $F = P\mu$.

Example.—To find the horsepower capacity of a friction drive of two wheels, 16 in. and 6 in., running at 300 and 800 r.p.m. Assume $\mu = .20$ and $P = 200$ lb.

$$\text{hp.} = \frac{2\pi RNP\mu}{33{,}000} \therefore$$

$$\text{hp.} = \frac{2\cdot 22\cdot 8\cdot 300\cdot 200\cdot 20}{7\cdot 12\cdot 33{,}000\cdot 100} = 1.5.$$

Note.—This shows the comparatively small horsepower for this transmission. A pair of gears of this size would transmit 20 to 200 hp., according to their materials.

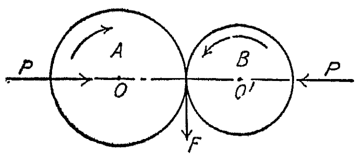


Fig. 138.

DESIGN OF FRICTION CYLINDERS

80. (1) The **driver** must be of some soft material and the driven of cast iron or some other hard material. If the materials were reversed, the driver would wear a flat spot on the driven wheel, when the latter is stationary in case of overloading. The materials used for drivers are paper, straw fiber, rubber, leather, leather fiber and tarred fiber filler. Material in the liquid state is subjected to heavy hydraulic pressure, and dried.

(2) Table I.—Working Values for Coefficient of Friction.*

Driver	Driven	μ	Driver	Driven	μ
Straw Fiber...	Cast Iron.....	.26	Paper........	Cast Iron.....	.20
Straw Fiber...	Aluminum....	.27	Tarred Fiber..	Aluminum....	.18
Leather Fiber.	Cast Iron.....	.31	Leather......	Cast Iron.....	.14
Leather Fiber.	Aluminum....	.30	Leather......	Aluminum....	.22
Tarred Fiber..	Cast Iron.....	.15	Wood........	Metal........	.25

*Values from Machinery's "Cyclopedia."

Note.—If the peripheral velocity of the wheels is greater than 1,000 ft. per min., or if the drive is to be started under load, smaller values than these should be used.

(3) For pressures on the various materials, for the purpose of making transmission, use the following values, per inch of contact width.

TABLE II.—PRESSURE VALUES PER 1 IN. OF CONTACT

	Pounds per inch of width		Pounds per inch of width
Straw Fiber	100	Leather	150
Leather Fiber	240	Wood	100–150
Tarred Fiber	240	Paper	150

Higher values than these must not be used, because higher pressures will compress the drivers to smaller diameters. These pressures can be carried by the wheels for long periods without compressing.

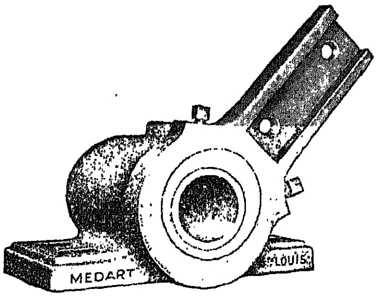


FIG. 139.—Eccentric take-up box. The Medart Co., St. Louis.

(4) A narrow face is preferable, because it is difficult to secure perfect contact in wide faces. The ratio of face to diameter should not exceed unity in the small pulley.

(5) Pressure devices are variously designed. A lever operated eccentric box, or thrust box, is the best way of controlling the pressure, since it can be thrown completely out when not in use, and can be made to vary with the load, increasing or decreasing as the load requires. Other devices of obtaining the

desired pressure are operated by a screw, sliding the bearing in guides, some are held in place by springs, and others are mounted in swinging supports.

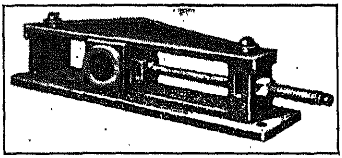


FIG. 140.—Screw take-up box. Jeffrey Mfg. Co., Columbus, O.

GROOVED FRICTIONS

81. To increase the transmission without increasing the bearing pressure, the expedient of grooving the wheels is sometimes resorted to. The **effective coefficient of friction** is increased inversely as the sine of half the groove angle; *i.e.*,

$$2\,F = \frac{P\mu}{\sin \frac{1}{2}\theta} \therefore \mu' = \frac{\mu}{\sin \frac{1}{2}\theta}.$$

(Let the student prove this, using the diagram Fig. 141*a*.)

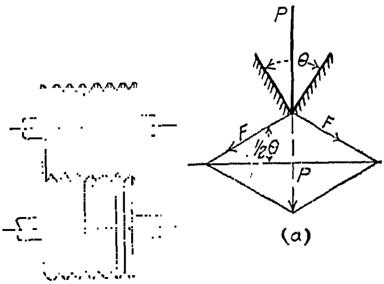


FIG. 141.

Grooved frictions are usually both of cast iron, with the tops turned off slightly to allow deeper penetration, if necessary. On account of the grinding action on both sides of the pitch cylinders, these wheels lose much of the advantage gained in grooving, and they are not common, because of wear and frictional loss. The principle is interesting, however, and applied to rope drive and cone clutches it results in enormous advantage.

FRICTION CONES

82. For angle drives, in which the axes intersect, friction cones take the place of cylinders. They are especially useful

in reversing drives. The writer is well acquainted with a drive of this kind on a wood-working machine at Armour Institute of Technology, that has done fairly continuous duty for 25 years, and is still in first class condition. By transferring contact from A to B, the shaft will turn in the opposite sense. Why?

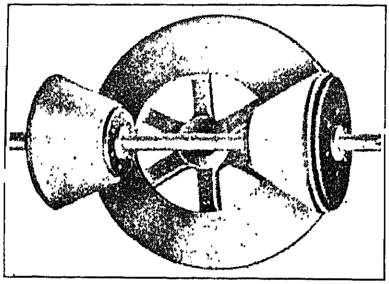


FIG. 142.—Bevel friction gearing. Iron and fiber wheels. Reversing cones. The Medart Co., St. Louis.

DESIGN OF CONICAL FRICTIONS

83. Rolling cones may be laid off on any pair of intersecting axes, to run at any relative speeds. They have practical limitations, which can only be determined by the designer, by the use of what is called common sense.

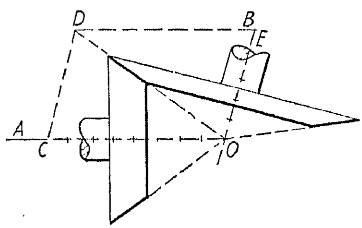


FIG. 143.—The layout of friction cones.

Example.—To lay out a pair of rolling cones on axes intersecting at 120 deg., to run respectively 750 and 1,200 r.p.m. The speed ratio is therefore $\frac{5}{8}$.

(1) Draw the axes AO and BO, intersecting at 120 deg.

(2) Lay off eight units on AO and five units on BO.

(3) Complete the parallelogram $OCDE$.

(4) The diagonal DO will be the element of contact. The cones may be drawn to any desired diameter, and altitude between bases.

Let the student prove that the diameters thus laid out will be inversely as the speeds of the cones.

84. Special Cases.—If *DO* (Fig. 143) is perpendicular to *AO*, the cone *A* becomes a plane, and the resulting combination is a disc, Fig. 144*a*, with cone drive. If the angle with *AO* becomes obtuse, the cone *B* rolls on the **inside** surface of *A*,

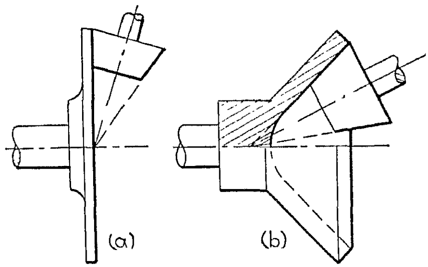


FIG. 144.

and *A* is called a **cup,** or **internal cone.** This is seldom practically applied.

85. Evans Friction Cone.—This speed changing device is used in many machines where a large number of speed ratios from slow to fast are wanted. The contact between the two cones (usually equal in size) is made by a narrow belt under compression. By shifting along the intervening space, the

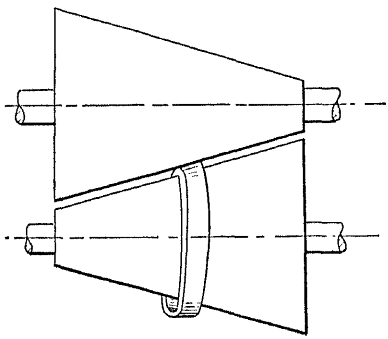


FIG. 145.

speed of the follower is changed from slower than the driver to a correspondingly faster speed, with any desired intermediate speed. Both flat and round belts are used, the round being theoretically better. Why? The disadvantage of the round is

that it is compressed more easily and therefore requires more attention than the flat belt.

DISC FRICTIONS

86. The frictions shown in Fig. 146 comprise a set of extremely sensitive speed changes. They are mostly used in very light machinery, such as sensitive drills and blue printing machines, but their usefulness is often extended to much heavier transmission, as in the case of railroad service cars manufactured by the Buda Company, and the Kelsey automobile. The form shown in Fig. 146*a* is the one usually employed in sensitive

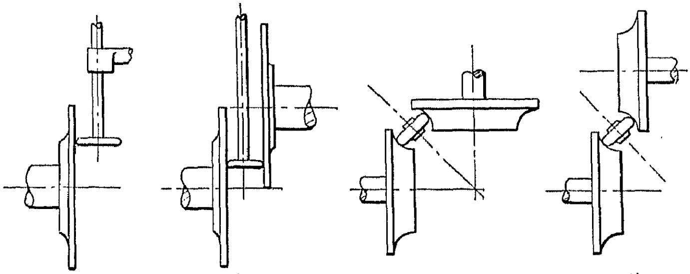


FIG. 146.—Disc frictions and toric frictions giving variable speeds.

drills, and there the drive is direct, the speed change being made by shifting the small wheel (soft surface) up and down on its axis. The other three employ the soft wheel as a contact maker, like the strap in the Evans cones. In *c* and *d* the change of speed is made by turning the soft wheel on its transverse axis. Both forms *a* and *b* can be used to reverse the drive by moving past the axis of the driven.

The Kelsey transmission is applied in the manner of Fig. 146*a*, a powerful spring giving the necessary pressure to the disc. The wheels are driven through a differential to internal pinion and gears on the rear axle, thus relieving the rear axle of the torque stress. This device does away with the clutch, gear box, and universal joint, all of which are agencies for power loss. The makers of the car claim that they have eradicated the former source of trouble which was present in older cars in which this device was used. This trouble consisted in wearing

ruts in the surface of the disc, when heavy pulls in sand, mud, or on hills resulted in slipping between the frictions. It is claimed that there will be no slip from these causes, because the spring is so powerful that the engine will stall before the frictions will yield.

Other forms of frictional transmission have been devised, but they are not important. The subject is quite thoroughly treated in Schwamb, Merrill, and James' "Elements of Mechanism."* For non-planar shafts, rolling hyperboloids have been suggested, and they could be made to work through any reasonable speed ranges, and at any desired angle, but, as far as this writer knows, such drives are always accomplished through helical gears.

PROBLEMS

Note.—In the following problems, whether they call for graphic or mathematical solution, solve the required velocities in ft. per min., and work to definite scales.

1. Beam X is actuated by the rotation of disc Y, mounted eccentrically at O. Y runs at ——— (10 to 100) r.p.m.

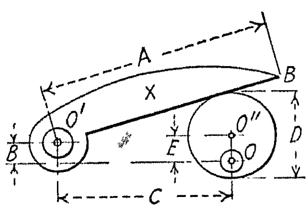


FIG. 147.—Problem 1.

DATA

	(a)	(b)	(c)	(d)	(e)
A	4	5	6	5	6
B	1	1	1	2	½
C	2	3	3	2	4
D	1	1¼	1½	1½	1¾
E	¼	¼	¼	⅜	½

	(f)	(g)	(h)	(j)	(k)
A	7	6	7	7	6
B	1	2	1	1	½
C	3	4	4	3	4
D	1¾	2	2	2	2
E	½	⅝	¾	¾	⅝

	(l)	(m)	(n)	(o)	(p)
A	6	6	8	6	7
B	1	2	1	1	2
C	3	3	4	3	4
D	2¼	2¼	2½	2½	3
E	¾	¾	⅝	⅞	1

Required.—(a) The angle of oscillation of X.
(b) The angular velocity of X at ——— deg. phase of Y.
(c) The velocity of slipping at the same phase.
(d) The linear velocity of P at the same phase.
(e) The velocity diagram of P for a complete cycle.

* SCHWAMB, P., MERRILL, L., and JAMES, W. L., "Elements of Mechanism," J. Wiley & Sons, New York.

2. Same problem. Substitute an ellipse of ———— in. major axis and ———— in. minor axis, for the eccentric circle. Mount the ellipse on one of its foci. (This will change the measurements of D and E only). Use any of the data for Prob. 1.

3. X and Y are two beams centered at O and O'. Y rotates uniformly at ———— (10 to 100) r.p.m. and carries a roller ½ to 1 in. diameter, as specified by the instructor.

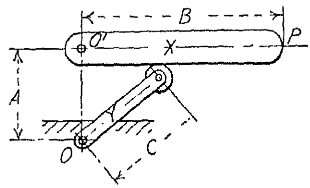


FIG. 148.—Problem 3.

DATA

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(j)	(k)	(l)	(m)	(n)	(o)	(p)
A	1	1	1	1¼	1¼	1¼	1½	1½	1½	1¾	1¾	1¾	2	2	2
B	4	5	6	4	6	5	5	7	6	5	8	7	6	8	10
C	2	2½	3	2	2½	3	2½	3	4	2½	3	4	3	4	5

Required.—(a) The angular velocity of X at ———— deg. phase of Y.
(b) The slipping velocity at the same phase, if the roller were not free.
(c) The linear velocity of P.
(d) The linear velocity of P for a complete cycle.

4. Take any of the data in Prob. 3. Attach a connecting-rod, twice the length of B, to P, and make it operate a slider on the line OO', extended.

Required.—(a) Slider velocity at ———— deg. phase of Y.
(b) Time ratio of advance to return of slider.
(c) Velocity diagram of slider for complete cycle.

5. Same as Prob. 4, except that path of the slider shall be horizontally through O or O', at option of the instructor.

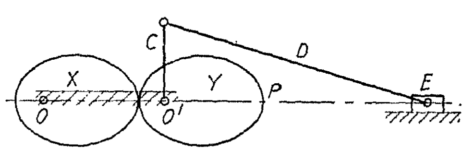


FIG. 149.—Problem 6.

6. Two equal ellipses, rotating on their foci, carry the crank C at right angles to OO'. Data: Major axes 3 in., minor axes 2 in., $C = 1\frac{1}{2}$ in., $D = 5$ in. X (driver) rotates uniformly at 70 r.p.m.

Required.—(a) Ratio of advance to return of E.
(b) Velocity diagram of E through complete cycle.
(c) Maximum linear velocity of P.

7. Same problem. Major axes ——— in., minor axes ——— in., C ——— in., D ——— in. (two and one-half to four times C), r.p.m. ———.

8. Same as Prob. **7**, making crank in line with OO', instead of at right angles.

9. Same as Prob. **8**, making E travel in a vertical path through O'.

10. Circular disc, of 2 in. diameter, is mounted eccentrically $\frac{3}{8}$ in. from its geometrical center O'. The disc runs at 40 r.p.m., and operates the mushroom follower as a cam does. At 60 deg. phase of the disc, what is the

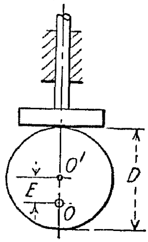


FIG. 150.—Problem 10.

linear velocity of the follower? What is the slipping velocity between them? Plot the follower velocities through a complete cycle.

11. Same problem. Make D = ——— in., E = ——— in., r.p.m. ———, and phase ——— deg.

12. Same as Prob. **10.** Substitute an ellipse, major axis ——— in., minor axis ——— in., center at focus, phase ——— deg., r.p.m. ———.

13. Same as Prob. **10.** Substitute a double logarithmic spiral. Mount the spiral on its center, and make the radius vector at zero degrees ——— in., and ——— in. at 180 deg. Mount the spiral on its center. What is the constant angle of this curve?

14. Same as Prob. **10.** Substitute a double involute, derived from a ——— in. circle, mounted on the center of the circle for the disc. Study the action of this pair. What kind of velocity would be imparted to this pair? Will there be a shock at either end of the stroke? Would a roller on the follower be of any advantage? Given ——— r.p.m. and ——— deg. phase of driver.

15. Lay out a curve, rotating about O', that will roll with the straight line, Fig. 151a. A = ——— in., B = ——— in. What is the angular velocity of the follower at ——— deg., if the driver runs at ——— r.p.m.?

16. With the same straightedge, determine A so that a closed curve will result.

17. Substitute an eccentric circle in this problem. A ——— in., B ——— in., r.p.m. ———, phase ——— deg., D ———.

18. Determine A in Prob. **17,** to make a closed curve for one revolution of the circle.

19. Determine A in Prob. **17,** to make a closed curve for two revolutions of the circle.

20. Substitute an ellipse (Fig. 151*c*), A ——— in., B ——— in., C ——— in., r.p.m. ——— in., ——— deg. phase.

21. Determine A in Prob. **20,** to make a closed curve for one revolution of the ellipse.

22. Determine A in Prob. **20,** to make a closed curve for two revolutions of the ellipse.

23. Substitute a double logarithmic spiral (Fig. 151*d*) in this problem. A ——— in., B ——— in., C ——— in.

24. Determine A in Prob. **23,** to make a closed curve for one revolution of the spiral.

25. Determine A in Prob. **23,** to make a closed curve for two revolutions of the spiral.

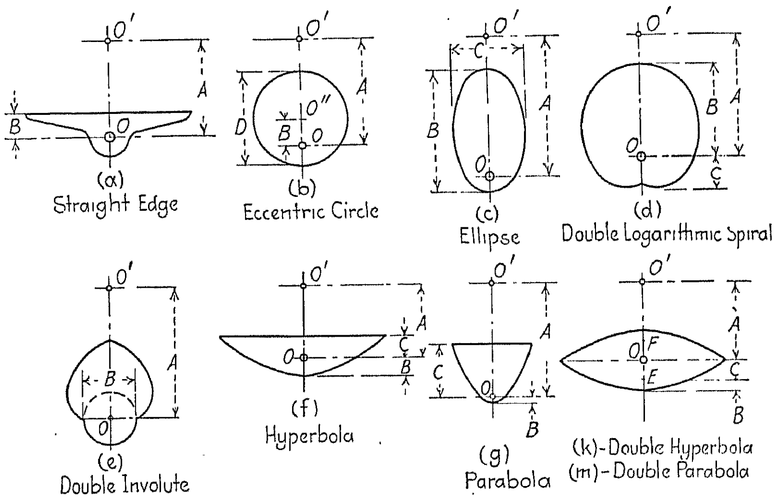


FIG. 151.—Rolling curve layouts for Problems 15–38.

26. Substitute a double involute (Fig. 151*e*), in Prob. **15.** A ——— in., B ——— in., C ——— in.

27. Determine A in Prob. **26,** to make a closed curve for one revolution of the involute.

28. Determine A in Prob. **26,** to make a closed curve for two revolutions of the involute.

29. Substitute a hyperbola with a straight edge, Fig. 151*f*, in this problem. A ——— in., B ——— in., C ——— in., r.p.m. ———, phase ——— deg.

30. Determine A in Prob. **29,** to make a closed curve for one revolution of the hyperbola.

31. Substitute a parabola with a straight edge, (Fig. 151*g*), in this problem. A ——— in., B ——— in., C ——— in., r.p.m. ———, phase ——— deg.

32. Determine A in Prob. **31**, so that it will make a closed curve for one revolution of the parabola.

33. Substitute a double parabola, Fig. 151k, in this problem. A ———— in., B ———— in., C ———— in., r.p.m. ————, phase ———— deg.

34. Determine A in Prob. **33**, to make a closed curve for a single revolution of the double parabola.

35. Determine A in Prob. **33**, to make a closed curve for a double revolution of the double parabola.

36. Substitute a double hyperbola, Fig. 151m, in this problem. A ———— in., B ———— in., C ———— in., r.p.m. ————, phase ———— deg.

37. Determine A in Prob. **36**, to make a closed curve for one revolution of the double hyperbola.

38. Determine A in Prob. **36**, to make a closed curve for two revolutions of the double hyperbola.

39. Design a pair of equal ellipses to be mounted on their foci, at 4 in. between centers, so that the follower will have a time ratio of its semi-revolutions of 3: 1. If the driver runs at 120 r.p.m., what is the maximum linear velocity of the ultimate point of the follower?

40. Same problem. Center distance ———— in., time ratio ————, r.p.m. ————.

41. Design a mechanism like the one given in Prob. **6**, whose stroke is 3 in. long, and the time ratio is 5: 2. If this mechanism were applied to a shaper or slotting machine, what would be its objections, if any?

42. Same problem. Stroke ———— in., time ratio ————.

43. Same as Prob. **41**, except that the mechanism be like the one in Prob. **9**.

44. Same problem. Stroke ———— in., time ratio ————.

45. Design a pair of three-lobed wheels, having logarithmic spiral outlines, to run on centers 3½ in. apart. Ratio of maximum to minimum angular velocities to be from 7: 3 to 3: 7.

46. Same problem. Outline ———— log. spiral or elliptical, number of lobes ————, center distance ———— in., angular velocity range from ———— to ———— (must be reciprocals).

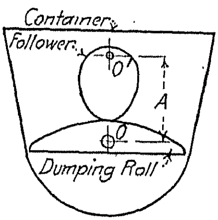


FIG. 152.—Problem 47.

47. Figure 152 is a sketch of the dumping mechanism of a dump cart. Given A = 28 in., let the student design the outline of the container, dumping roll and follower. Would there be positive drive?

48. Same problem. $A =$ ——— in.

49. Friction wheels. Given $OO' = 23$ in.; r.p.m., 480 and 210, respectively. Materials, wood and cast iron. Efficiency 98 per cent. Determine the pressure, width of face, and diameters, necessary to transmit 4 hp.

50. Same problem. $OO' =$ ——— in., r.p.m. ——— and ———, materials selected by instructor ——— and ———, efficiency ——— per cent, hp. ———.

51. Friction wheels. Given center distance ——— in., r.p.m. ——— and ———, efficiency ——— per cent, materials ——— and ———. Determine width of face, μ, bearing pressure, and greatest horsepower that could be transmitted by such a pair with the usual limitations as to width.

52. With data selected from **49, 50,** or **51,** design a friction drive, and made a working drawing of it, together with an appliance for regulating the bearing pressure. Make assembly, complete details with dimensions, sectional views if needed, title, and bill of material.

53. Cast iron grooved frictions; groove angle 35 degrees, $\mu = .15$, efficiency 80 per cent. Required: the diameters to transmit 5 hp. at 300 and 120 r.p.m., respectively, under a bearing pressure of 400 lb.

54. Same problem. Groove angle ——— deg. (20—40), μ ——— (.12 to .25), efficiency ——— per cent, ——— hp., r.p.m. ——— and ———, bearing pressure ——— lb.

55. Same problem. Groove angle ——— deg., μ ———, efficiency ——— per cent, r.p.m. ——— and ———, bearing pressure ——— lb. Required: the hp. transmitted.

56. Same problem. Groove angle ——— deg., μ ———, efficiency ——— per cent, speed ratio ———, hp. ———, bearing pressure ——— lb. Required: the diameters and r.p.m. of both wheels.

57. Same problem. Groove angle ——— deg., μ ———, efficiency ——— per cent, r.p.m. ——— and ———, center distance ——— in. Required: the diameters and the bearing pressure.

58. Design friction cones to run on axes intersecting at 75 degrees, at speeds of 700 and 210, pressure of 200 lb. in the direction of the axis of the smaller cone. Required: the diameters of the large bases and width of face to transmit 3½ hp. at 90 per cent efficiency, the materials to be selected by the student. Make a layout of the cones and give the face angles.

59. Same problem. Axial angle ——— deg., r.p.m. ——— and ———, axial pressure ——— lb., hp. ———, efficiency ——— per cent.

60. Design a bevel friction reversing drive to transmit 5 hp. at speeds of 450 and 200 r.p.m., at 90 deg. axial angle. Drive from small wheel to large at 450 lb. axial pressure. Select your own materials. Make a working drawing of the assembly, with provision for shifting and applying the necessary pressure. Also make a detail drawing of all parts, fully dimensioned, with titles and bill of material.

61. Same problem. Hp. ———, r.p.m. ——— and ———, pressure ——— lb., efficiency ——— per cent., axial angle 90 deg.

62. The Fellows Gear Shaper Company, by using cutters of mating shape, are able to generate in quantity production very odd-shaped pieces, in their gear shapers. The action of the cutter and the product resembles

rolling, but is not pure rolling. Let the student design a cutter that will generate one of the outlines here given by this process. Cutter and product

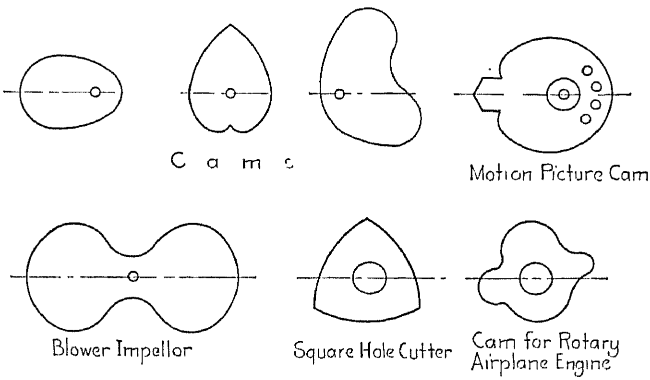


FIG. 153.—Problem 62. Suggested outlines to be generated by shaper cutters.

must traverse equal angles in equal time intervals. The instructor may supply dimensions and details of the outlines.

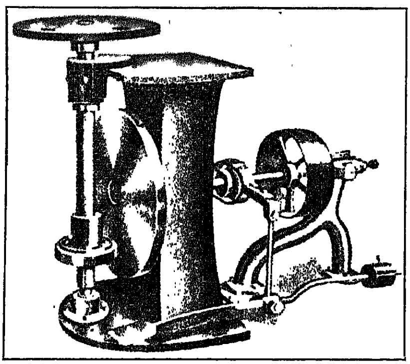


FIG. 154.—Moulder's jolly. Patterson Foundry & Machine Co., East Liverpool, Ohio.

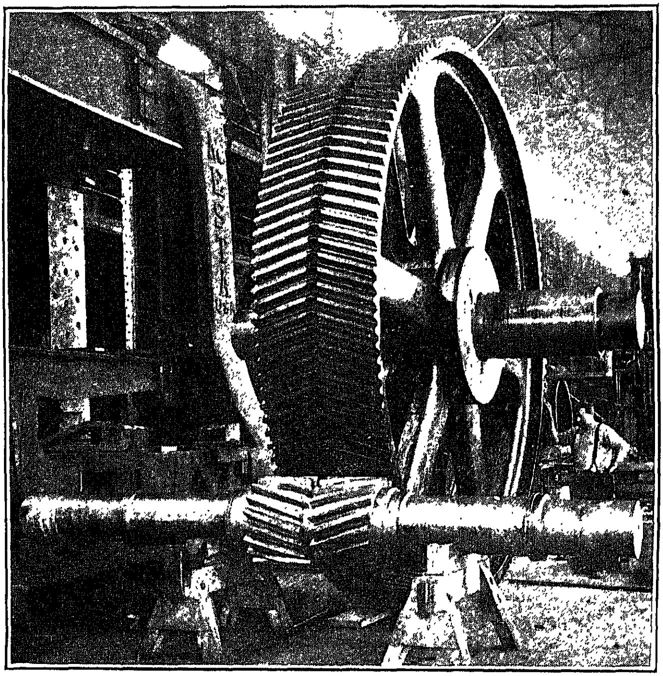


FIG. 155.—Large herringbone gears for heavy work in steel mills. Mesta Mfg. Co., Pittsburgh.

CHAPTER V

TOOTHED GEARING FOR PARALLEL SHAFTS

87. A pair of **spur gears** (gears working on parallel shafts) may be regarded as a pair of rolling cylinders provided with teeth to give them **positive drive.** The change from friction to tooth transmission almost entirely eliminates bearing pressure, and transmission uncertainty, and provides a means of speed reduction which enables small motors to drive heavy and powerful machines with the utmost precision.

The complete story of gear development would be a long and interesting account of an upward struggle lasting for centuries. It is a long engineering stride from the wooden wheels with rude wooden pegs fitted into holes, which served well our ancestors in their hoists, corn grinders, and wine presses, to the helical bevel gear, accurate to .0001 in., made of chrome vanadium, or some other marvellous alloy of steel. For a century and a half the mathematician, the mechanical engineer, the draftsman, the chemist, and the metallurgist, through the most searching study, have united their researches and ingenuity to develop the manufacture of gears to their present perfection and cheapness.

In the development of any mechanical commodity, two items for its improvement are needed: (1) **Improvement in the design,** aiming at the best running qualities, the best distribution of the material, and the best functioning of the related parts; (2) **Improvement in the materials** to provide the greatest strength, durability, elasticity, toughness, rigidity, or whatever quality is needed, with the smallest amount of material.

To the attainment of such ideals in the development of gears, a large share of the best scientific and engineering brains have contributed, in the aggregate, countless centuries of study and experiment to the design and production of gear wheels. **Theory has been the basis of this work,** always years in advance of commercial production, and today a clearer knowledge of

theory is more needed by the engineer than has ever been demanded of him in the past.

THEORY OF GEARS

88. When positive drive, exact speed ratios, heavy loads, great speed reductions (or increases), and steadiness are required, and when the distance between shafts is not too great, the only transmission thought of is the toothed wheel, called a **gear.** The necessity that frequently exists for **exact speed ratio** is shown very conspicuously in the synchronizing mechanism of the fighting airplane. The machine gun is geared to the revolving propellor shaft, and is timed so accurately that the bullets pass between the revolving blades. Nothing but an exact speed ratio would be possible for such accurate timing.

Constant Angular Velocity Ratio.—The prime essential in the design of tooth outlines is the maintenance of a constant angular

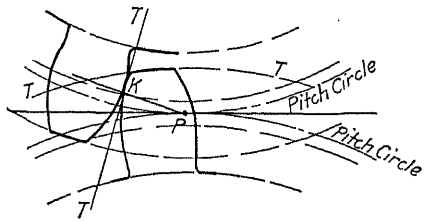


FIG. 156.

velocity ratio between the pair of mating gears. For example: two gears are to run at speeds of 200 and 100 r.p.m. For the sake of smoothness of running and efficiency, this **exact ratio** must be maintained, without varying between 200 to 99 and 200 to 101 r.p.m. If tooth outlines were such that a constant angular velocity were not maintained, there would be maximum and minimum velocities set up in the follower for every engagement of teeth. Thus, a gear of 40 teeth, running at 100 r.p.m., would impart to its follower 4,000 accelerations and retards every minute, or 66 every second. This would result in violent shaking and great wear, resulting in early destruction of the machine. Before the era of high speeds and heavy loads, this quality was comparatively unimportant, but today poor gear design is fatal.

How Constant Angular Velocity Ratio Is Effected.—Referring to Art. **69**, it will be noted that this ratio obtains when the

intersection I of the common normal with the line of centers OO' is a stationary point. This means that smooth running is dependent on the tooth outlines being of such a character, that their normals must always pass through a certain point on the line of centers. Figure 156 shows two teeth in contact at K, having a common normal PK. To obtain perfect results, the point P must be stationary through the entire duration of contact, although K is shifting at every phase. This point P is also the point of tangency of the pitch circles, and is called the **pitch point.**

THE TOOTH OUTLINES

There are two curves in use that possess this quality, the **Cycloid** and the **Involute.** These curves are the only ones that have found any consideration as gear outlines. Other curves

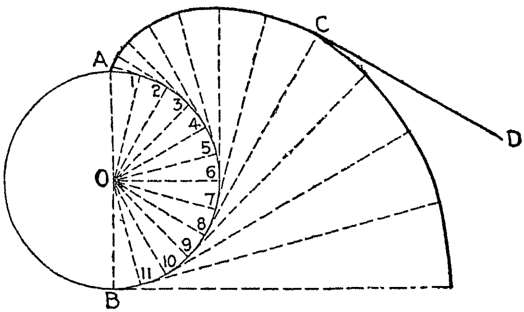


FIG. 157.—Construction of an involute.

might be used, but thus far have not. Figure 157 shows these curves and their normals at various points.

Terminology.—Two gears on parallel shafts, engaging between the shafts, are called **Spur Gears.** The smaller of the pair is called the **pinion,** and the larger the **gear.** Gears of a small number of teeth are usually called pinions. A gear of infinite radius is called a **Rack,** and this term is extended to include gears of very large diameter, such as the toothed rims of bridge turntables, concrete mixers, or similar structures. The rack is designed both as a stationary and as a moving member, but is more often stationary. **Internal gears** are those in which contact is outside of both centers. Gears with helical grooves and teeth, engaging on parallel shafts, are called **Twisted gears.**

89. Pitch Circle.—It is obvious that for each tooth on one wheel there must be a recess on the mate, and therefore each toothed wheel in every gear system is provided with alternate

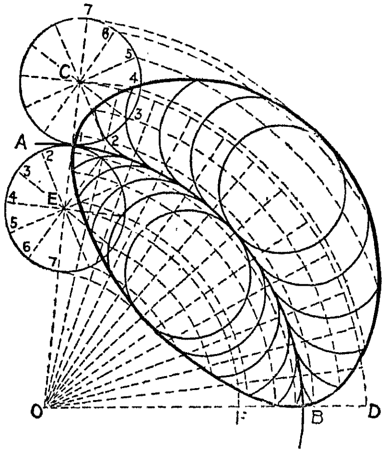


FIG. 157a.—Construction of an epicycloid and a hypocycloid.

projections and recesses, which extend above and below an imaginary surface, called the **pitch cylinder,** or more conveniently, the **pitch circle.** This circle, invisible and, in a material

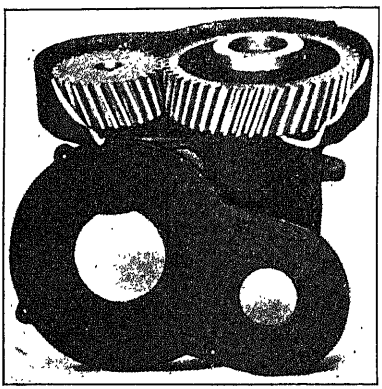


FIG. 158.—Twisted gears used on the spindle of a drill press. Colburn Machine Co., Cleveland, O.

sense, non-existant, is the most important line or dimension in the wheel, both from the designer's and the mechanic's standpoint. The pitch diameters determine the speed ratio, just as

do the diameters of two friction cylinders in their transmission, and the size of the gear is that of its pitch diameter.

90. Circular Pitch.—In any pair of mating gears, the length of the arc of the pitch circles from a point on one tooth to the corresponding point on the next tooth must be equal. This arc length is called the **circular pitch** (c.p.), and it must be an exact divisor of the pitch circumference.

Thus

$$\text{c.p.} = \frac{\pi D}{N},$$

D being the pitch diameter in inches, and N the number of teeth.

Example 1.—A 10-in. gear has 40 teeth. Required its circular pitch.

Ans. $\text{c.p.} = \frac{\pi D}{N} = \frac{\pi \times 10}{40} = .7854$ in.

Example 2.—The pitch of a 24-toothed gear is 1 in. What is its pitch diameter?

Ans. Pitch circumference $(\pi D) = \text{c.p.} \times N = 24$ in., $\therefore D = \frac{24}{\pi} = 7.639$ in.

Note.—If the circular pitch has an even value (1, 1½, 2 in., etc.), the diameter will have a decimal value, and if the diameter is even (10, 12, 14 in.), the c.p. will have a decimal value.

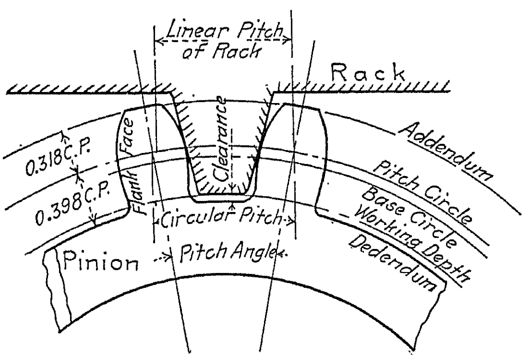


FIG. 150.—Gear tooth dimensions and definitions.

91. Tooth Dimensions.—In any system of **interchangeable** gears, it is necessary that the teeth have the same dimensions, thickness, height, etc., as well as the same pitch. The dimensions have been fixed as multiples of the circular pitch. For standard gears, they are:

The **Addendum,** the height of the tooth outside the pitch circle,

$$= \frac{1}{\pi} \text{ c.p.} = .318 \text{ c.p.}$$

The **Dedendum,** the height inside the pitch circle,

$$= \frac{5}{4} \cdot \frac{\pi}{10} \cdot \text{c.p.} = .398 \text{ c.p.}$$

The **Thickness** of the tooth (measured on the pitch circle) = ½ c.p. in cut gears. Cast gears, on account of surface imperfections, and the impossibility of casting teeth perfect in thickness, outline, and spacing, are made .48 c.p. in thickness.

Note.—For convenience, draftsmen usually lay off the addendum = 3/10 c.p., the dedendum = 4/10 c.p., and the thickness 5/10 c.p.

92. Diametral Pitch.—The relation between circular pitch and the pitch diameter is complicated by the factor π, and this has proved awkward for shop men in former years. The adoption of **diametral pitch** as the standard for sizes has greatly simplified calculation.

Diametral Pitch is the **ratio of the number of teeth to the diameter.** That is, d.p. $= \frac{N}{D}$. Thus, a 24-in. gear having 48 teeth has a diametral pitch of 2,—not a dimension, but a ratio. It is spoken of as 2-pitch.

Since

$$\text{c.p.} = \frac{\pi D}{N}, \text{ and d.p.} = \frac{N}{D},$$

their product is π. Therefore

$$\text{c.p.} = \frac{\pi}{\text{d.p.}} \text{ and d.p.} = \frac{\pi}{\text{c.p.}}$$

Table III.—Pitches.

D.P.	C.P.	D.P.	C.P.
1	3.1416	5	.6283
1½	2.0944	6	.5236
2	1.5708	7	.4488
3	1.0472	8	.3927
4	.7854		

Example.—What are the d.p. and c.p. of a 22-in. gear having 66 teeth?

Ans.

$$\text{d.p.} = \frac{N}{D} = \frac{66}{22} = 3.$$

$$\text{c.p.} = \frac{\pi}{\text{d.p.}} = \frac{3.1416}{3} = 1.05 \text{ in. (nearly).}$$

CYCLOIDAL TEETH

93. The first scientifically designed gear outlines made use of the cycloid, and such gears proved very serviceable until a better system was developed. It was noted that, unless the pitch circles were in almost perfect tangency, the common normal

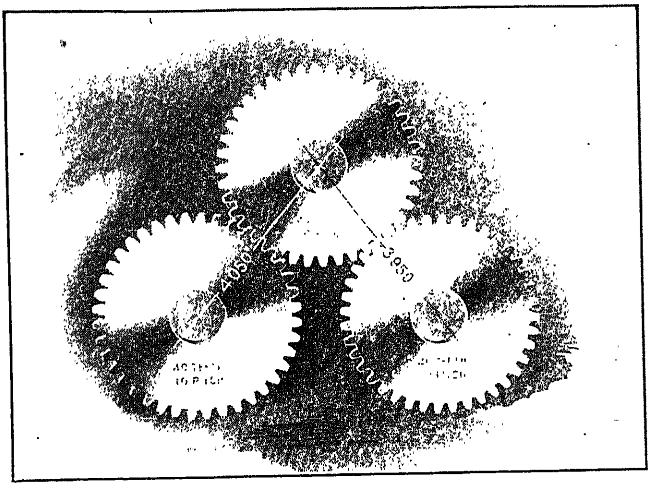


FIG. 160.—Involute gears of equal size engaging at different center distances. Fellows Gear Shaper Co., Springfield, Vt.

did not intersect OO' in a stationary point. This resulted in transmission losses and heavy wear. Where shifting gears were employed and where the bearings became worn, this condition of perfect tangency was impossible to obtain, hence designers set about to produce an outline which would not be thus affected. The involute gear was found to give remarkable results in this respect, and its success was so pronounced that the cycloidal outline has been practically discarded for cut gears. On account of their strength, many cast gears of this outline are still used in heavy, slow-moving machinery; for example, hoists, bascule bridges, turn tables, locomotive cranes, and

other machines that are used in exposed places. It is only a question of time when the cycloidal outline will be entirely superseded by the stub tooth outline, or something similar.

94. The Diameter of the Generating Circle for Cycloidal Gears.—Some kind of a cycloid may be generated by rolling any sized circle on any line, straight or curved. For **interchangeable gears,** however, particular sized circles must be used for **each pitch.** That is, the epicycloid of one gear must be generated by the same sized circle that generates the hypo-

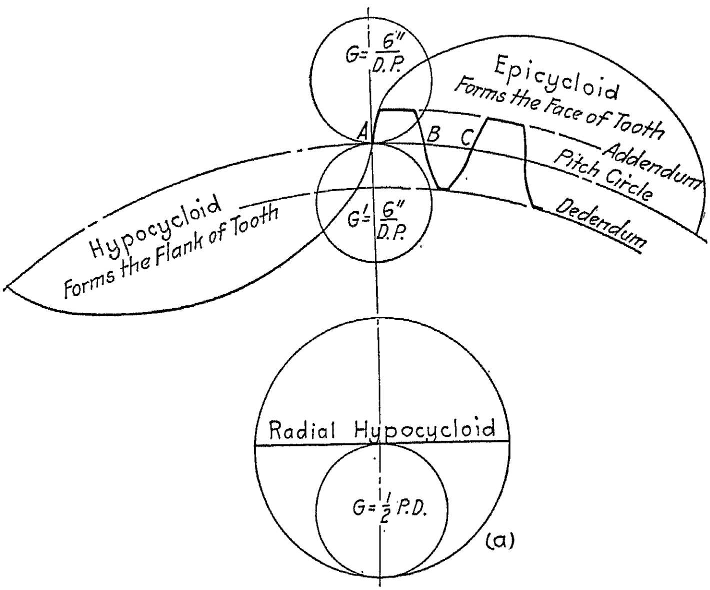


FIG. 161.—Showing how tooth outlines are formed from epicycloids and hypocycloids generated on the pitch circle. Below is a radial hypocycloid generated by a circle half the pitch diameter.

cycloid of its engaging gear. Where one gear works perfectly with all others of the same pitch, it is said to be **interchangeable.** It is therefore essential to interchangeability that both the epicycloid and hypocycloid for all gears of any pitch shall be generated by a certain sized circle.

To determine what should be the proper diameter to use, a study of the cycloid was made. In Fig. 161, showing teeth made by cycloidal outlines generated by equal circles, note that a circle of one-half the diameter of the pitch circle generates what is called a **radial hypocycloid**; *i.e.*, a straight line through

the center of the pitch circle. If this hypocycloid is used for the flanks of teeth, the tooth is said to have **radial flanks.** If the generating circle is less than one-half the pitch diameter, it generates **spreading flanks,** if larger, the flanks are **undercut.** Figure 162 shows the shape of the three outlines.

The undercut tooth was discarded, because (1) it is weak at the root, and (2) it was impossible to cut by the milling machine methods that then prevailed. These early investigators decided that radial flanks should be the limit in narrowness of flanks, and that 12 should be the lowest number of teeth. This established the rule:

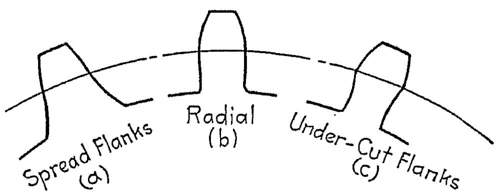


FIG. 162.

The generating circle for any gear must be one-half the diameter of the 12-toothed pinion of that pitch. From this it is evident that

$$G \text{ (generating circle)} = \frac{6 \text{ in.}}{\text{d.p.}}.$$

Example.—What is the diameter of the generating circle for a cycloidal gear of 72 teeth, and 18 in. d.p.?

$$\textit{Ans.} \quad \text{d.p.} = \frac{N}{D} = \frac{72}{18} = 4. \qquad G = \frac{6 \text{ in.}}{\text{d. p.}} = \frac{6 \text{ in.}}{4} = 1\tfrac{1}{2} \text{ in. } D.$$

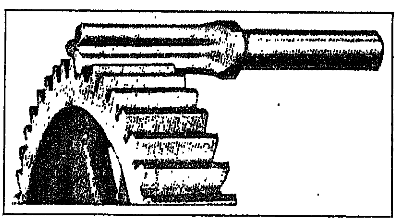


FIG. 163.—Three-toothed pinion. Boston Gear Works, Boston.

Note.—Although the early gear designers decided against undercut teeth, the Boston Gear Company and Niles-Bement-Pond Company (making the Maag gear) have recently put on

the market **generated** gears having 3 to 10 teeth. The Boston Gear Company claims the following advantages for these gears, as compared with larger pairs.

Small diameter; small weight; small cost; peripheral speed reduced; minimum of noise and vibration for equal r.p.m.; no keys necessary; less expense of mounting; large integral shaft permits heavier loading; great speed reductions for space occupied; greater width of face, greater width at root of tooth; finally, a substitute for worm gear reduction, giving greater efficiency and no end thrust.

Note.—In order to give strength at the root of the teeth of gears of less than 12 teeth, they are generated on enlarged pitch diameters.

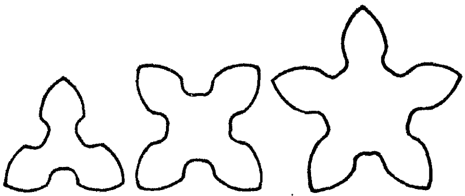


FIG. 164.—Outlines of small pinions. Boston Gear Works.

INVOLUTE GEARS

95. As the involute of a circle is generated by a point on a tangent as it rolls on the circle, so a point, moving in a tangent direction as the circle turns on its axis, will generate an involute on the revolving plane. Figure 165 illustrates two circles on which a cord is wound, tangent therefore to both. As the cord winds on one and off the other, the wheels are rotated about their centers and any point on the cord will generate an involute on the rotating plane of each wheel. The two involutes are thus tangent, and the cord is their common normal. Note that, no matter where the involutes are, the common normal passes through a stationary point on the line of centers.

Note.——(1) The circles on which the involutes are generated are not tangent, and are therefore not the pitch circles. They are called **Base Circles.**

(2) The farther apart the base circles are, the greater will be the angle ϕ, between the common normal (commercially known as the pressure line) and the tangent to the pitch circles. This

angle is called the angle of action, but more often the **Pressure Angle,** by which term it will be known in this work.

(3) The pressure line always passes through the pitch point.

Note.—For many years the pressure angle was always made 14½ deg., but now it is very often made 20 and 22½ deg.

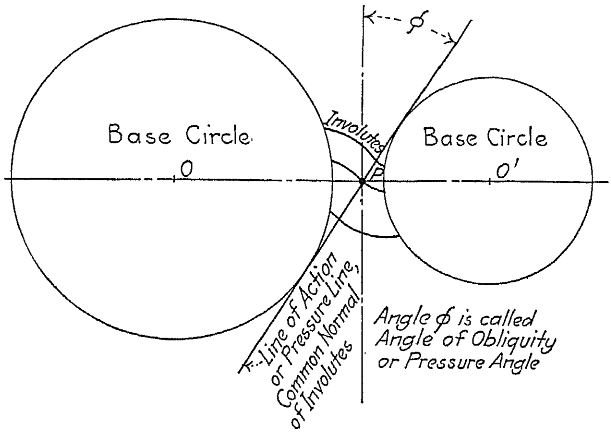


Fig. 165.

96. A Comparison of the Outlines.—In Figure 166 are shown a typical cycloidal and a typical 14½-deg. involute tooth. The cycloidal shows two curves meeting at the pitch circle, and the involute shows a continuous curve from the base circle to the tip. Since no involute can penetrate its base

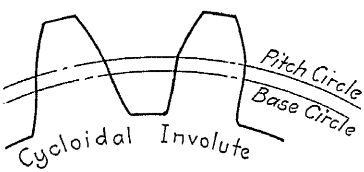


Fig. 166.

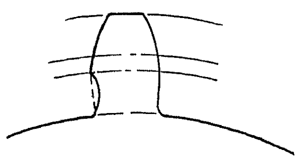


Fig. 167.—Wear on radial flank due to interference.

circle, there must be some means of completing the flank between the base and dedendum circles. This is done by making it **radial.** This design causes the chief objection to the involute system, called **interference.** An involute does not act properly on a radial flank, and wear hollows out the lower portion of the tooth, as shown in Fig. 167. This defect is not present in teeth

made by the **generating** process, which will be described later, but before that process was perfected, it was a very serious defect in the involute system. In 14½-deg. involute gears of more than 57 teeth, the radial flank disappears. Why?

MODERN DEVELOPMENTS IN GEAR DESIGN

97. It has been observed that the cycloidal tooth was the earliest scientific design, and that the 14½-deg. involute gradually superseded it for all purposes requiring smooth running. Before the end of the last century, the involute was established as the standard, except in cases where great strength was more desirable than perfect running quality.

In 1898 a revolutionary design was put forward by the Fellows Gear Shaper Company, called the **Stub Tooth.** This called for a 20-deg. involute, and a shorter addendum and dedendum by about 25 per cent. This change from standard design was slow in gaining recognition. Two objections, based on fact, were urged by engineers; (1) that the new pressure angle increased the pressure on the bearings and diminished the transmission; and (2) the shorter tooth decreased the duration of contact. Although both of these objections are valid, they are slight, and are overwhelmed by the numerous advantages of the system.

The accompanying figure shows the strength and advantage of the stub tooth over the standard, which is estimated to be

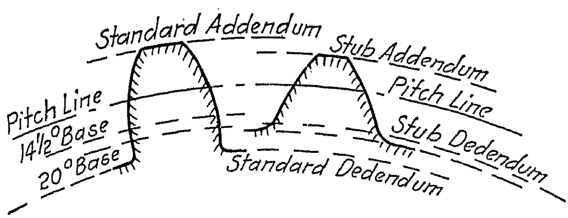


FIG. 168.—Comparative outlines of a 24-toothed gear of standard 14½-degree involute proportions, and of 24-toothed stub tooth gear, 20-deg. involute.

about 75 per cent. Automobile engineers have recognized this point, and the new design is universally adopted for their transmissions, the most severe service of any trains of gears. In machinery for manufacturing plants, it is also coming into universal use. Its complete adoption is probable before long, unless some better design is devised.

The **Maag Gear** (Niles-Bement-Pond) is very similar to the Fellows stub, the main difference being in the method of manufacture. The Fellows machines employ a pinion-shaped cutter, which can be used as well on internal gears, and the Maag method uses a rack-shaped cutter, which can only be used to cut external gears. This method seems to be especially adapted to heavy work.

The Simmons Method-Hob Company is responsible for a hob-generated tooth of 22½-deg. pressure angle, with rounded clearance.

Advantages.—The advantages of modern design are obvious to any one who studies the situation: (1) Greater strength, (2) absence of interference, (3) greater accuracy in outline, (4) cheaper production (less metal to be removed), (5) smaller pitch allowable for equal transmissions, (6) smoother running quality, because of less slippage, and (7) longer life on account of its closer approximation to pure rolling.

98. Proportions of Modern Gear Teeth.—The only proportions that have been altered are the addendum and dedendum. In the stub tooth gears these dimensions are based on a lower pitch than its circular pitch and thickness. For example, a gear of 3-pitch has its addendum and dedendum derived from 4-pitch.

Example.—To find the thickness, addendum, and dedendum of a stub-tooth gear of 24 teeth and 8-in. diameter.

$$\text{d.p.} = \frac{24}{8} = 3, \therefore \text{ c.p.} = 1.047 \text{ in., and the thickness} = \tfrac{1}{2} \text{ c.p.} = .524 \text{ in.}$$

$$\text{Addendum} = \frac{1.000}{\text{d.p.}} = \frac{1.000}{4} = .250 \text{ in. (based on 4-pitch).}$$

$$\text{Dedendum} = \frac{1.250}{\text{d.p.}} = \frac{1.250}{4} = .3125 \text{ in.}$$

Note.—The common pitches for stub tooth gears are $3/4$, $4/5$, $5/7$, $6/8$, $7/9$, $8/10$, $9/11$, $10/12$, and $12/14$. These do not mean fractional expressions, but that the first number is the actual diametral pitch, and the second is the pitch on which to base addendum and dedendum calculations. The system employed by the Nuttall Gear Company is better, and it is to be hoped that manufacturers will unite on a similar standard for their dimensions. The Nuttall system is:

$$\text{Addendum} = .250 \times \text{c.p., and Dedendum} = .300 \times \text{c.p.}$$

The Simmons firm has a still different set of dimensions, using

the **module.** The module is the reciprocal of the diametral pitch; *i.e.*, module $= \frac{D}{N}$. The addendum for their $22\frac{1}{2}$-deg. involute tooth is $\frac{7}{8}$ of the module, and the dedendum is $\frac{1}{4}$ longer; *i.e.*, $1\frac{3}{32}$ of the module. By this reckoning the addendum of the 24-tooth 8-in. gear would be $\frac{7}{8} \times \frac{1}{3} = .2917$ in., somewhat longer than the stub tooth addendum.

DURATION OF CONTACT, DETERMINATION OF THE ADDENDUM, LINE OF ACTION

99. The **point of contact** between two mating gears must always lie on the generating line of the tooth outline. (Why?).

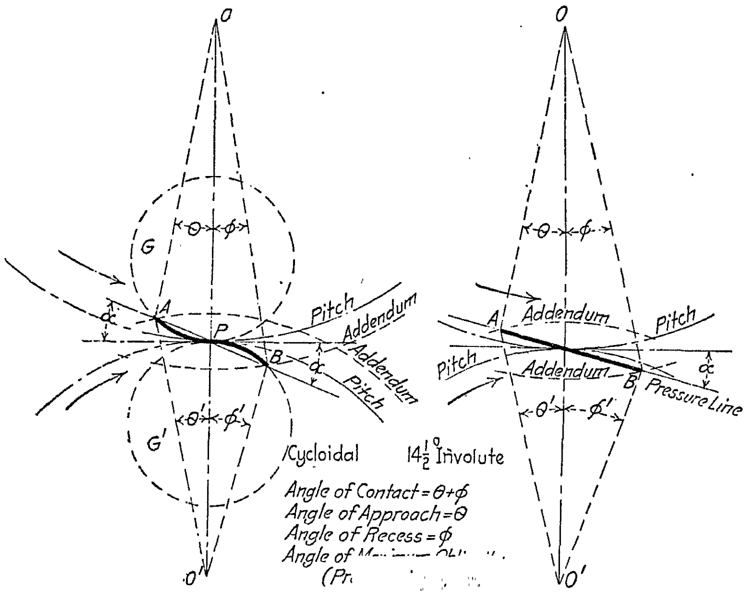


FIG. 169.—Duration of contact in gears.

In the cycloidal system, this is the generating circle (G and G', Fig. 169), and in the involute system it is the tangent to the base circles (AB, Fig. 169). The point of contact therefore travels along these lines (shown heavy in Fig. 169) from the point where the addendum of one wheel cuts this line to the point where the other addendum cuts it. These points are marked A and B in the figure, and subtend the angle of contact

duration from each center. This angle was formerly called the **Angle of Action,** but now the **Contact Angle.**

100. Determination of the Addendum.—The addendum was determined by the contact angle. It was necessary to adopt a standard proportion for the various tooth dimensions, and the natural basis was the circular pitch. It was decided that the contact angle should equal or exceed the pitch angle, because a new pair should be mating before the working pair quits contact. To this end a pair of pitch circles for 12-toothed gears, together with their generating circles were all drawn tangent. Laying off the pitch angle from one center marked the point where the addendum should cross the generating circle, and the result of this operation established the addendum as about 3/10 of the circular pitch, and the value $\frac{1}{\pi}$ c.p., or .318 c.p. was made official. This value was adhered to for both cycloidal and involute systems, until the introduction of the stub tooth gear, which reduced the addendum to about .250 c.p.

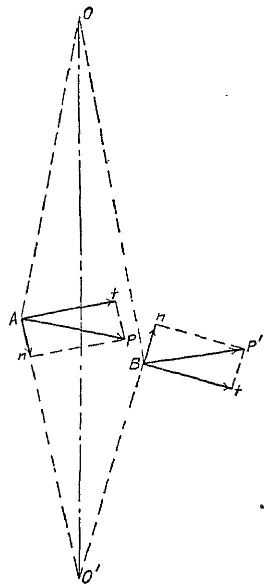


FIG. 170.

101. Line of Action, Angles of Approach and Recess.—The **line of action,** often called the pressure line is the line from the point of contact to the pitch point. (Why is it called the pressure line?) This line makes an angle (α) with the tangent at P, called the **angle of obliquity,** and commercially the pressure angle. The result of this angle between the normal to the teeth at the contact point and the tangent to the pitch circles, is that the pressure acting in the direction of the normal is resolved into two components, the **tangential,** or **turning component,** and the **radial** component, known as **bearing pressure.**

In Fig. 170 these components are shown at the beginning and end of the contact between two cycloidal gears. At A, the beginning of contact, we have the drive exerted in the direction Ap, along the common normal. This is resolved into At, perpendicular to the radius of the driven gear, and An, in the

direction of that radius. The component At acts to turn the driven gear, and is the effective driving power of the transmission at that point, and the component An is the force acting on the bearing of the gear, and is waste force. The resolution at B is the same.

It will be noted that, in cycloidal gears, the angle of obliquity varies from a maximum at A or B, to **zero** at P. This shows that at the pitch point there is neither normal component nor bearing pressure, and that all the force is exerted to turn the follower. In the involute system this is not the case, but the obliquity is constant throughout the duration of contact.

The contact angle is divided into two portions, the **angle of approach** (θ) and the **angle of recess** (ϕ). If the components of the transmitted pressure are taken at points in these angles, it will be found that the transmission is smoother in recess than in approach. This fact is taken advantage of sometimes in the design of special gears, by making the driven wheel with short, or no addendum and long dedendum, and the driver the opposite. This design shortens the approach angle and lengthens the smoother running recess. An example of this design was encountered in the works of the Foote Gear Company, Chicago, where a large order of these special gears was being filled for an automobile firm. They were designed to give a short approach angle and long recess with long addenda on one wheel and short on the other. These gears were bevel gears of 5-pitch, and the addendum on one was .120 in. on a 51-toothed wheel, and .280 in. on a 13-toothed wheel. Which of these should be the driver, and which the driven to obtain the proper results?

It is obvious that **interchangeable** gears cannot be made to take advantage of this principle, but **must be made standard.**

A similar pair of spur gears is produced by the Allbaugh-Dover Company, in which the addendum on one wheel is zero, and the dedendum of the other is zero + the clearance. The tooth outline of this pair is a 20-deg. involute stub, and it is used for speed increase with fine results. Such gears as these are classified as freaks, which is the usual title for anything not standard. Their justification is found in the results they produce, in spite of the handicap of greater cost. However, in large production, the initial expense of special tools is soon reduced to a negligible amount. Many standard articles were once freaks.

INTERNAL GEARS

102. Where parallel shafts are to be geared on a **short center distance,** the problem of making the gears of sufficient size, strength, and running qualities is answered by the **Internal Gear,** formerly called the Annular Gear. The internal gear is one in which the teeth are on the inside of the rim, and the addendum and dedendum are on the opposite sides of the pitch circle from the usual spur gear teeth.

Refer to Fig. 172. The center distance on the external gear, $X = R + r$, and the center distance on the internal gear, $Y = R - r$. From this it is evident that with equal pitch circles and an equal number of teeth, the internal pair has a smaller center distance. Notice, also, that the internal gear rotation is in the

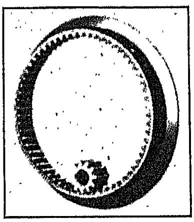


FIG. 171.

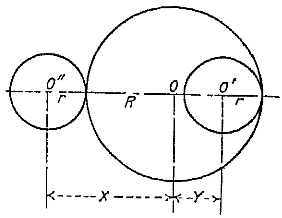


FIG. 172.

FIG. 171.—Internal gear and pinion. Fellows Gear Shaper Co.
FIG. 172.—Showing the difference of the center distances of an internal and an external gear pair of equal size and pitch.

opposite sense to that of the external gear. In spur gearing the two wheels rotate in opposite directions, and in internal gearing both rotate in the same direction.

Another advantage the internal gear possesses is a larger tooth contact. It will be noticed that, contrary to external gears, the pitch circles have their curvature in the same direction, which results in a considerably larger angle of contact. This results in a larger number of teeth carrying the load, in consequence of which smaller pitches can be used, and much easier running qualities obtained. The reason for the latter is that there is more nearly a rolling action between the pinion and gear, and almost no slippage between the teeth. The internal gear is cut in the same way, and by the same cutter, as the external gear of the same class.

103. Internal Gears Used as Clutches.—If the pinion is made the same size as the gear, there is no relative motion

between them, and they revolve together as one piece. In automobile transmission, such an arrangement, or a modification of it, is used to drive the car on high gear; *i.e.*, direct drive. The engaging and releasing motion is in the direction of the axis, and the teeth of the pinion slide into the grooves of the internal gear.

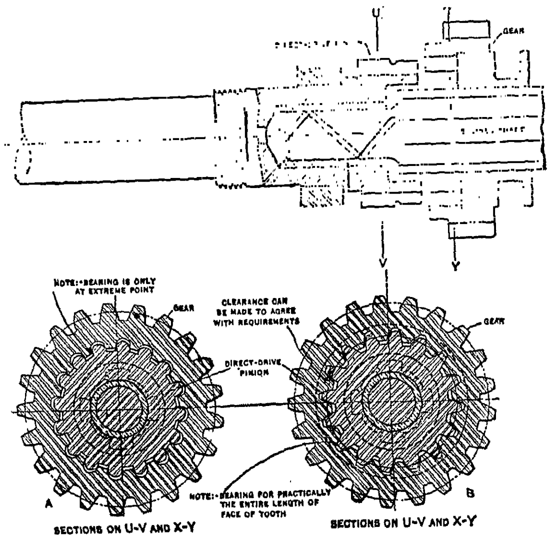


FIG. 173.—Internal gears used as a clutch. Fellows Gear Shaper Co.

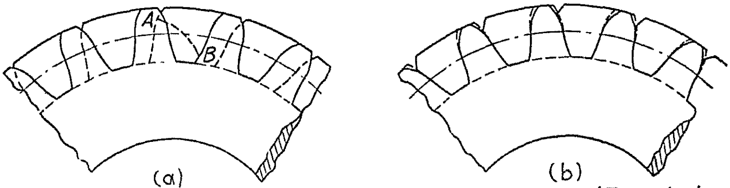
TWISTED GEARS

104. In Art. **101** it was noted that when the contact is at the pitch point that there is no bearing pressure and all the force is exerted to turn the follower. This means 100 per cent efficiency, as nearly as such can be attained; *i.e.*, pure rolling, no sliding, and all the power of the driver delivered to the driven. This ideal condition is brought about by cutting the grooves in a helix, instead of straight across the face. The effect of the helical groove is to change the contact line from a straight line longitudinally with the groove, to a helical line, such as is shown from A to B in Fig. 174a. Notice that the line AB always crosses the pitch circle (properly the "pitch cylinder"). This improves the running quality somewhat, but a second operation brings about a vast improvement.

The second operation is this: the face of the tooth is cut back slightly, beginning at the pitch point, and continuing to the

tip. The dotted lines in Fig. 174*b* show this. This operation is called relieving the tooth, and the effect is to prevent any point on the face from coming into contact with the other tooth; *i.e.*, **contact is permitted only on the pitch cylinder** (pitch point).

The influence on the running quality, that is effected by this operation, is that the contact is reduced from line to (theoretical) **point contact.** Since the contact can only be on the surface of the pitch cylinder, and the pitch cylinder intersects each tooth in a helical **line,** the contact moves along that line from point to point, making the gears a **rolling pair.** This achieves the ideal condition in transmission, **pure rolling, positive drive, constant angular velocity ratio,** and **no wasted power,** a combination not found elsewhere in any pair known in kinematics.



Line ... of ... relieved.

Same ... original ... only Point-Contact at the Pitch-Surfaces

FIG. 174.

This ideal condition is never absolutely attained in practice, but so nearly, that twisted gears are specified for all fine work. The advantage may be summed up, as follows:

(1) No abrasion, hence minimum of wear;

(2) No bearing pressure, hence no loss from bearing friction;

(3) No power diverted from the drive, hence no waste of power;

(4) Quieter running and perfect freedom from vibration.

Note.—These gears are sometimes called helical, and sometimes spiral, but twisted seems to give the necessary distinction, if helical is used to denote gears on non-planar shafts. Modern methods of manufacture produce twisted and helical gears as cheaply as plain spur gears with straight grooved teeth.

HERRINGBONE GEARS

105. Twisted gears develop a tendency to side-shift, on account of the tangential component of the driving pressure. This results in end thrust and is an objection. The objection

is overcome by the use of double helical or herringbone gears, consisting essentially of two gears for each single gear, with grooves cut in right- and left-handed helices. This neutralizes the side shift and consequent end thrust. Various designs in cut and cast herringbone gears are turned out, most of them in one piece, and their use is becoming very extensive. The grooves are made both in matched and staggered grooves, sometimes with a space between the right and left sides. Sometimes two gears of opposite helices are keyed to the same shaft, but more often herringbone gears are made in a single unit. Internal gears are sometimes cut in the herringbone style, which makes them of the maximum efficiency.

For reasons of expediency it is best to adhere to a standard helical angle for twisted and herringbone gears. This angle, as nearly as can be ascertained, is 23 deg.

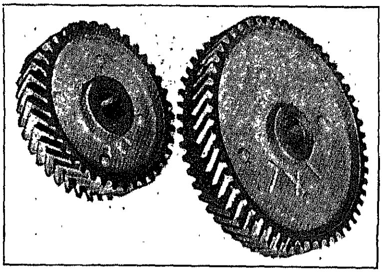


FIG. 175.—Herringbone gears. Foote Bros. Gear Co., Chicago.

METHODS OF DRAWING TOOTH OUTLINES

106. Cycloidal Teeth, Exact Method.

(1) Draw the pitch circle, and the addendum and dedendum circles.

(2) Lay off the c.p. for as many teeth as you wish to draw.

(3) Bisect these arcs for teeth and grooves.

(4) With generating circle G describe the epicycloid, and with G' the hypocycloid, beginning both at A. To draw these curves, see French's "Engineering Drawing," page 48, or Smith's "Practical Descriptive Geometry," page 164.*

(5) Use that portion of the epicycloid between the pitch and

* FRENCH, THOMAS E., "Engineering Drawing," McGraw-Hill Book Co., New York.

SMITH, WILLIAM G., "Practical Descriptive Geometry," McGraw-Hill Book Co., New York.

addendum circles for the face, and of the hypocycloid between the pitch and dedendum circles for the flank.

(6) Through *B*, *C*, and the other pitch points of the teeth, duplicate the curves thus drawn, preferably by transferring them, rather than by generating the curves each time.

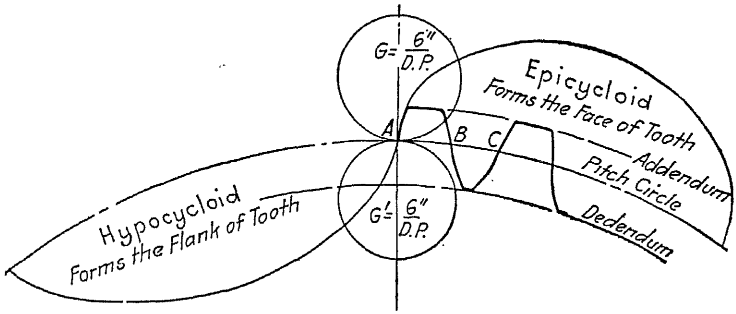


FIG. 176.

107. Cycloidal Teeth—Approximate Methods.—As the exact method is laborious (and not entirely exact), it is not often used by draftsmen, because several approximate methods have been devised that will enable draftsmen to draw gear teeth that serve the purpose of representation satisfactorily. Gear teeth are cut by special cutters made by a small number of tool companies, and when gears are made with these cutters, no drawing,

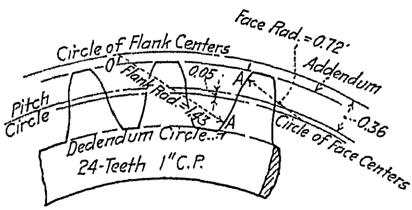


FIG. 177.—Drawing a cycloidal tooth by the Grant 3-point odontograph.

whether exact or inexact, can alter the shape of the teeth. Therefore, only on very rare occasions does it become necessary for the engineer to make an exact drawing of a gear outline. Probably not one engineer in a thousand ever even sees one. The conclusion is, then, that a good-looking approximate drawing will answer all practical purposes. The only method worth using is the Grant 3-point odontograph, whereby the teeth outlines are drawn with the compasses, set at proper centers and radii.

Operation.—(1) Draw the pitch, addendum, and dedendum circles.

(2) Lay off the pitch points as in (2) and (3), Art. **106.**

(3) Using the table here given, determine the distance of flank centers for the pitch and number of teeth in the given gear, and draw the circle of flank centers, using as a radius the radius of the pitch circle plus the flank center distance.

(4) Compute the distance of face centers in the same way and draw the circle of face centers, using as a radius the radius of the pitch circle minus the face center distance.

(5) With radii taken from the table, and computed for this pitch, draw the face arc from the pitch circle to addendum, passing through A, from O as a center, and draw the flank arc from O' as a center.

TABLE IV.—THREE POINT ODONTOGRAPH. STANDARD CYCLOIDAL TEETH. INTERCHANGEABLE SERIES.
(From Geo. B. Grant's "Odontics.")

Number of teeth		For one diametral pitch. For any other pitch divide by that pitch				For one inch circular pitch. For any other pitch multiply by that pitch			
		Faces		Flanks		Faces		Flanks	
Exact	Intervals	Rad.	Dis.	Rad.	Dis.	Rad.	Dis.	Rad.	Dis.
10	10	1.99	.02	− 8.00	4.00	.62	.01	−2.55	1.27
11	11	2.00	.04	−11.05	6.50	.63	.01	−3.34	2.07
12	12	2.01	.06	∞	∞	.64	.02	∞	∞
13½	13–14	2.04	.07	15.10	9.43	.65	.02	4.80	3.00
15½	15–16	2.10	.09	7.86	3.46	.67	.03	2.50	1.10
17½	17–18	2.14	.11	6.13	2.20	.68	.04	1.95	0.70
20	19–21	2.20	.13	5.12	1.57	.70	.04	1.63	0.50
23	22–24	2.26	.15	4.50	1.13	.72	.05	1.43	0.36
27	25–29	2.33	.16	4.10	0.96	.74	.05	1.30	0.29
33	30–36	2.40	.19	3.80	0.72	.76	.06	1.20	0.23
42	37–48	2.48	.22	3.52	0.63	.79	.07	1.12	0.20
58	49–72	2.60	.25	3.33	0.54	.83	.08	1.06	0.17
97	73–144	2.83	.28	3.14	0.44	.90	.09	1.00	0.14
290	145–300	2.92	.31	3.00	0.38	.93	.10	0.95	0.12
∞	Rack	2.96	.34	2.96	0.34	.94	.11	0.94	0.11

Note.—With one setting of the compass draw all the face arcs, and with a second setting draw all the flank arcs. In this way a gear with all the teeth may be drawn in a few minutes.

Note.—In most shop assembly drawings of gears, the teeth are not drawn, the gears being indicated by pitch circles, with notes giving diameters, pitch, and number of teeth. In detail drawings, the outside diameter and many other dimensions are given, but, even there, the teeth are seldom drawn.

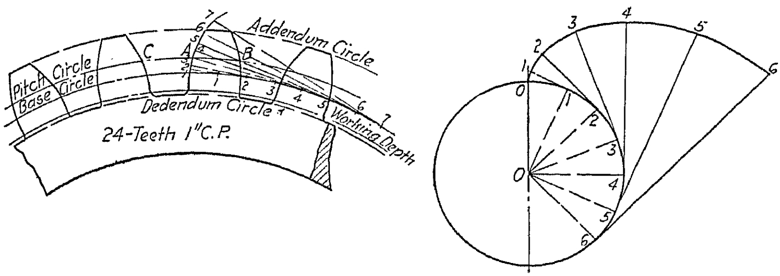


FIG. 178.—Exact method of drawing involute teeth. Method of drawing an involute on right.

108.—Involute Teeth—Exact Method.—(1) Draw the pitch, addendum, and dedendum circles, and lay off the pitch points A, B, C, etc.

(2) Draw the base circle.

Note.—For the $14\frac{1}{2}$-deg. involute, the base circle radius is approximately $\frac{29}{30}$ of the pitch radius (cos. $14\frac{1}{2}$ deg. $=$ .9682), and for 20-deg. involute (stub tooth) it is $\frac{15}{16}$ (cos. 20 deg. $=$.9397) of the pitch radius.

(3) Draw through A, B, C, etc., an involute from the base circle to the addendum. See Fig. 178*a* for method. Each tangent to the base circle is equal in length to the arc from the point of tangency to the origin of the curve.

(4) Complete the outline by drawing a radial line from the base circle to the dedendum, ending with a fillet inside the working depth circle.

Note.—The **working depth** is the limit of penetration of the engaging tooth, and = dedendum minus the clearance.

109. Involute Teeth—Approximate Method.—(1) Draw the pitch, addendum, dedendum, and base circles.

(2) Lay off the pitch points.

(3) Obtain the face and flank radii from Grant's Involute Table. With these radii, from O and O' (on the base circle) as centers, draw the face AB, and the flank AC (Fig. 179), and draw the remainder of the flank radial, ending in a fillet at D.

Note.—Above 36 teeth, one radius will draw the entire curve from B to C.

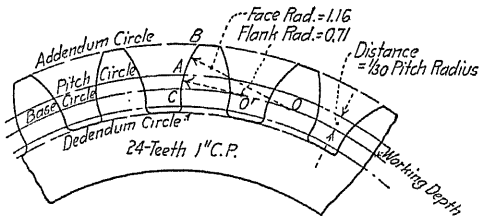


FIG. 179.—Drawing 14½-deg. involute teeth with the compass.

TABLE V.—INVOLUTE ODONTOGRAPH. STANDARD INTERCHANGEABLE TOOTH, CENTRES ON THE BASE LINE.

Teeth	Divide by the diametral pitch		Multiply by the circular pitch		Teeth	Divide by the diametral pitch		Multiply by the circular pitch	
	Face radius	Flank radius	Face radius	Flank radius		Face radius	Flank radius	Face radius	Flank radius
10	2.28	.69	.73	.22	28	3.92	2.59	1.25	0.82
11	2.40	.83	.76	.27	29	3.99	2.67	1.27	0.85
12	2.51	.96	.80	.31	30	4.06	2.76	1.29	0.88
13	2.62	1.09	.83	.34	31	4.13	2.85	1.31	0.91
14	2.72	1.22	.87	.39	32	4.20	2.93	1.34	0.93
15	2.82	1.34	.90	.43	33	4.27	3.01	1.36	0.96
16	2.92	1.46	.93	.47	34	4.33	3.09	1.38	0.99
17	3.02	1.58	.96	.50	35	4.39	3.16	1.39	1.01
18	3.12	1.69	.99	.54	36	4.45	3.23	1.41	1.03
19	3.22	1.79	1.03	.57	37–40	4.20		1.34	
20	3.32	1.89	1.06	.60	41–45	4.63		1.48	
21	3.41	1.98	1.09	.63	46–51	5.06		1.61	
22	3.49	2.06	1.11	.66	52–60	5.74		1.83	
23	3.57	2.15	1.13	.69	61–70	6.52		2.07	
24	3.64	2.24	1.16	.71	71–90	7.72		2.46	
25	3.71	2.33	1.18	.74	91–120	9.78		3.11	
26	3.78	2.42	1.20	.77	121–180	13.38		4.26	
27	3.85	2.50	1.23	.80	181–360	21.62		6.88	

Note.—Gears of 57 teeth and over have no radial flanks in the 14½-deg. involute, while stub tooth gears have no radial flanks on gears of 24 or more teeth. This is so, because the base circles coincide with the working depth circles at these sizes, and for larger numbers of teeth the base circle is inside the working depth circle.

110. Stub Teeth.—To draw by exact and approximate methods.—The exact method differs in no way from that of the 14½-deg. involute, except the following dimensions: addendum, dedendum, and base circle. These dimensions for most sizes

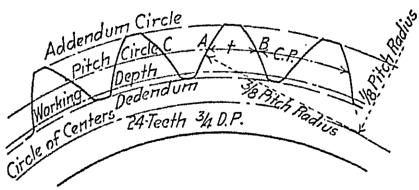


FIG. 180.—Drawing stub teeth with the compass.

will be found in Art. 98. The base circle radius is $^{15}/_{16}$ of the pitch radius.

Example.—What are the dimensions of a 24-toothed gear of ¾ diametral pitch?

Ans. For 3-pitch, c.p. = 1.047 in., ∴ the thickness at the pitch line = ½ c.p. = .524 in. This is also the thickness of the stub tooth.

The addendum and dedendum are figured from 4-pitch = .7854 in. c.p.

$$\text{Addendum} = \frac{1.000}{4} = .250 \text{ in.}$$

$$\text{Dedendum} = \frac{1.250}{4} = .3125 \text{ in.}$$

$$\text{Base Circle Radius} = \frac{15}{16} \times 4 \text{ in.} = 3\tfrac{3}{4} \text{ in.}$$

Approximate Method.—(1) Draw the pitch, addendum, dedendum, and working depth circles.

(2) Lay off the pitch points *A*, *B*, *C*, etc., each spaced an arc = ½ c.p.

(3) Draw the circle of centers with a radius of ⅞ of the pitch radius.

(4) With the center on this line and a radius equal to ⅜ of the pitch radius, draw an arc from the working depth to the addendum, and connect by a fillet to the dedendum.

Note.—This method is not official, but the outline thus drawn

will approximate a true outline except in gears of 12 teeth, or less. Such outlines should be **generated** by the draftsman. For gears having 13 to 23 teeth, the base circle must be drawn, and the outline between the base circle and the working depth, a very short space, must be blended with the fillet. It should be remembered that these approximate methods are merely for illustrations, and not intended to be workable in the actual gear.

METHODS OF GEAR MANUFACTURE

111. Gear wheels are made of many materials, cast iron, steel (of many varieties), brass, rawhide, wood, fiber, etc., depending

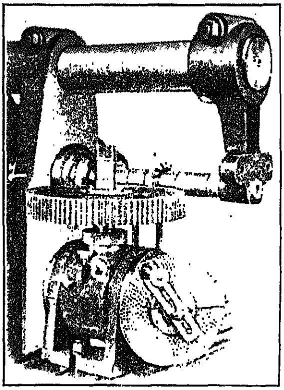


FIG. 181.—Milling gear teeth. Kempsmith Mfg. Co., Milwaukee.

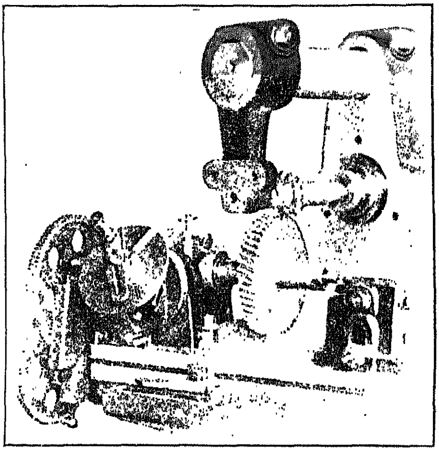


FIG. 181a.—Milling twisted or helical teeth. Kempsmith Mfg. Co., Milwaukee.

on their purpose. **Cast iron gears** are suitable for outdoor work, and for heavy, slow moving machinery. They are usually cast with the teeth formed, and modern foundry methods are so good that these gears can be used with no machine work on the teeth. For finer work, and especially on wheels of less than 12 in. diameter, the gears are usually cast in a blank (plain rims), and the teeth are cut in a milling machine or shaper. Medium sized gears are frequently cast with the teeth formed, and then finished in the machine, which requires the removal of only a small amount of metal. This is of doubtful advantage, because high speed steel cutters will remove large quantities of cast iron very

quickly, and there is less damage to the tool than when a thin cut is taken. Large castings are "gashed" with roughing cuts, and then finished.

Steel Gears are made from forged blanks, cast blanks, and blanks cut from rolled bars. They are made in many grades for their special duties, from common steel having a tensile strength of 50,000 lb. per square in., to alloys of vanadium, chromium, nickel, and molybdenum, which, when heat treated, will show a tensile strength of 250,000 lb. and upwards. The gears in the all-geared drill press shown in Chapter IX, are said to be made of steel having a tensile strength of 225,000 lb. per square inch. Automobile gears are made of similar steel, heat treated, and finished by grinding and sand blasting.

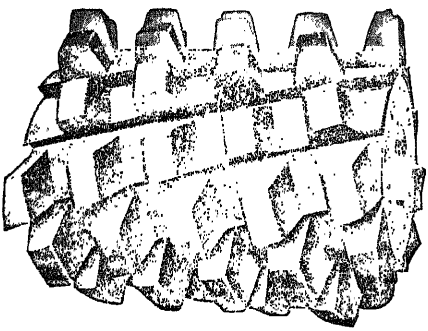


Fig. 182.

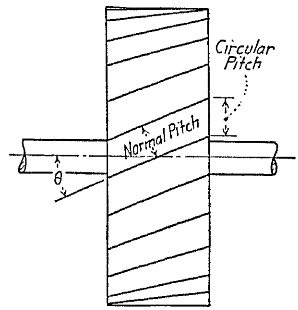


Fig. 183.

Fig. 182.—Gear hob. Brown & Sharpe Mfg. Co., Providence.

Fig. 183.—The helical pitch lines of a twisted gear. θ = helical angle, made 23 deg. in this figure.

Machining the Teeth.—Two general methods are followed, **forming** and **generating.** The forming method is the older, and is usually done on a milling machine. A special milling cutter, shaped to cut a groove of the approximate outline of the tooth, and called a gear cutter, is employed. After the blanks are machined, the shaft hole drilled, and the keyway slotted, the blanks are fixed on an arbor to which an index plate is attached. If the blanks to be cut are plain wheels (not cluster gears, or irregular in other ways), a number of them can be fastened on the arbor, and all cut together. The index plate enables the operator to turn the arbor through the pitch angle after one groove has been cut, or it may be set to perform automatically. When the cutter has passed over the work and has

made a groove all the way, the feed is reversed and the work returned to its original position, the work is revolved through an angle equal to the pitch angle, and is sent forward for a second groove to be cut, and so on until all the teeth are cut. In cutting twisted gears (also helical gears) on the milling machine, the arbor must be set so that the central plane of the cutter will make the helical angle with the right section of the arbor. The arbor is then rotated, synchronizing with its advance past the cutter, so that a helical groove of the proper lead is cut. In these gears, the cutter must fit the **normal** pitch, not the circular pitch of the gears.

Note.—In twisted gears both members of the pair must have the same circular pitch and the same normal pitch. In helical gear pairs the normal pitch must be the same, but the circular pitches may differ.

The table of gear cutters here appended is taken from the Brown & Sharpe catalogue. An inspection will show that a single cutter is used to cut gears of several numbers of teeth; *e.g.*, No. 3 involute cutter may be used to cut gears having 35 to 54 teeth, 20 different outlines in all. Needless to say, these outlines are not accurate in a scientific sense. They are commercially satisfactory, and answer the purpose, and this explains why gears must be broken in. The gears must work together for a time, and wear each other into perfect outlines before smooth running is accomplished.

TABLE VI.—GEAR CUTTERS

Cycloidal			Involute
A cuts 12 teeth	J—21–22	R— 50– 59	No. 1—135–rack
B 13	K—23–24	S— 60– 74	2— 55–134
C 14	L—25–26	T— 75– 99	3— 35– 54
D 15	M—27–29	U—100–149	4— 26– 34
E 16	N—30–33	V—150–249	5— 21– 25
F 17	O—34–37	W—250 up	6— 17– 20
G 18	P—38–42	X—Rack	7— 14– 16
H 19	Q—43–49		8— 12– 13
I 20			

The cycloidal system requires 24 cutters for each pitch, and the involute 8 cutters for each pitch.

Generating Teeth.—Most gears for fine work are **generated**; *i.e.*, they are cut in special gear shapers or hobbing machines, which give both the work and the cutter motion, while the machining is in process, identical with what they would have, if they were two similar gears in engagement. The figures

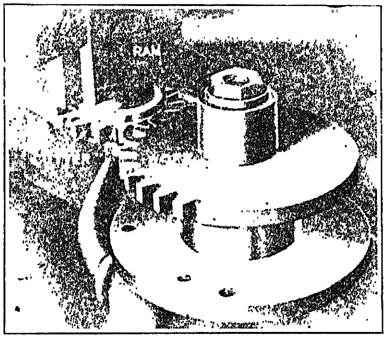


FIG. 184.—Generating cutter as used on the Fellows shaper.

here shown, taken from industrial literature, illustrate clearly the operation of gear generating machines. In the Fellows shaper, the cutter is made exactly like a pinion, such as may be in engagement with the gear about to be made. The cutter

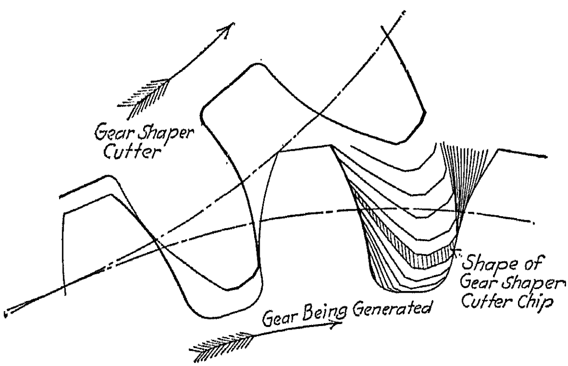


FIG. 185.—The cuts made by a generating cutter as it rotates with the work. Fellows Gear Shaper Co.

differs in one respect from the gear, in that its **addendum is long,** so as to cut into the clearance space. It is mounted on a vertical ram, and both work and cutter rotate between cuts at the proper speed ratio of two similar gears.

The Maag gear is made in the same way as is the Fellows,

but the cutter used is rack-shaped. Helical grooves present no difficulties to the Maag system, but internals cannot be man-

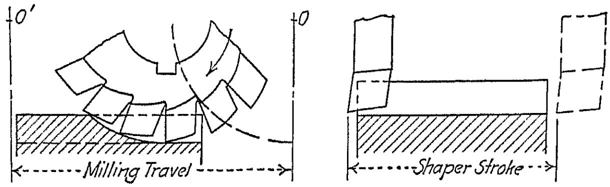


FIG. 186.—Comparison of cuts made by milling machine and shaper tool.

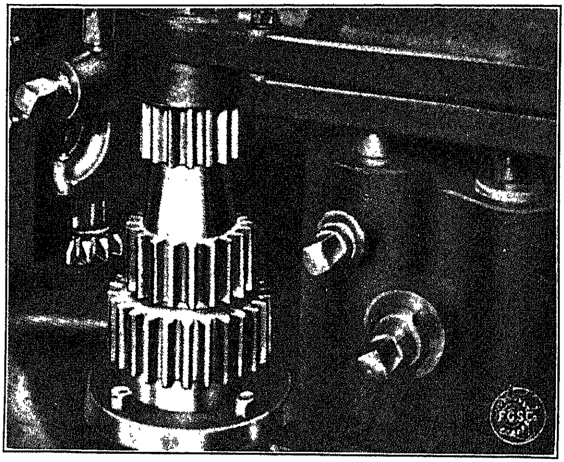


FIG. 187.—Cutting a cluster of gears on a Fellow shaper.

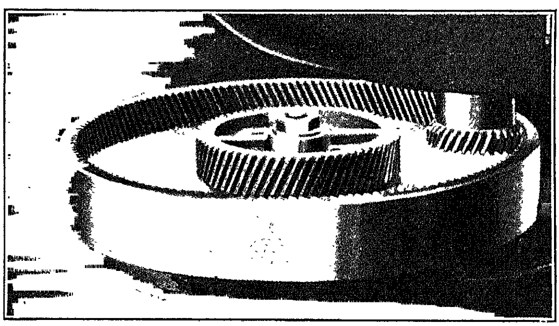


FIG. 188.—Cutting an internal twisted gear on a Fellows shaper.

aged. Very large gears are cut in the Maag process, and this seems to be its greatest opportunity, whereas the Fellows seems

to be particularly adapted for smaller and more specialized work. Where gears have to be cut close to shoulder, the shaper method only can be used, since a milling cutter must pass well beyond the groove at both ends of the cut, whereas the shaper needs only a small clearance at either end. Makers of machines of both types claim advantages for their product, and it is not the intention of this text to enter into discussion over it.

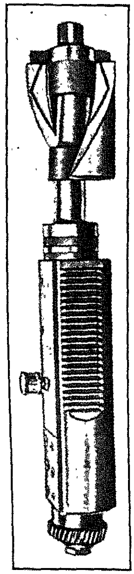


FIG. 189.—Attachment for giving the helical motion to a shaper cutter. Fellows Gear Shaper Co.

The hobbing method of cutting spur gears is largely used. It has the advantage of continuous cutting, whereas both the shaper and milling machine are obliged to give idle travel to their tools. The Simmons Method-Hob Company, as has been noted heretofore, manufactures a hob which cuts a tooth of its own design.

ELLIPTICAL GEARS

112. **Elliptical gears** differ from all other gears in that they **do not transmit a constant angular velocity,** and **do not want to.** Their reason for existence is the fact that they impart a rapid change of velocity. In the preceding chapter, attention was called (and the student may have observed it through his experience with some of the problems) to the fact that astonishing ratios of advance to return can be attained through the agency of rolling ellipses, and that there are at least two im-

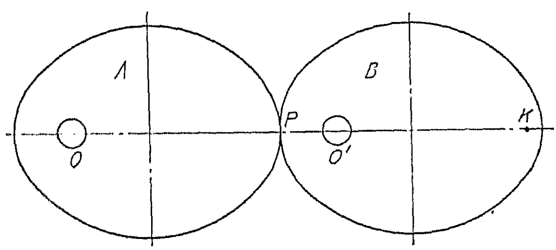


FIG. 190.—Pitch lines of elliptical gears.

portant fields where this property can be utilized to great advantage, **quick return motion** and **sudden impact.** The point

K, Fig. 190, has such a great acceleration and maximum velocity, that it can be used most effectively for heavy blows.

Thus far, the elliptical gear has not received much attention from machine designers, owing to lack of knowledge and appreciation of its qualities, and to the fact that its production was formerly expensive. This comes from the fact that each tooth in a quadrant must be different in outline, owing to the changing radius of curvature of the pitch line. In making cast gears this difficulty is slight, extending only to the making of the pattern. For rough work it is easy to obtain good results.

The difficulty in cutting elliptical gears in a milling machine is met by dividing the circumference of the pitch ellipse into

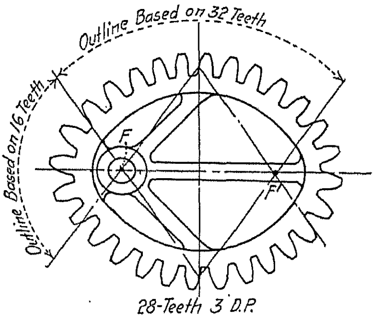


FIG. 191.—The method of laying out the pitch sections of an elliptical gear.

four section. These sections are approximately circular, and thus **two cutters** can be made to give satisfactory teeth, one for those on the end sections, and one for the center sections. Sub-dividing will make for more accuracy.

The generating process, however, meets with no difficulty whatever in making elliptical gears as cheaply, and as closely to theoretical perfection, as spur gears. The expense of making accurate cutters would be more than that of standard cutters, but that would be a small item in large production. This should open a very promising field to the engineer and designer of the future.

PROBLEMS

1. Draw an epicycloid and a hypocycloid, using $1\frac{1}{4}$-in. circles rolling on 5-in. pitch circles. If these curves were intended for standard gears, what should be the number of teeth?

2. Same problem. Generating circles ———— in., pitch circle———— in.

3. Draw an involute (180 deg.) on a base circle of 3 inches. If the pitch circle were $3\frac{1}{8}$ in., how many teeth should be cut if the d.p. were 8? What is the pressure angle?

4. Same problem. Base circle ———— in., pitch circle ———— in.

5. Two involute gears are generated on base circles of $3\frac{1}{8}$ and $1\frac{5}{8}$ in. diameter, on centers 5 in. apart.

Required.—(a) The pitch diameters of the gears.
(b) The pressure angles.
(c) If the dedendum is twice as far in as the smaller base circle, how many teeth should there be?

6. Same problem. Base circles ———— in. and ———— in., center distance ———— in.

7. Draw a cycloidal rack and 16-toothed pinion, d.p. = 4. Use exact method.

8. Same problem. Pinion ———— teeth, d.p. ————.

9. Same problem. Fourteen-and-one-half-deg. involute rack and pinion, ———— teeth, d.p. ————.

10. Same problem. Stub tooth rack and pinion, ———— teeth, pitch ———— ($\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{7}$, $\frac{6}{8}$, $\frac{7}{9}$) etc.

11. Draw any of Prob. **7** to **10** by the odontograph method.

12. Two cycloidal gears are mounted on centers at 14-in. distance, to run at 350 and 140 r.p.m. D.p. = 3. Shaft diameter = $\frac{1}{8}$ p.d. Hubs = twice the shaft diameter.

Required.—(a) Pitch diameters.
(b) Circular pitch.
(c) Tooth dimensions.
(d) Outside diameters.
(e) Size of generating circles.
(f) Draw the gears complete, with dimensions. Draw one tooth by exact method, remainder by odontograph.

13. Same problem. Center distance ———— in., r.p.m. ———— and ————, d.p. ————.

14. Same problem. $14\frac{1}{2}$-deg. involute gears. Center distance ———— in., r.p.m. ———— and ————, d.p. ————. (e) Instead of size of generating circles give size of **base circles**.

15. Same problem as **14.** Substitute 20-deg. stub tooth.

16. Substitute internal gear in Prob. ———— (**13, 14, 15**).

17. Given the data for Prob. ———— (**12, 13, 14, 15** or **16**), and substitute the following requirements:

(a) Maximum obliquity or pressure angle.
(b) Angles of recess and approach for each gear.
(c) Angle of contact for each gear.
(d) Show extent of contact on teeth of each wheel.
(e) Show the velocity of slippage at beginning and quitting contact.
(f) Show the percentage of driving component and bearing pressure to the total motive force of the driver, at the beginning and end of each tooth contact.

18. Draw a twisted gear and pinion, using the data and requirements of Prob. ———— (**12, 13, 14, 15** or **16**). Draw front and side views. Make the

lead of the helix $1\frac{1}{10}$ the c.p., and the face of the gear 3 times the circular pitch. How great is the helical angle? How great the normal pitch?

19. Design two elliptical gears having cycloidal teeth, of center distance 6 in., d.p. = 6, and the maximum and minimum angular velocities of the follower 2: 1 and 1: 2. Make a complete working drawing of one gear.

20. Same problem. Center distance ——— in., d.p. ———, angular velocities of follower, maximum ——— and minimum ——— (reciprocal).

21. Same problem. $14\frac{1}{2}$-deg. involute teeth.

22. Same problem. 20-deg. involute stub teeth.

MACHINE DESIGN PROBLEMS

Note.—In designing gears to carry a certain load at a certain speed, it is necessary that the student shall understand the use of if not the derivation of a few equations. The following equations will be needed in the solution of these problems.

(1) C.p. $= 4.1\sqrt{\frac{W}{sn}}$, where $W = \frac{2}{3}$ the tooth pressure calculated from the horsepower equation, $s =$ the safe stress per square inch, and n is a factor =face of gear ÷ c.p.

(2) C.p. $= 2.35\sqrt{\frac{W}{sn}}$, when stub tooth gears are specified.

(3) **Shaft diameter:** $D = 1.72\sqrt[3]{\frac{T}{S}}$ where T is the twisting moment,

$$T = PR = 63{,}000\frac{\text{Hp.}}{N} \quad \text{where } N = \text{r.p.m.}$$

Machine steel for shafting is used, and its safe shear (S) is from 6,000 to 10,000 lb. per square inch.

(4) **Hub diameter** = $2D$ (Shaft diameter).

(5) **Rim thickness** and **spoke** or **web thickness** are usually taken at $\frac{1}{2}$ c.p., or a little over that.

(6) **Square keys** = $\frac{1}{5}$ to $\frac{1}{4}$ the shaft diameter in breadth.

(7) **Hubs** are usually made $\frac{1}{10}$ p.d. longer than the gear face, to give the teeth a clearance, separating them from the gear guards, beams, etc.

Although these problems are somewhat outside the ordinary scope of kinematics, they are given for those who care to employ them, and to be disregarded by those who prefer to wait until Machine Design is studied as a separate subject.

23. Design a pair of $14\frac{1}{2}$-deg. involute spur gears, on shafts 18 in. between centers, running 420 and 120 r.p.m., transmitting 40 hp. Material, cast iron. Calculate the dimensions as far as possible, and use standard pitches, shafts, keys, always with regard to safety. Make complete working drawings of the wheels, with necessary title, bill of material, and notes for the instruction of the workmen. For cast iron, $s = 3{,}500$.

24. Same problem. Center distance ——— in., r.p.m. ——— and ———, hp. ———, material ———, s ———.

Note.—The safe tensile strength of steel runs from 5,000 to 25,000 lb. per square inch, depending on its quality.

25. Same problem, cycloidal gears.

26. Same problem, stub tooth gears, 20-deg. involute.

27. Same problem, internal gears, ——— outline ($14\frac{1}{2}$-deg. involute, cycloidal, or stub 20-deg involute).

BIBLIOGRAPHY

The best and most recent information on gearing is to be found in commercial publications, of which the following will be found especially helpful and authoritative.

Brown & Sharpe Manufacturing Company, Providence, R. I., "Practical Treatise on Gearing."

Philadelphia Gear Works, Philadelphia; "A Treatise on Gear Wheels," by George R. Grant.

Fellows Gear Shaper Company, Springfield, Vermont, "Commercial Gear Cutting," "The Involute Gear," "The Stub Tooth Gear," "The Internal Gear," "The Helical Gear."

CHAPTER VI

GEARS ON NON-PARALLEL SHAFTING

Non-parallel shafts are either **intersecting** or **non-planar.** This divides them into two groups, as follows:

Intersecting shafts..	Bevel Gears.
Non-planar shafts..	Worm Gears. Helical Gears. Hyperboloidal Gears.

BEVEL GEARS

113. This term is used to include all gears working on shafts whose center lines intersect at any angle. Their pitch surfaces

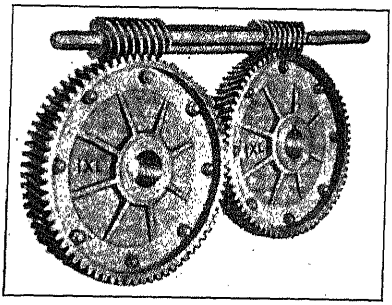


Fig. 192.—Tandem worm gears. The worms are triple thread, right- and left-hand, so that the gears mesh. This eliminates end thrust and prevents back pressure. Foote Bros. Gear Co., Chicago.

are **tangent cones,** and their velocity ratios are obtained in exactly the same manner as in the case of rolling cones. To lay out the pitch cones for bevel gears, mounted at any center-line angle and for any given velocity ratios, the method employed is exactly the same as that given for rolling cones in Art. **83.** There is no limit to the angles at which the center lines may intersect, but in commercial practice there are certain standard angles. On account of lack of demand for internal bevel gears, there are none manufactured, so far as this investigator knows. Their design, however, is not freakish, nor would they be diffi-

cult to produce, if the necessity for using them should arise. They present no new problems of design or manufacture, but are not of sufficient importance to discuss in these pages.

Classes of Bevel Gears.—Commercially, the term "bevel gears" is not employed for all bevel gears, but the varieties are classified, as follows:

(1) **Mitre Gears:** 90-deg. shafts, equal speeds.

(2) **Bevel Gears:** 90-deg. shafts, unequal speeds.

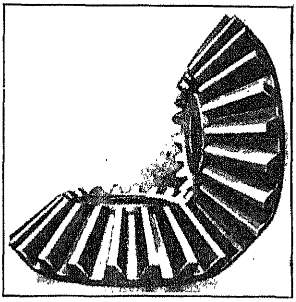


FIG. 193.—Mitre gears. Jeffrey Mfg. Co., Columbus, O.

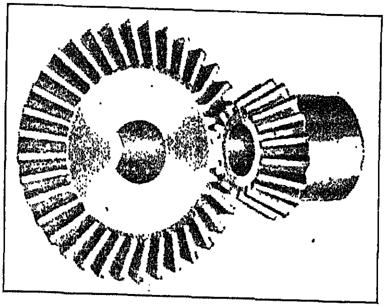


FIG. 194.—Bevel gears. Boston Gear Works, Boston.

(3) **Angle Gears:** acute or obtuse angled shafts, equal or unequal speeds.

(4) **Crown Gears:** in which the pitch cone of the larger gear is a plane (altitude = zero).

(5) **Spiral Bevel Gears:** gears of any of the above varieties, having curved grooves.

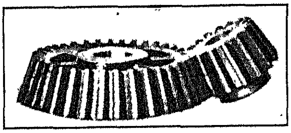


FIG. 195.—Angle gears. The Link Belt Co., Chicago.

NON-INTERCHANGEABILITY OF BEVEL GEARING

114. Scientifically, each bevel gear is designed for its mate, and for no other, but in practice this limitation is not always imposed. The layouts in Fig. 196 will illustrate how the bevel gear is thus limited. Figure 196*a* shows two gears, *A* and *B*, designed to work together on 90-deg. shafts. If it is desired to alter the speed ratio, without changing the angle of transmis-

sion (*e.g.*, to gear in B', shown by the dotted lines), the center line must be moved. The figure shows what poor design this would bring about, for the two pitch cones would then have separate apices, and it is obvious that proper design should have a common apex for both cones. The figure shows that if the pitch cone of B is changed in size, and therefore in its inclination to its axis, that the tangency of the pitch cones will be lost.

Figure 196*b* shows how interchangeability is possible, to a limited degree, **by changing the transmission angle.** This shift is not practicable in most transmissions, and is impossible in many cases. The practical method of avoiding this difficulty is

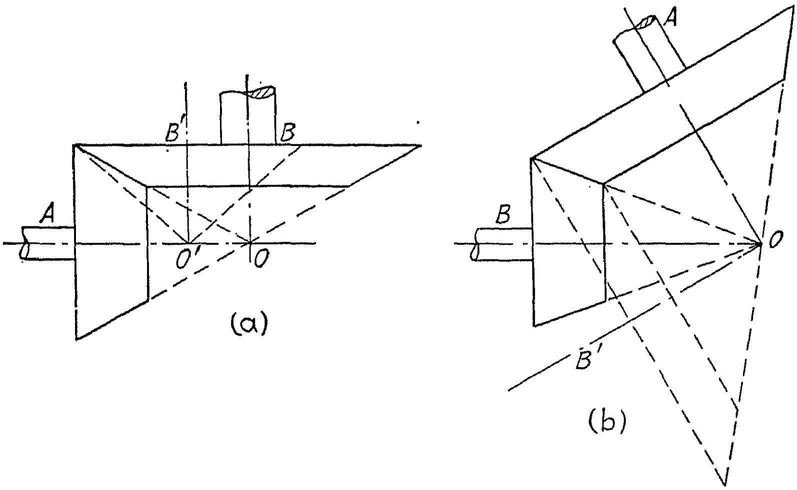


FIG. 196.

to use bevels that are designed to mate, and making the speed changes before the transmission is applied to the bevels. A good instance of this is in the automobile, where the speeds are changed **between** the motor and the differential.

DESIGN OF BEVEL GEARS

115. Bevel gear design may be viewed in two ways; from the purely kinematical standpoint, and from that of machine design. A kinematic problem requires of the student only the drawing of the outline and a few simple calculations of tooth dimensions from complete data furnished him. A machine design problem requires the working out of such items as the

proper pitch, length of face, diameters of shafting, hubs, keys, length of hubs, thickness of rims, etc., from assumptions relating to load, speed, and the materials best suited to the character of the work. Since pure kinematics in this case is nothing more than a drawing exercise, it is deemed advisable to examine the design under the conditions found in commercial work. The subject of **Machine Design** concerns itself with the **derivation** as well as the **application** of the equations, but **Engineering Kinematics** takes the derivations for granted, and confines itself to the **application** only. Sufficient material will be found in the problem section to satisfy teachers who prefer to limit themselves to kinematic considerations and those who wish to employ engineering data.

DATA REQUIRED

(1) The r.p.m. of each shaft.
(2) The shaft angle.
(3) The circular or diametral pitch.
(4) The number of teeth, or the pitch diameters.

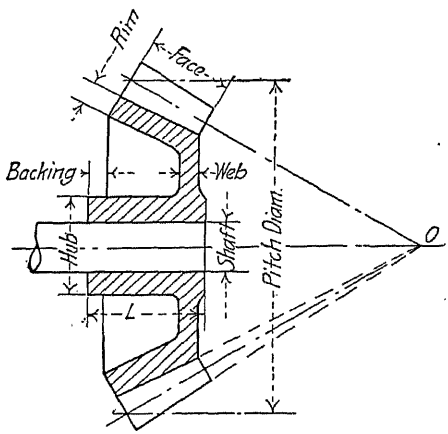


FIG. 197.—The dimensions of a bevel gear which require calculation.

Note.—Since there is no center distance, because the shafts are not parallel, circular pitch is used in bevel gearing about as much as diametral pitch. The convenience of diametral pitch applied to spur gearing is not as pronounced here. An examination of catalogs will prove that each are about equally employed.

Dimensions to be Calculated.—From the assumptions just given, a pair of bevel gears can be worked out, drawn, and made. The sketch, Fig. 197, gives the principal dimensions to be determined, assuming that the pitch cone angles have been previously worked out as in Art. **83.**

(1) **Pitch Diameter.**

$$D = \frac{N \times \text{c.p.}}{\pi},$$

if the circular pitch is given, or

$$D = \frac{N}{D.P.},$$

if the diametral pitch is given.

Note.—If the diameter is given instead of the number of teeth, these equations may be transposed.

Note.—In commercial practice D is the large diameter of the pitch cone. In theory it should be the mean diameter for exact calculations.

(2) **Face.** This dimension is to some extent arbitrary, and is made $= n \times$ c.p., where n is a factor, ranging from 1 to 5, limited by the slant height of the cone. The reason for making this factor a variable comes from the fact that the strength of the tooth depends on its **length** and **thickness.** In light transmission and good materials a small value may be assigned to n, and if the transmission is so heavy that teeth sufficiently strong would be clumsy, they reduce the thickness by increasing the length. In commercial gear catalogues of **cast iron** gears, this value is found ranging from 1½ to 4½. **Steel** gears are made shorter, and 1 is probably the most common value for n.

(3) **Rim thickness** and **Web thickness.** These dimensions are usually made = 0.5 c.p. in cast iron gears. This is stronger than necessary, but it is desirable for castings to be uniform in the thickness of their various walls, to avoid imperfections due to uneven shrinkage. In steel gears these dimensions have no set value, and may be made to suit the necessities of the machine.

(4) **Shaft Diameter.** This depends on the power transmitted, and is usually figured to the nearest standard diameter above the diameter actually calculated. The stress imposed on the shaft is mainly **torsion,** but if the bearings are not close to the hub, there is a bending moment that must be considered, which

necessitates a larger shaft. Where twisting alone is considered, the shaft is calculated: $D = 1.72\sqrt[3]{\frac{T}{S}}$, where T is the **twisting moment** (torque). $T = P \times R$, tooth pressure times pitch radius, and s is the safe shear per square inch. In the horsepower equation,

$$T = PR = \frac{63{,}000 \text{ hp.}}{\text{r.p.m.}}$$

Shafting is usually of machine steel, and 8,000 is a reasonable safe shear. In stock cast iron gears, the rule for shafting is $D = 2 \cdot$c.p. for gears less than 25 teeth, and $D = 3 \cdot$c.p., for larger gears.

Automobile gears are mounted on "splined" shafts; that is, shafts having integral keys. This means that the shaft is not cut into for the key, which adds considerably to its strength and that of the key. The material for these shafts is very high grade, and the safe torque is 20,000 lbs., or higher.

(5) **Hub Diameter.** Arbitrary, usually $= 2D$.

(6) **Hub Length.** Arbitrary, usually $= 1.3 \times$ face.

(7) **Backing.** Depends on the design of the machine, as to clearance desired, and the size of the gear. In stock cast iron gears it is made $= \frac{1}{10} D$.

Note. D in all these equations is **pitch diameter.**

CALCULATING CIRCULAR PITCH

The size of the tooth depends on the character of its service, the load it must carry, and the quality of the material. The pitch circle velocity often enters into the computation, because the impact and heating effects increase rapidly with the speed. For ordinary work, cast iron gears operating at peripheral speeds of less than 1,000 ft. per min., the following equation is satisfactory:

$$\text{c.p.} = 4.1\sqrt{\frac{W}{ns}}$$

W is the tooth pressure and $= \frac{2}{3}P$ in the equation, hp. $= \frac{2\pi RNP}{33{,}000}$, n is the arbitrary factor $= \frac{\text{face}}{\text{c.p.}}$, and s is the safe tensile strength. Most designers use $W = \frac{2}{3}P$, because in well designed gears there should be a division of the load over 2 or more teeth, and

this figure is considered conservative. The larger the number of teeth in the gears, the larger will be the number dividing the load. The resourceful designer will take advantage of this fact.

A safe value for circular pitch of **stub teeth** is to make the pitch = $\frac{2}{3}$ of the value determined by the foregoing equation, or the Lewis equation which follows.

The **Lewis** formula is more complicated, and is much used by designers where the job calls for heavy loads carried at high speeds. For the $14\frac{1}{2}$-deg. involute and the cycloidal systems,

$$\text{c.p.} = \sqrt{\frac{W}{ns(.124 - \frac{.684}{N})}}$$

To use this equation properly, a hand book should be employed so that variations for each style of outline and each number of teeth can be utilized. The safe stress varies for different velocities. The following list of safe values is taken from Professor H. L. Nachman's "Elements of Machine Design." *

TABLE VII.—SAFE VALUES FOR STRESS IN GEAR TEETH

Velocity of pitch line in ft. per min.	100 or less	200	300	600	1,000	1,500	2,000
S for cast iron....	6,000	4,500	3,600	3,000	2,500	2,000	1,800
S for machine steel	15,000	12,000	10,000	9,000	8,000	6,000	5,000

For excessive shock, as in rolling mill machinery, rock crushers, punches, bull dozers, these values should be decreased. For alloy steels properly heat treated, values can be used as high as five times the values for machine steel.

Trial Calculations.—Frequently engineers will find that the data supplied to them will be insufficient to use in solving the foregoing equations; that is, **two unknowns** will exist. For example, the r.p.m. and the hp. of two wheels may be given, which would leave both the pitch and the number of teeth to be determined. It is obvious that 24 and 48 teeth will give the same speed ratio as 36 and 72, but, if the same pitch for both pairs were used, the latter pair would be larger, stronger, and

* NACHMAN, H. L., "Elements of Machine Design," J. Wiley & Sons, New York.

better running. If the latter pair were given a smaller pitch, the teeth would be weakened but the tooth pressure would be less, so that there would be compensating advantages. The expedient resorted to is that of **trial** calculations; *i.e.*, assuming a value for one unknown and solving for the other, until a pitch is found that will be strong enough for the job. Problems involving this kind of thinking on the part of the student are presented at the end of this chapter, and are examples of every day problems in industrial engineering.

DRAWING BEVEL GEARS

116. Tredgold's Approximation.—When it is necessary to draw a fairly accurate representation of a bevel gear, the following method is employed.

(1) Draw the center lines and lay out the projections of the pitch cones, according to their specifications as to axial angle and respective speeds.

(2) Draw the normal cones OA and $O'B$, perpendicular respectively to the pitch cones. The normal cone contains the tooth outline, and the problem resolves itself into one of **descriptive geometry,** that of locating the projection of points on the surface of a cone.

(3) Develop the surfaces of the normal cones.

(4) Draw a single tooth on each surface so developed, as though it were a spur gear of that pitch and diameter.

Note.—The diameter of this developed cone will be greater than the diameter of the base of the pitch cone, therefore the tooth outline will be that of a larger number of teeth than actually obtains in the gear.

(5) Draw the end views of the pitch cones, together with the addendum and dedendum circles of both bases of the frustum.

(6) Divide the pitch circle into pitch points as in spur gears, and lay off the tooth-widths, obtained from the outline on the normal cones, on the addendum and dedendum circles in each tooth space.

(7) Draw radial lines outlining the teeth, from the points in the outside addendum, pitch, and dedendum circles to the corresponding inside circles, and draw the tooth profiles at each end of these radial lines.

Note.—These tooth profiles will be foreshortened, and apparently stubbier than the actual teeth. Why?

(8) Project these points to the side elevations of the gears, and connect corresponding points with lines running toward the common apex of the pitch cones.

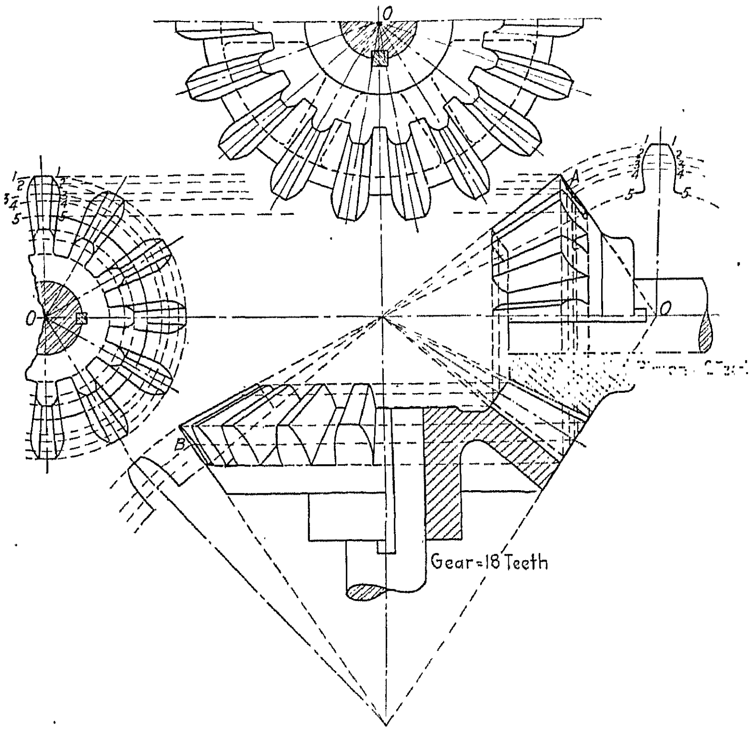


FIG. 198.—Bevel gears drawn by Tredgold's approximation. Each gear shown in half section.

WORKING DRAWINGS OF BEVEL GEARS

117. In commercial drafting the tooth outlines are seldom drawn except for catalog work. Detail drawings are made similar to Fig. 199*a*, which gives all the information necessary to the shop for its manufacture, by dimensions and notes. For layout sketches and large assemblies of machines, a mere skeleton, like Fig. 199*b*, showing the pitch cones and normal cones (and not always even drawn to scale), is all that is required.

THE MANUFACTURE OF BEVEL GEARS

118. Although milling the teeth in bevel gears is still in considerable practice, the generating of the teeth is supplanting

it to a very large extent. The **milling process**, shown in the illustration, is substantially as follows. The blank to be cut

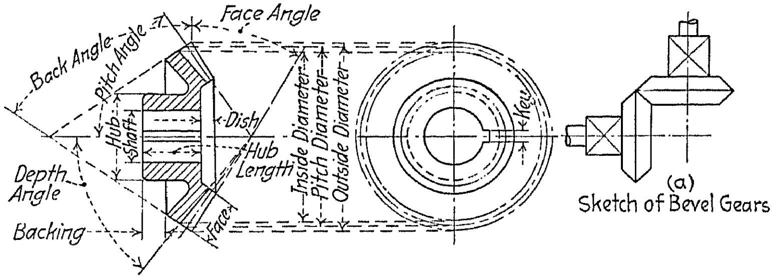


FIG. 199.—Working drawing of a bevel gear, showing all dimensions necessary for production. Sketch (a) shows manner of representing a bevel gear in an assembly drawing.

is placed on an arbor in such a position and at such an angle that the cutter edge passes through the imaginary apex of its

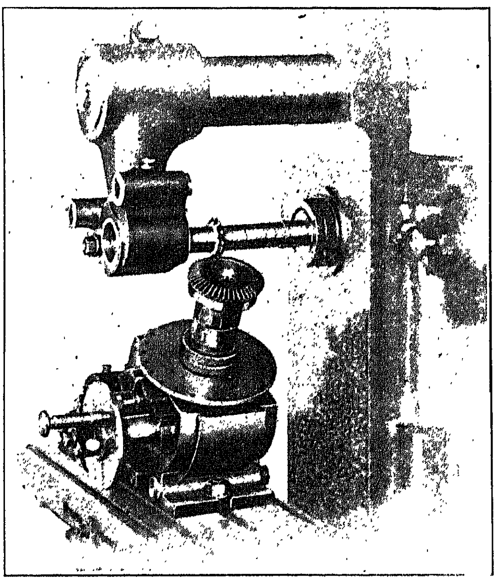


FIG. 200.—Milling the teeth of a bevel gear. Brown & Sharpe Mfg. Co., Providence.

pitch cone, as the blank is fed past the cutter. Inasmuch as the groove of a bevel gear is tapering toward the apex, the opening at the smaller base must be smaller than the opening at the

larger base. This means that the cutter cannot be made larger than the small end of the groove. The cutter then is made to form the tooth outline first on one side of the tooth (or groove), and then take another cut on the other side. This gives only an approximation to the theoretical surface of the tooth, because the theoretical surface is conical, and the outline at any point

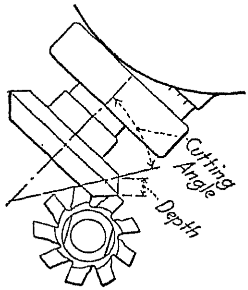


Fig. 200a.—Diagram showing how the gear blank is mounted for milling. Kempsmith Mfg. Co., Milwaukee.

is a different one from that of the adjacent section. The result is, that the surface as cut by a milling cutter can only be a compromise with the proper surface.

The high speeds, heavy duty and increased efficiency, demanded of modern transmission, require more accurately cut

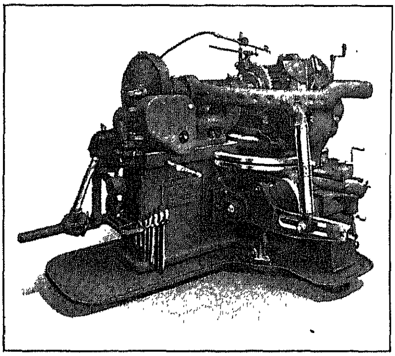


Fig. 201.—Bevel gear generator. The Gleason Works, Rochester.

teeth than milling machines can turn out, and **generating machines,** mostly of the shaper type, have been devised. Of these, the best known are the Gleason and the Bilgram planers, both of which operate on the principle of making the point of the tool travel directly toward the apex of the pitch cone on

every cutting stroke. The blank, as shown before, must be mounted so that the point of the cutter passes through the apex in the direction of the groove. The tooth outline is generated by revolving the tooth on its axis between cutting strokes. The depth of the cut is regulated by swinging the holder about an axis through the apex.

Another method accomplishes this, not by swinging the gear blank about the apex, but by causing the tool to move in the proper outline. This is done by rolling the tool carrier over a template, having the outline of a magnified gear profile, so that the depth of the cut is changed as the point of the cutter travels in the line of the involute.

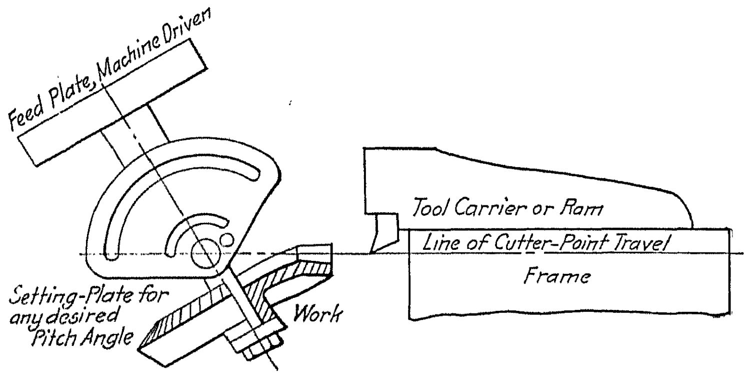


Fig. 202.—Mounting a gear blank in a shaper.

Probably the most advanced type is the double-cutter generator. In this machine, two cutters with straight edged blades cut on both sides of the tooth, as shown in Fig. 203. The angle of these cutters is that of the rack of the involute of that type, and the blank is rotated as the cutters travel alternately to and from the apex. This generates a tooth in much the same manner as the shaper-generator operates on spur gear outlines.

Although the teeth generated by these straight edged cutters are called involute teeth, they are in reality a slight departure from the true outline, being of the type known as octoid. The reason can be given in this way: As the action of the cutter in the spur gear generator is that of a rack or pinion engaging the gear that is being cut, so the bevel gear cutter is supposed to operate as a **crown gear** engaging with the work in process. The crown gear differs from the involute rack in this respect:

the involute spur gear rack has straight line teeth, and the crown gear has involute-shaped outlines for its teeth. This is true, because the crown gear pitch surface must be regarded as a cone, and as such, its tooth outlines will be curved when properly made. This technical distinction has not imposed any objections on the octoid tooth, and the running qualities of the gears made by this method are excellent. Teeth made by the template

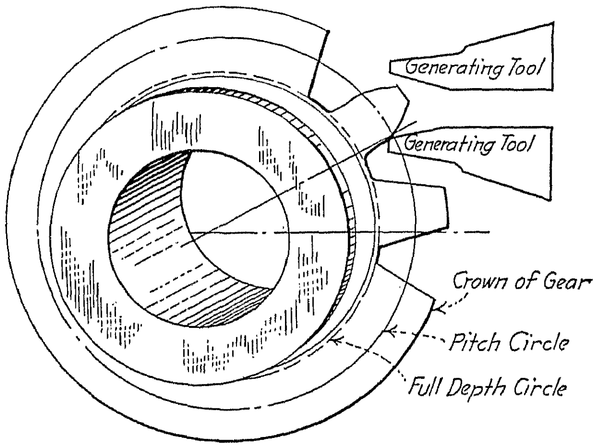


FIG. 203.—Generating bevel gear tooth outlines on a Gleason planer. Both outlines are generated simultaneously.

method of guiding the cutter may be made truly involute, if a correctly designed template is used. Cycloidal teeth are no longer used for bevel gears, so far as this writer knows.

SPIRAL BEVEL GEARS

119. With the introduction of special machines for cutting grooves in the shape of a conical helix, instead of in a straight line, the manufacture of spiral bevel gears has become commonplace in the automobile industry. The term spiral is not a misnomer in the case of bevel gears, because the center line of the groove projects on the circular view as a **Spiral of Archimedes.** Figure 206 shows the two views of a conical helix, and this should be self-explanatory in its application to gear design.

The spiral bevel gear is believed to possess the same superiority over the straight grooved bevel, as the twisted gear or herringbone obtains over the ordinary spur. Probably the ad-

vantage is there, but if not, no harm is done, for it is at least as good, and costs no more to produce.

There are two methods of cutting the spiral bevel gear:

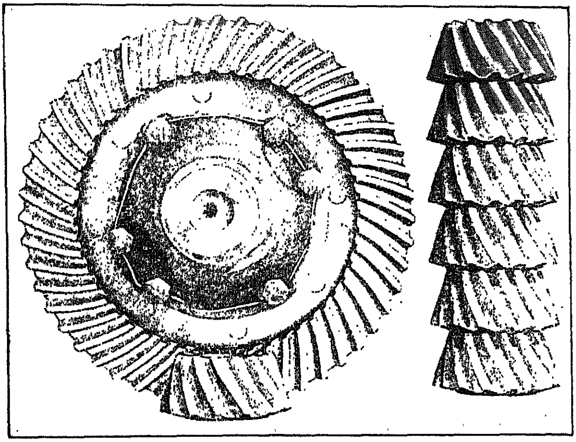


FIG. 204.—Spiral bevel gears. Foote Bros. Gear Co.

(1) The same as that employed in the shaper-generator method of cutting straight grooves, except that the blank is

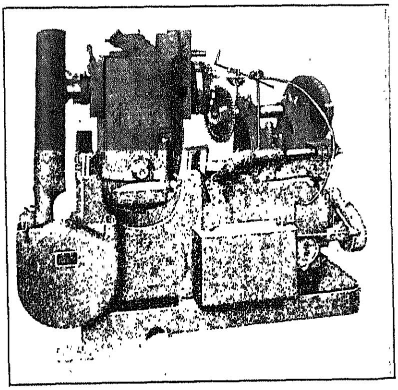


FIG. 205.—Gleason spiral bevel gear generator.

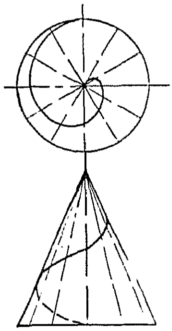


FIG. 206.—Conical helix.

rotated on its axis during the cutting stroke, in order to cut a helical groove.

(2) An approximate method, sometimes called gashing. The gear blank, G (Fig. 207), is mounted on an arbor, so that its

pitch cone is tangent to a vertical plane. The cutter (or knife) revolves in the vertical plane. The radius of revolution of the point of the cutter is such, that the point will pass through three points on the conical helix, as the gear blank is rocked about its apex. While this groove is circular, and therefore not a true helix, it should be remembered that the frustum of the cone for such gears is only a small fraction of the entire cone, and the error in making a helix in this manner is slight. This method of production is a milling method and is rapid in operation, and the resulting gear is as near a perfect one as most machine-made parts can be.

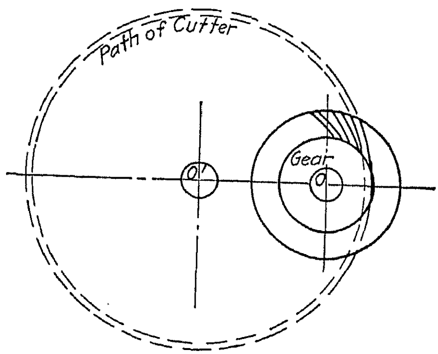


FIG. 207.—Diagram of approximate method of cutting spiral bevel gears. The circular arc approximates that of the conical helix.

HEAT-TREATED GEARS

120. Modern transmission, with its heavy loads, high speeds, and small wheels with a large number of teeth, is very destructive of gears, chiefly through **wear.** Stripped gears are known, but the greatest failure is because of worn teeth. Heat-treated gears of chrome-nickel, or 3½ per cent nickel steel, will give remarkable service. The term heat treatment includes carbonizing, annealing, hardening, and case-hardening, and the treatment varies with the chemical constituents of the steel and with the purpose for which it is intended. A pair of gears of high grade alloy and heat-treated properly will outwear four pairs of common steel gears, and probably more than that. Since the cost is not four times as much, it is by far the best plan to adopt gears made of high-tension alloy steel, carbonized on the surface, and heat-treated to give a wear-resisting shell, with a tough center.

HELICAL GEARS

121. When transmission is desired from one plane to another at **low speed ratios,** the helical is usually employed. Helical gears are made the same as twisted gears, and when the helices are right- and left-handed, and the circular pitch and helical angles the same, they can be mounted as twisted gears. Generally, however, the two gears have radical differences, and should not be confused. One point of variance should be understood: Two twisted gears must be opposite handed, have the same helical angle, and the same circular pitch, whereas the members of a helical pair have the same direction of helix, but may have differing circular pitches and helical angles.

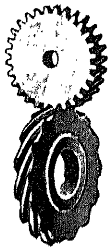


FIG. 208.—Helical gears. Foote Bros. Gear Co.

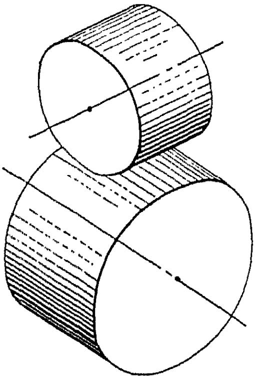


FIG. 209.—Pitch cylinders of a helical gear pair. Point contact.

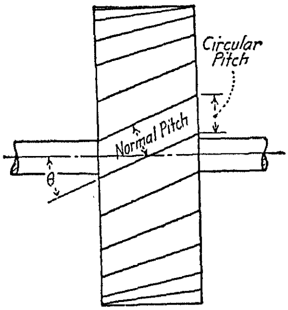


FIG. 210.—Normal pitch and circular pitch of a helical gear.

Two cylinders on parallel axes have **line contact,** whereas cylinders on non-planar axes (Fig. 209) have **point contact.** When this latter pair is projected on a plane parallel to their axes, the angle between the axes may be any desired angle. The speed ratio may also be any reasonable amount. In practice, **most** helical gears are mounted at 90 deg., and the speed ratio is usually **unity.** Theoretically there is no limit to either, and other angles and ratios are found in actual machines. The speed ratio seldom exceeds 3:1. Helical gears are sometimes called **screw gears,** and many firms call them **spiral gears.**

122. Relation of Speed Ratios and Diameters.—In contrast to spur and bevel gears, the velocity ratio of a pair of helical

gears is not necessarily in the inverse ratio of their diameters. Since the **actual** width of tooth and groove in each pair is the **normal** (perpendicular) **pitch** (Fig. 210), it is obvious that both gears must have the same normal pitch. Since they may have different helical angles, it is evident that they may have different circular pitches. From Fig. 210, the relation of the two pitches is $n.p. = c.p. \cos. \theta$. Therefore, as the number of teeth is the pitch circumference divided by the circular pitch, the speed ratio

$$= \frac{D' \cdot \cos \theta'}{D \cdot \cos \theta}.$$

The sum of the helical angles of the pair is equal to the axial angle ϕ.

HELICAL GEAR DESIGN

123. The design of helical gears in relation to strength is accomplished no differently from that of spur gears. There are other conditions that alter the running design and the proportions. The assumptions necessary are as follows:

(1) The angle θ, projected angle of the axes;
(2) The distance L between the axes;
(3) The speed ratio.

In following up the design from this data, there is considerable latitude allowed the designer to vary the remaining dimensions. The following dimensions must be determined, and since each is dependent on the other, two of them must be assumed, and the third worked out from the assumptions. This often results in impossible conditions, which compel changes in the assumed dimensions in order to effect a possible working basis. The variable values to be determined are the following:

(1) The helical angle θ, of **each** gear:
(2) The diameters, or the number of teeth in each gear:
(3) The normal pitch.

Two of the three must be assumed to determine the third. Probably it is wisest to choose the normal pitch first, then the diameters. This (from the speed ratio given in the original data) will determine a workable number of teeth, and with this information, the circular pitch can be worked out, and the helical angles determined from the normal pitch. The determination may be accomplished by mathematical calculations or by graphical methods.

124. Example.—Data.—$\phi = 90°$
$L = 6$ in.
r.p.m. = 200 and 100.

Assumption.—Normal pitch = 8 d.p.

DIAMETERS EQUAL

Required.—The helical angles, θ and θ', and the number of teeth in each.

(1) Since the speed ratio = $\frac{2}{1}$, the number of teeth in A must be twice that in B.

Therefore, since $N = \dfrac{\pi D}{\text{c.p.}}$, and $\text{c.p.} = \dfrac{\text{n.p.}}{\cos\,\theta}$, and that D, n.p., and π are the same for both gears,

$$N : N' :: \cos\,\theta : \cos\,\theta'.$$

Therefore,

$$\cos\,\theta = 2\cos\,\theta'.$$

(2) $\phi = 90$ deg., $\therefore\ \theta' = 90\text{ deg.} - \theta.$

$$\therefore\ \cos = \theta\, 2\sin\,(90\text{ deg.} - \theta).$$

(3) From an examination of the table of natural sines and cosines, we find that cos 26 deg. 30 min. = .89493, and cos 63 deg. 30 min. = .44620, hence these angles will give approximately a 2 to 1 speed ratio.

(4) To obtain the number of teeth in each, obtain the circular pitch of each, and divide the pitch circumference by the pitch thus found.

$$\text{c.p. of } A = \frac{.3927}{.89423} = .4388 \text{ in.}$$

$$\text{c.p. of } B = \frac{.3927}{.44620} = .2194 \text{ in.}$$

$$D^A = 6 \text{ in. } \therefore\ N^A = \frac{\pi \cdot 6}{.4388} = 42.9 + \text{teeth}$$

$$D^B = 6 \text{ in. } \therefore\ N^B = \frac{\pi \cdot 6}{.2194} = 85.9 + \text{teeth},$$

say 43 and 86 respectively.

Comment.—This calculation shows the necessity for approximations and compromises. Inasmuch as the tooth numbers cannot be made integers from the data and assumptions, and as a fractional tooth is impossible, one of two values must be changed. The two that are easiest to change are the distance between centers and the normal pitch. If it is possible to change the center distance a small amount, this should be done to accommodate 43 and 86 teeth. This is best, because standard cutters should be used wherever possible. If, however, no change in this distance is permissable, calculate a normal pitch that will give the correct number of teeth. Other dimensions may be altered, the pitch diameters, for example, and in many cases it will be best to do this. If the exact speed ratio is unimportant, it can be changed to suit. This is a case like many others in engineering, where the common sense of the man must decide what compromises or substitutions must be made to give what seem to be the best results.

GRAPHICAL METHOD

125. The following method of determining graphically the diameters of a pair of helical gears, to run at a given speed ratio at a given center distance, and a given normal pitch, is taken in substance from Barr's "Kinematics of Machinery." The reader is referred to that work for proof and more extended treatment.

(1) Draw two lines Ox and Oy, of indefinite length, making the included angle equal to the axial angle ϕ.

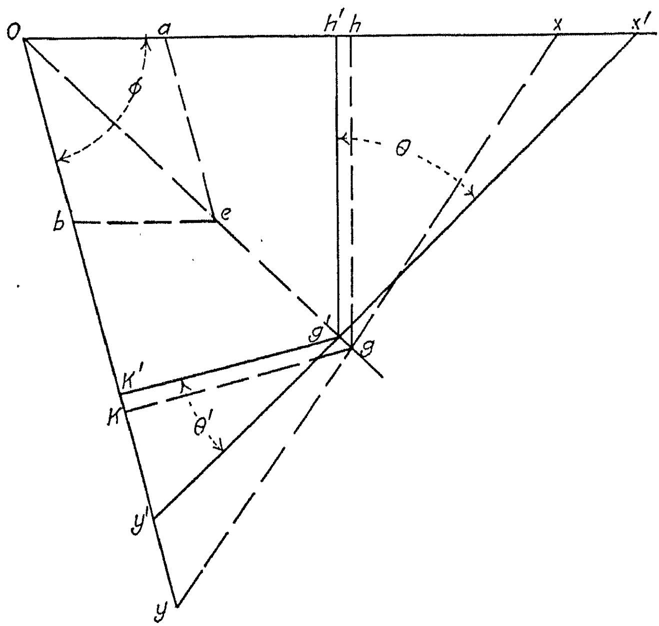


FIG. 211.—Graphical method of determining helical angles and diameters of helical gears.

(2) Lay off on Ox the distance Oa, and on Oy the distance Ob, proportional respectively to the given angular velocities.

(3) Construct the parallelogram $Oaeb$, and draw the diagonal Oe, continuing it indefinitely.

(4) Take a length $= 2L$, the center distance, $(L = R + R')$. Lay this length off, so as to terminate in Ox and Oy, so that it will intersect Oe at g, so that Og will be a maximum (determined by trial).

(5) Draw gh and gk, respectively $\perp$ to Ox and Oy.

(6) Multiply gh and gk by the normal diametral pitch to obtain **approximate** values for N and N'.

(7) For **actual** values, take the largest whole numbers **below** the values obtained in (6), that will satisfy the equation

$$\frac{\omega}{\omega'} = \frac{N'}{N}.$$

(8) Compute $\frac{N}{\text{d. p.}}$ and $\frac{N'}{\text{d. p.}}$, and lay them off $= h'g'$ and $k'g'$, respectively $\perp$ to Ox and Oy.

(9) Through g' draw $x'y'$ equal in length to xy.

Conclusion.—The respective diameters are equal to $g'x'$ and $g'y'$, and the helical angles will be θ and θ'.

Note (1).—This method is also approximate, and the same compromises and corrections will be necessary as in the method preceding this.

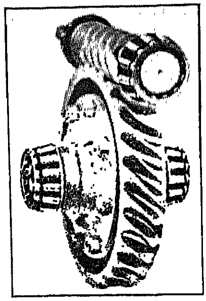


FIG. 212.—Worm and gear. Timken Bearing Co., Canton, O.

Note (2).—It seems well to repeat that there is no difference **in appearance** between **twisted** and **helical** gears, that under certain conditions one can be used for the other. Two equal gears, having respectively right- and left-hand helices, will drive on parallel axes. If a third gear be introduced, a duplicate of either, the resulting combination of the two that are alike will be a helical gear driving on non-planar axes. The axial angle at which these gears operate will be the sum of their helical angles.

THE WORM GEAR

126. The Worm Gear is a variety of helical gear, essentially the same, but differing widely in purpose, design, manufacture, and speed ratio. It consists of two members, the **worm** (driver) and the **gear** (follower), which rotate in planes at right angles to each other. Aside from the incidental and unimportant fact that its axes are non-planar, its chief importance is that of speed reducer. Speed ratios as high as 126 to 1 are listed in catalogs. The significance of this ratio for a single pair can be seen from a comparison of a typical **spur gear** pair giving the same reduction. The worm gear of 126 to 1 reduction shows (Fig. 213) a height of the housing of 54 in. (4 ft. – 6 in), and

the width about the same. A spur gear pair, with a 12-toothed pinion and a 1,512-toothed gear (!), of the same pitch, would be over 40 ft. in height and breadth. The spur pair would be more efficient, but the pinion would wear out quickly, and the compactness of the worm pair would decide immediately any question of their practicability.

Note.—The equivalent of this great reduction is also effected by the employment of a gear train, without using an abnormally large wheel. This subject is taken up later.

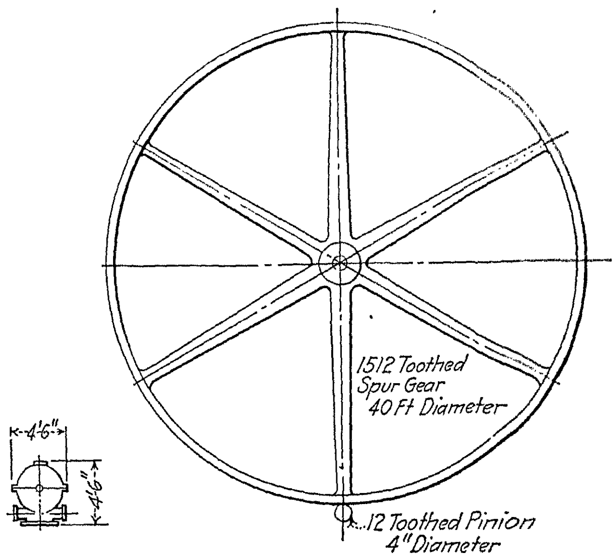


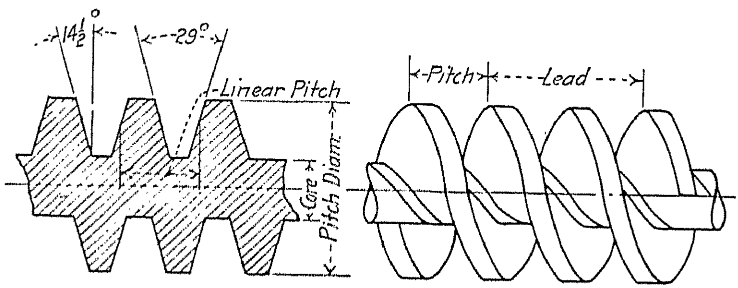
Fig. 213.—Comparison of the sizes of a worm gear and spur gear pair of equal functioning.

THE WORM

127. The worm is a screw thread, mounted on bearings, so that its motion is that of plane rotation. The outline is about the same as that of the acme thread, and its axial section is the same as the outline of an involute rack. The helix may be right or left-handed, and may have any desired lead. The groove is cut at a 29-deg. angle, the helicoidal angle being $14\frac{1}{2}$ deg. **Pitch** in a screw thread is the distance between cuts, see Fig. 214. **Lead** in a screw thread is the pitch of the helix; *i.e.*, **the advance of a point on the helix in one revolution about the axis.** In a **single** thread, lead equals pitch; in a **double** thread, lead equals

twice the pitch, etc. The lead is the corresponding multiple of the pitch, and in a multiple thread (triple, quadruple, etc.) there are that number of grooves cut in the metal.

Speed Ratio.—When a worm and gear are in engagement, the worm moves the gear through an angle determined by the lead of the helix. If the lead is **one**, the gear is moved through an angle whose arc is equal to the circular pitch of the gear. If the thread is **double** (lead equals 2), the gear turns through **twice the pitch**, etc. Therefore, the equation is $\frac{Worm}{Gear} = \frac{N}{M}$, where N is the number of teeth in the gear, and M is the multiple of the thread in the worm.



Axial Section of a Single-Thread Worm Pitch and Lead of a Double-Thread Worm

FIG. 214.

Example.—A single-thread worm is in mesh with a 48-toothed gear. The r.p.m. of the worm is 480, how fast does the gear turn?

$$\frac{Worm}{Gear} = \frac{N}{M} \therefore \text{r.p.m. of Gear} = \frac{480 \cdot 1}{48} = 10.$$

If the worm had been double-thread, the r.p.m. would be 20. The reduction decreases as the lead increases.

EFFICIENCY OF WORM GEARING

128. The efficiency in transmission from the worm to the gear is mainly dependent on the helical angle and the coefficient of friction. Since the **developed helix** is a straight line, the problem resolves itself into that of a body on an inclined plane. It is shown in Physics that the angle of repose is the angle whose tangent $= \mu$, the coefficient of friction; *i.e.*, $\beta = \tan^{-1}\mu$. The **static friction** must be overcome in starting, but when under way, it is only the **kinetic friction** that need be considered, and

it is the coefficient of kinetic friction that is taken into account here. In well lubricated gears this is about 0.05, where a steel worm drives a cast iron gear.

Let E = the efficiency, μ = coefficient of kinetic friction, α = the helical angle, and $\beta = \tan^{-1}\mu$.

$$E = \frac{\tan \alpha}{\tan (\alpha + \beta)}.$$

When E = zero, α = zero. E is also zero, when $\alpha = 90$ deg. $- \beta$. E will be a maximum when $\alpha = \frac{90 \text{ deg.} - \beta}{2}$. With $\mu = .05$, the maximum efficiency is attained when $\alpha = 43$ deg. 30 min.

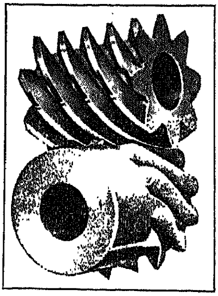


FIG. 215.—Low reduction, high efficiency worms. Boston Gear Works.

From this it is evident, that to increase the helical angle toward the limit 43 deg. 30 min., means to raise the efficiency. The conclusion is that multiple threads will give the most efficient service. However, one of the advantages of worm gearing, high speed ratio, is lost by multiplying the threads. In order to obtain a large helical angle it is necessary to keep the worm diameter as small as possible in order to avoid making it clumsy. There are limits to this operation, in that the core may not be made too small. Many worms are drilled and mounted on shafts, which puts a further limitation on this design. The helical angle $\alpha = \tan^{-1} \frac{\text{lead}}{\pi D}$.

Self-Locking Property.—If the helical angle is small, it is self-locking; *i.e.*, it cannot be driven from the gear to the worm. This is of great advantage in elevators, other hoists, and in automobile steering arrangements, because it eliminates slipping and reversing. The worm can drive in either direction, but holds against opposing force without effort. The only case in the writer's knowledge where the gear is the driver, is that of the cream separator, in which case the helical angle is large.

The tandem worm gears in the illustration at the beginning of this chapter furnish an instance of great safety in elevator design. The worms and gears are right- and left-handed, and the design not only prevents the load from getting beyond control but acts like a herringbone gear in the elimination of end thrust.

The worm gear is of immense advantage where enormous pressures are to be applied; *e.g.*, crushing machinery, large presses, heavy lifts, and it is used to a great extent in truck rear axles. In the latter case there is no necessity for large speed reduction, so the worms are made quintuple and sextuple thread to secure good running condition.

TABLE VIII.—THEORETICAL EFFICIENCY
(Neglecting losses from end and side thrust.)

Coefficient of friction	Helical angle α							
	5°	10°	15°	20°	25°	30°	35°	40°
0.02	81.3	89.5	92.6	94.1	95.0	95.4	95.5	95.7
0.04	68.4	80.9	86.1	88.8	90.4	91.4	92.1	92.4
0.06	59.0	73.8	80.4	84.0	86.1	87.5	88.0	88.4
0.08	51.9	67.8	75.4	79.6	82.2	83.8	84.8	85.4

This table, taken in part from Nachman's "Elements of Machine Design," and partly calculated by the writer, shows that the coefficient of friction is very important, especially in worms of low lead. Worm gears need to be well made and well lubricated.

DRAWING WORM GEARS

129. The sketch shows the essential parts of the assembly and housing, as it is usually drawn. No dimensions are given, as dimensions do not properly belong to an assembly drawing, excepting a few important ones, such as the shaft center distances, the overall and bolt-center dimensions. All the information regarding horsepower, speeds, capacity, number of gear teeth, etc., should be on the drawing in the form of notes. A bill of material is an essential item on an assembly, giving a list of all the constituents of the assembly with information as to materials, number required, etc., and references to the detail drawings of each part. The bill of material is of great importance to the purchasing department, as orders for supplies are made from it.

In commercial drawings the teeth of the gear are seldom drawn, and the worm helices are usually drawn as straight lines. The worm shaft is provided with radial and end thrust bearings.

In the better class of transmission, ball or roller bearings are usually provided for these places, and on the cheaper gears, bronze bushings or babbitted bearings are employed.

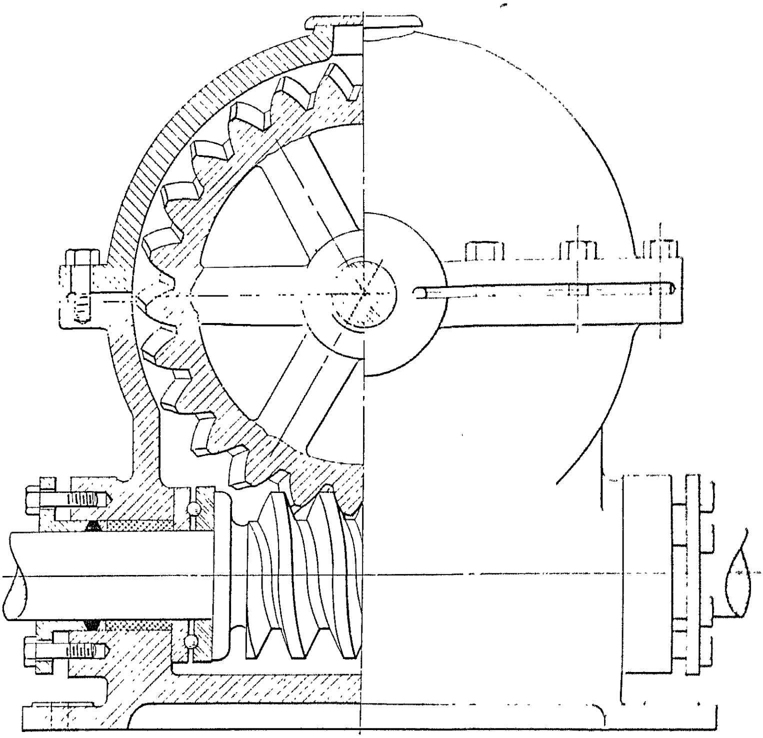


FIG. 216.—Section of a worm, gear, housing, and bearings.

WORM GEAR DESIGN

130. The following items constitute the necessary data for this purpose:

Effective horsepower to be transmitted, hp.

R.p.m. of worm, N^W. (This may have to be altered.)

R.p.m. of gear, N^G.

Materials of each, with safe stresses.

Coefficient of friction, assumed.

With these to start, the calculations for the principal dimensions can then be made from the following equations. As some of these involve more than one variable, trial calculations are necessary. By making assumptions and substitutions, and

checking the results until values are obtained that will satisfy the equations, very good results can be achieved.

(1) **Circular pitch** of worm and gear: c.p. $= 4.1\sqrt{\frac{W}{ns}}$. Assume $n = 2$, $W =$ tooth pressure, $\frac{2}{3}P$ in the horsepower equation. Also assume the multiple of the worm thread and the number of gear teeth.

Note.—Some or all of these assumptions may be such that it is advisable to change them. This necessity may not be apparent until all the calculations have been made, and the work will have to be done over entirely.

(2) **Diameter** of the core. $D = 1.72\sqrt[3]{\frac{T}{s}}$.

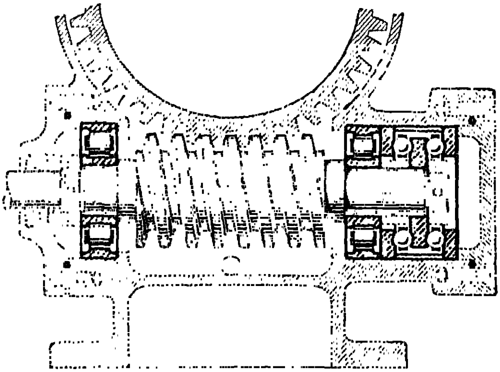


FIG. 216a.—Commercial drawing of a worm gear assembly. The Norma Co. of America, New York.

$T = 63{,}000\,\frac{H}{N}$. The core of the worm $=$ p.d. $-$ twice the dedendum $=$ p.d.$^W - 0.8$ c.p.

(3) **Pitch diameter** of the worm, the **helical angle,** and the **efficiency.**

P.d.$^W =$ core $+ 0.8$ c.p.

Helical angle $\alpha = \tan^{-1}\frac{M \times \text{c.p.}}{\pi D}$

Efficiency of worm transmission $= \frac{\tan\alpha}{\tan(\alpha + \beta)} \times 100.$

The general efficiency of the mechanism will be 5 to 10 per cent less than the theoretical efficiency.

Note.—This calculation may change the value of the gross horsepower, and necessitate fresh assumptions.

(4) **Gear shaft diameter.**

$$D = 1.72\sqrt[3]{\frac{T}{s}}. \qquad T = 63{,}000\,\frac{H}{N}.$$

With these dimensions the drawing can be made, and the minor dimensions can be taken without calculation, from past experience with similar proportions.

THE MANUFACTURE OF WORMS AND GEARS

131. The "milling method" (*i.e.*, cutting the grooves on a milling machine) is probably the most effective means of pro-

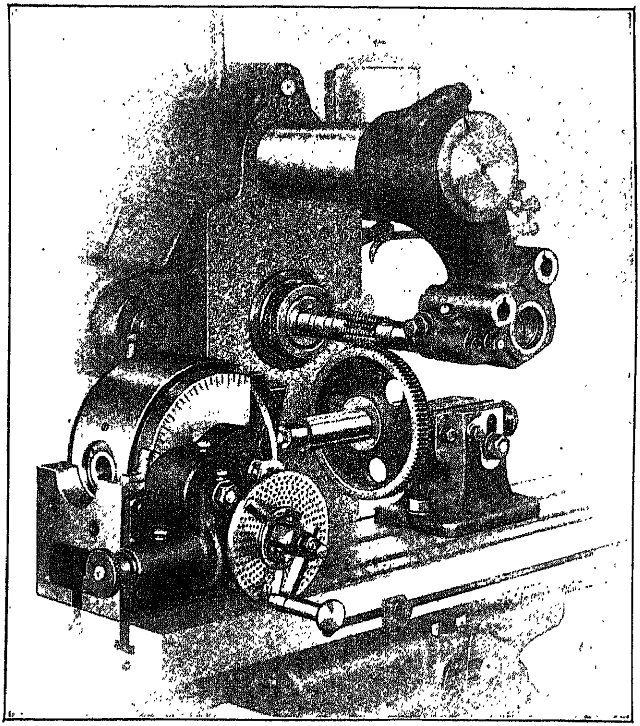


Fig. 217.—Hobbing a worm gear. Brown & Sharpe Mfg. Co.

ducing **worms** for worm gearing. Helical gears, twist drill grooves, and worms are produced in this manner; *i.e.*, by

mounting the cutter at the proper angle to the work, feeding the work past the cutter, and revolving it at the proper rate. Multiple cutters (similar to hobs) are used for multiple threads. Worms can be cut on lathes after the manner of an ordinary screw thread, and small ones can be turned out on automatic screw machines.

In large assemblies the worm is usually made separate from the shaft, and is drilled and key-seated to take the shaft. In small pairs the worm is often made from a solid bar, turned on lathe or screw machine, with the necessary extensions and shoulders. Where the worm and shaft are made separately,

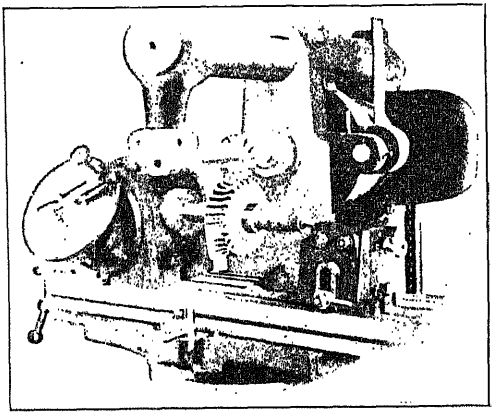


FIG. 217a.—Gashing a worm gear. Kempsmith Mfg. Co.

the core diameter is about twice as large as that of the solid core, as calculated in the previous paragraph.

Worm gears are usually "hobbed." The hob is a tool made in the shape of the worm that is to run with it, gashed and relieved like a milling cutter. The blank is first "gashed" by an ordinary milling cutter, similar to the manufacture of a spur or helical gear. This gashing serves a double purpose, it removes most of the metal, thereby saving the wear on the more expensive hob, and acts as a guide for the hob in keeping the teeth properly spaced. After the wheel has been gashed, the work and the cutter are mounted in the milling machine in the positions the worm and gear are to occupy, as shown in the illustration, and are both set in motion at their proper velocity ratios. After the hob is fed into its proper cutting depth, the

grooves are machined automatically, until all are cut. This process cuts the concave, helical groove peculiar to worm gears.

Not all worm gears are hobbed, however, and some designers find it advantageous to engage with their worms other forms of teeth. The illustration here given shows two other designs, and sometimes ordinary spur gears are used. In the illustration

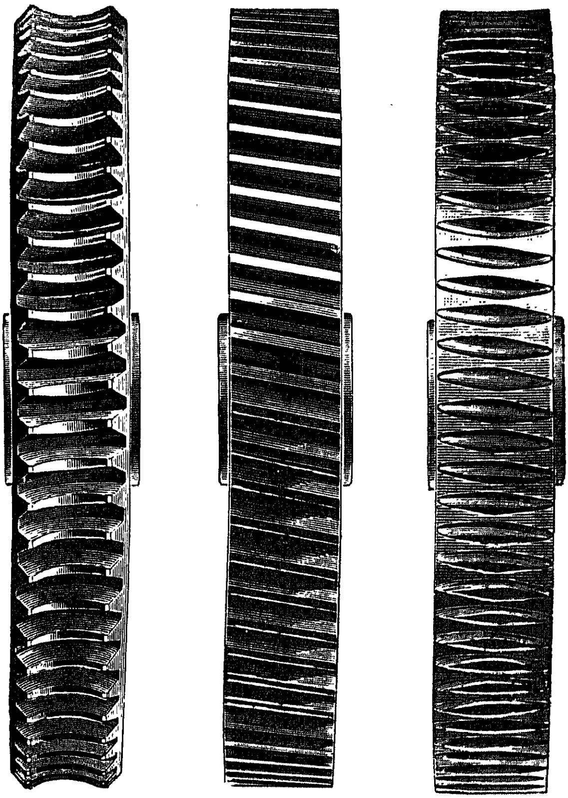


FIG. 218.—Worm gear faces, three types. Brown & Sharpe Mfg. Co.

of the three styles of worm gears, Fig. 218, taken from the Brown & Sharpe "Practical Treatise on Gearing," the first cut is that of the concave-grooved hob-cut gear. This ensures a perfect fit between the pair, with surface contact which means a minimum of wear, but it also necessitates perfect alignment of the two. Most heavy work, as well as much of the lighter work, is carried over this type.

The second design is used where the concave cannot be obtained. The teeth are not hobbed, but are cut with spur gear cutters, straight faced, and set at the helical angle of the worm. The contact is a point, and with heavy loads the wear is very great. The alignment need not be perfect.

The third design is hobbed, but not so deeply as the regular design. It is used for indexing, and in gear cutting machines where the workman has occasion to turn the work spindle by hand, it is not so rough to grasp.

One prominent ironmaster gave as his opinion that a worm operating an ordinary spur gear, with its shaft set at the helical angle of the worm, would achieve the best performance. He proved his belief in this opinion by installing all the worm gears in his works in that manner.

HYPERBOLOIDAL GEARS

132. When the shaft axes very nearly intersect, the distance between them is too small to drive with either helical or worm gears. Bevel gears are not available unless the axes do intersect. The recourse then is to a type called **Skew Gears,** the pitch surfaces of which are **rolling hyperboloids of revolution.** The hyperboloidal surface is generated by revolving a line about a non-planar axis. If this line be revolved about another non-planar axis, the two surfaces thus generated will be tangent along the common element. Gears have been designed to utilize this condition, having straight teeth (grooves) and therefore **line contact.** The necessity for such a gear, however, is very rare, and it is so seldom that a compromise cannot be arranged with bevel or helical gears, that it is virtually a freak, and is therefore outside the scope of this text.

PROBLEMS

Note.—In these problems, it is left to the option of the instructor to assign involute standard or stub teeth, or cycloidal. **No load calculations** are necessary, if the pitch and speed ratios are given. Thus the instructor may limit his assignments to pure kinematics, or may make engineering problems of them. A very valuable problem and worth the time it requires, is to make a large and complete working drawing of a worm gear assembly, with housing and bearings, in half-section, together with title, bill of material and all instructions as to horsepower, r.p.m., pitch, and number of teeth. The calculations, made out in a neat and business-like manner, should accompany the drawing.

1. **Bevel Gears.**—Shaft angle 75 deg., r.p.m. 90 and 360, d.p. 4, least number of teeth in pinion 15.

Required.—Pitch diameters, circular pitch, number of teeth, pitch angles, addendum angles, dedendum angles, normal cone angles of each wheel, and a drawing in half-section of the pair in mesh.

2. Same problem. Shaft angle ——— deg., r.p.m. ——— and ———, d.p. ———, least teeth in pinion ———.

3. **Bevel Gears.**—Shaft angle 105 deg., 15 horsepower, at 150 and 350 r.p.m. Material nickel steel, hardened ($s = 15{,}000$). Make $n = 3$.

Calculate the circular pitch and design for the nearest even diametral pitch. Also the shaft diameters, and design the proportions in accordance with these.

Make a complete working drawing, fully dimensioned, of the two gears engaged, half-section.

4. Same problem. Shaft angle ——— deg., hp. ———, r.p.m. ——— and ———, material ——— ($s =$ ———), $n =$ ——— (1 to 4).

5. **Helical gears.**—Center distance 4 in., axial angle 90 deg., r.p.m. 200 and 450, normal pitch 8 d.p.

Required.—Pitch diameters, circular pitches, helical angles, and numbers of teeth.

Make an assembly drawing of the two gears in two views.

6. Same problem. Center distance ——— in., axial angle ——— deg., r.p.m. ——— and ———, normal pitch ——— d.p.

7. **Helical Gears.**—Center distance 5½ in., axial angle 70 deg., r.p.m. 450 and 200, horsepower transmitted 1½, material machine steel case-hardened ($s = 7{,}000$), $n = 2\frac{1}{2}$, and the efficiency 90 per cent.

Required: the normal pitch, circular pitches, pitch diameters, helical angles, and numbers of teeth. Make an assembly drawing of the pair in two views.

8. Same problem. Center distance ——— inches, axial angle ——— deg., r.p.m. ——— and ———, hp. ———, material ———, $s =$ ———, n ———, and efficiency ——— per cent.

9. **Worm Gears.**—R.p.m. 20 and 500, multiple of worm triple, circular pitch 1¼ in., the core diameter of the worm 3 in.

Required.—Number of teeth, helical angle of the worm, the theoretical efficiency, pitch diameter of the gear, and the center distance. Make a drawing of the pair.

10. Same problem. R.p.m. ——— and ———, multiple of worm thread ———, circular pitch ——— in., core diameter ——— in.

11. **Worm Gears.**—Effective horsepower transmitted 25, r.p.m. 10 and 400, worm single thread, cast iron gear, steel worm. Safe tensile strength 3,500 lb. per square inch for cast iron, and 10,000 for steel. Safe shear for cast iron 4,000, for steel 9,000. Coefficient of friction 0.04.

Required.—Circular pitch, number of teeth, pitch diameter of worm and gear, helical angle of worm, core diameter, theoretical efficiency, and center distance.

Make an assembly drawing as Prob. 9, or in full as described in the note.

12. Same problem. Horsepower ———, r.p.m. ——— and ———,

——— thread, material ——— and ———, safe tensile strength ——— and ———, safe shear ——— and ———, coefficient of friction ——— (0.04 to 0.10).

13. Design a worm gear to operate a 10-ton hoist. The load is raised by winding a cable on a 12-in. drum. The worm gear is mounted on the drum shaft, and the worm is coupled direct to a motor revolving 1,800 times per min. The maximum speed of the load is to be 90 ft. per min. The efficiency of the machine is 60 per cent.

14. Same problem. Capacity of hoist ——— tons, drum ——— in., motor r.p.m. ———, maximum speed of load ——— ft. per min., efficiency ——— per cent (40 to 75).

Note.—Probs. **12** and **13** may be used as the basis of the expansion of this problem as given in Chapter IX.

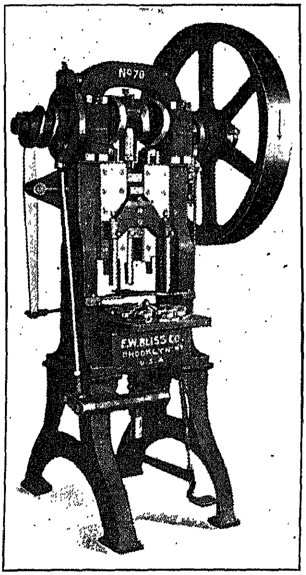


Fig. 219.—Punch press showing cams. The E. W. Bliss Co., Brooklyn, N. Y.

CHAPTER VII

CAMS

133. The workman operating a machine producing a large number of pieces exactly alike goes through a set of motions, such as pulling levers, pressing foot treadles, turning screws, chucking the work, and the like, which are duplicated for every piece turned out. The man does not furnish the driving power, but **at a certain time** he pulls the proper lever or feeds in a certain tool, turns an index plate, changes a tool or a speed of cutting, stops some movement and reverses it, or starts another, whatever the occasion requires. These processes are repeated in exact order in making the next piece. This mechanical repetition becomes a habit, and the workman goes through the round hour after hour in a regular succession of motions.

Human labor is the most expensive power in use, so that doing by human hands, work that can be done by mechanism, is appalling waste and inefficiency. In every well-planned industry we find a wonderful assortment of labor-saving devices, most of which do the work more accurately than would be accomplished by hand. There will be found in such plants, jigs for holding parts to be milled, drilled, or punched; there are multiple drills with sometimes as many as 50 spindles, capable of drilling 50 holes at one time, milling machines which can face all the plane surfaces of an automobile cylinder in one operation, and presses large and small which do the work of a thousand hammers in as many hands.

Looms, linotypes, printing presses, screw machines, gear cutters, and many other marvels of labor-saving machines can be seen by anyone, and no factory of any importance can be entered without seeing in every direction instances of man's ingenuity in the design of machines and in methods of production.

Of the agencies employed for these wonders of production through automatically controlled machinery, one of the greatest and least conspicuous is the **Cam.** A cam is a mechanical unit

having simple and regular motion which imparts to a follower irregular or intermittent motion, with or without periods of rest, making its displacements at regular intervals of time.

Time the Important Element in Control.—Take the case of the **cam shaft** in an automobile. Its function is to open and close each valve at the exact time to draw in and shut off the supply of fresh gas, and to expel and shut out the exploded gas.

In the time of Watt, the steam engines then in use required the services of a boy to open and close the steam and exhaust

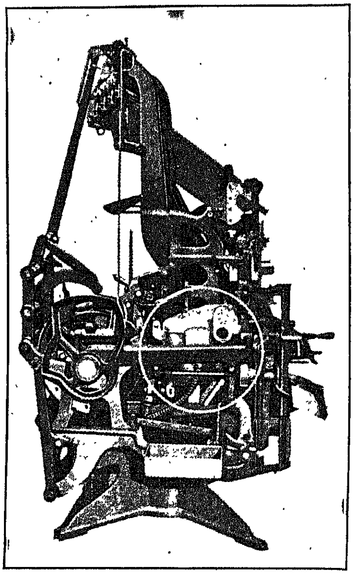


FIG. 220.—A linotype machine showing cams of complicated outline. The Cutler-Hammer Co., Milwaukee.

ports. He had to attend closely to his work, or there would be trouble, and one day he rigged up a rude mechanism whereby the engine would take care of itself. The cam shaft in an automobile opens and closes the valve ports in excess of a hundred times every second at high speeds in some cars.

The **automatic screw machine** is far more complex in its operation and its product is of great variety, yet the **timing** of its operations does not have to be more exact. Very complicated articles are made on these machines without a single operation performed or directed by man. Take the case of the hollow knurled screw, shown in the sketch, Fig. 222. Work of this

type is cut from the bar; that is, it is turned out of rolled steel or brass rods. These rods are fed through the head stock and held in a chuck for turning. The rods are of various lengths, mostly 18 to 20 ft., and a large number of the parts can be made from one rod. The operations on such a piece are about as follows:

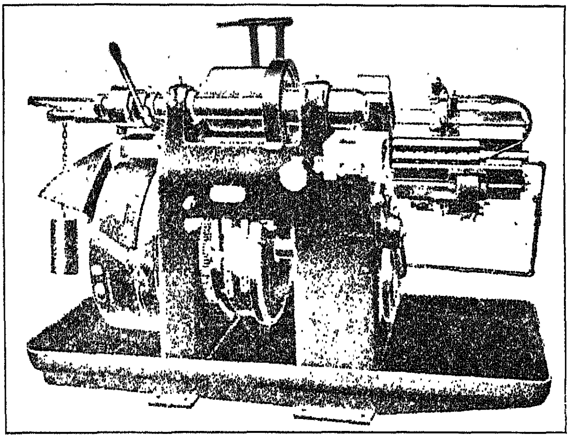


FIG. 221.—An automatic screw machine showing cams. National Acme Co., Cleveland.

(1) The stock is fed through the head stock a certain distance, and is gripped in that position, leaving enough protruding to make the part, and allow for cutting it off.

(2) The forming tool in the cross feed is started forward. The forming tool brings the piece to the proper contour.

(3) The turret, which takes the place of the tail stock is moved forward and the die, which cuts the thread A, is fed on the work. This tool is brought into position while the forming tool is withdrawn.

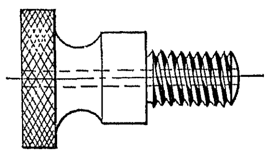


FIG. 222.

(4) The die is expanded, and the turret withdrawn.

(5) As the die is withdrawn, the knurling tool, to dent the surface C, is set to work by the cross feed cams. During this operation the turret is turned on its vertical axis through 60 deg., and a drill for the axial hole is brought into position, and started forward.

(6) The drill is withdrawn.

(7) The turret is revolved to its original position, and the cut-off tool is fed in and out by the cross feed, and the finished piece drops into the pan.

Each of these operations takes **a certain time,** and the next one starts as soon as one is finished. The **cam** is the agency for achieving this result. In a screw machine (of which there are many designs, some of them three spindle) there are three cams, (1) for the stock feed, (2) for the cross feed, and (3) for

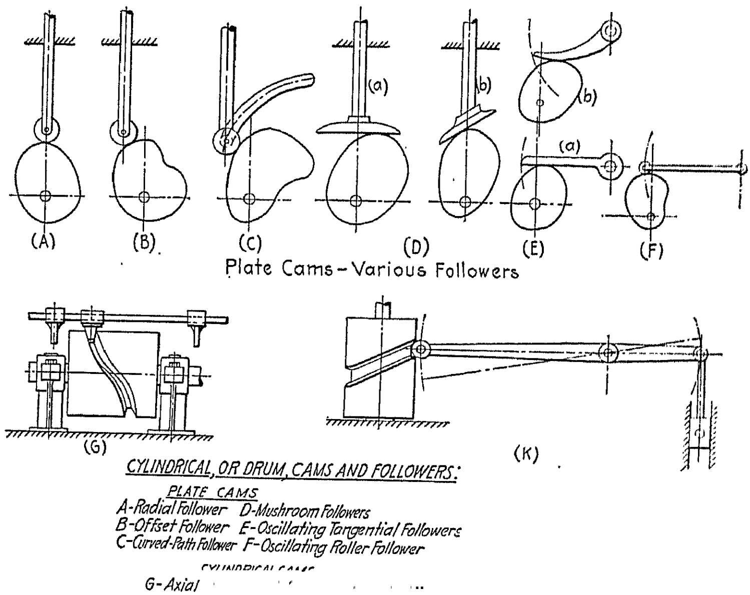


FIG. 223.—Varieties of cams for different followers.

the rotation and feed of the turret. In some machines there are more. In a factory equipped for this kind of work, one man can look after several machines, keeping them supplied with bar stock, and inspecting the parts from time to time.

134. Varieties of Cams.—Cams are of numerous varieties, but practically all may be grouped into two main classes, **Plate** and **Cylindrical Cams.** Their distinction is that the **plate cam** moves its follower in the plane of its revolution, and the **cylindrical cam** moves its follower in the plane of its axis. The cylindrical cam acts like a worm in worm gearing.

Inside these classes, the varieties are determined by the peculiarities of their followers.

Note.—These cams will hereafter be referred to as cam A, cam B, etc.

The cams shown in Fig. 223 are all of the commonest types and perform their functions by complete and uniform rotation. No necessity exists for giving them irregular motion, for they can be designed to impart whatever irregularity is desired in the follower.

Positive return in plate cams is obtained by grooving the face of a disc, as is illustrated in Fig. 224. The groove is of the

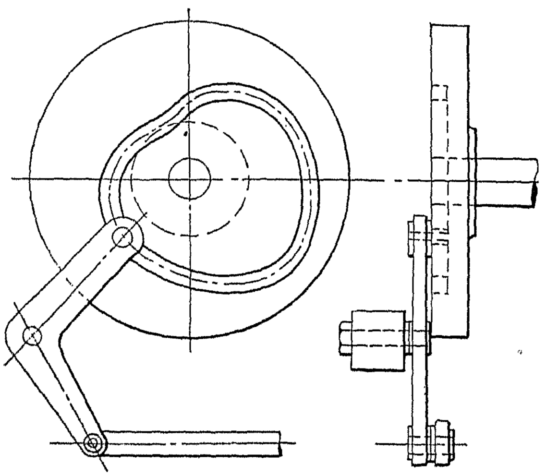


Fig. 224.—Plate cam grooved for positive return of the follower. This cam is usually called a face cam.

same outline as would be required in a plain cam giving the same performance, as for example A, B, C, or F. The roller of the follower moves in the groove. Such a cam is called a **Face Cam.**

135. Cam Design.—The first consideration (1) is the action of the follower. It may be necessary to give it a quick up-and-down motion with a long period of rest. Such a case would require a design like Fig. 225*a*. It might require a complicated series of back-and-forth motions of different lengths and durations. This would call for a cam like Fig. 225*b*, or if a smooth trip up and down its path, with gradual start and finish, forward and advance in equal time, the cam would resemble Fig. 225*c*.

If the follower is not required to make sudden changes in velocity or direction, the **tangential type** may be preferable, or if some oscillator, like a shear, be required, the choice lies between *E* and *F*.

(2) The length of the path, or the angle of oscillation being known, it is necessary to determine the time of arrival at the various points in the path.

(3) In order to impart smooth action in starting, stopping, and reversing, where such is necessary, the character of the motion must be determined.

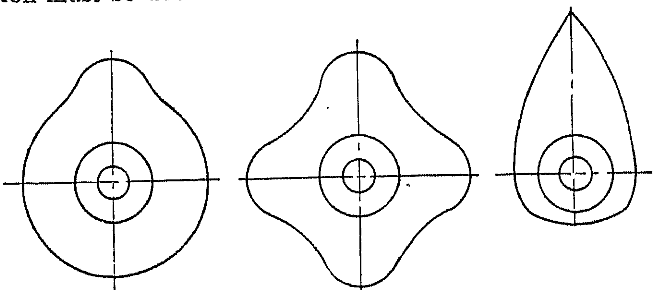


FIG. 225.

CAM DESIGN VELOCITIES

Uniformly accelerated for starting: for **equal** time units lay off successive space intervals proportional to 1: 3: 5: 7; etc.

Uniform motion for continuing: equal spaces in equal time units.

Uniformly retarded for stopping: for **equal** time units lay off successive space intervals proportional to 7, 5, 3, 1.

Harmonic, for quick forward and return strokes, where no sustained uniform velocity is required; space intervals laid off by projecting equal arcs of a semicircle on its diameter. The diameter thus used would be the path of the follower.

Base Circle.—The largest circle that can be drawn inside the cam from the center of cam rotation. Its radius is the least distance from the cam center to the cam outline.

With the data assumed from the foregoing items, problems can be laid out, as follows:

PROBLEM

To design a Plate Cam (A), base circle 3 in., to move the point follower 6 in. up and down. The time and velocity table is as follows:

TABLE IX.—DESIRED CAM MOVEMENTS

Time (deg.)	Follower (in.)	Velocity
30	1 up	Uniformly accel.
60	4 up	Uniform
45	1 up	Uniformly ret.
30	Rest	....
165	6 down	Harmonic
30	Rest	....

A specimen "Time Table."

Note.—The first column gives the amount of cam rotation necessary to move the follower to certain points in its path. In slow machines the amounts are sometimes given in time units, seconds, in proportion to the angles traversed. In the above column the sum of the angles is 360 deg. In exceptional cases it is 720 deg.

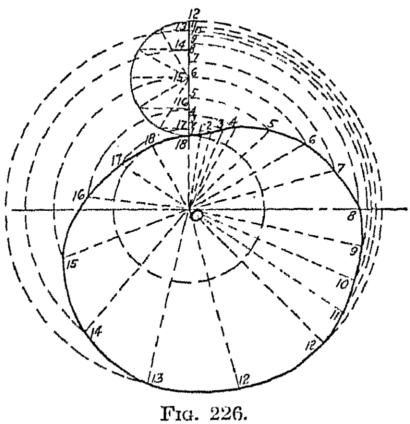


FIG. 226.

Solution.—(1) Draw the base circle.

(2) Draw the path of the follower.

(3) Divide the 6-in. path into 1-, 4-, and 1-in. sections.

(4) Divide the base circle into the angles specified in the Time column, commencing at the top.

(5) Divide the lower 1-in. space of the path into segments proportional to 1, 3, 5, 7.

(6) Divide the first 30-deg. arc of the circle into four equal arcs to correspond to the unequal segments of the path.

(7) Divide the 4-in. space and the 60-deg. arc into the same number of equal segments (four in this case).

(8) Divide the upper 1-in space into segments proportional to 7, 5, 3, 1.

(9) Divide the 45-deg. arc into four equal arcs to correspond.

(10) Draw a semicircle on the 6-in. path as a diameter.

(11) Divide the semicircumference into 6 equal arcs.

(12) Project these arcs on the path.

(13) Divide the 165-deg. arc into six equal parts.

(14) Draw radii from the cam center through all the divisions and subdivisions on the base circle.

(15) Make these radi' equal in length to the distance from the cam center to the corresponding point on the path.

(In a cam like this, the radii should measure the displacement of the follower.)

(16) Draw a smooth curve through the ends of all these radii.

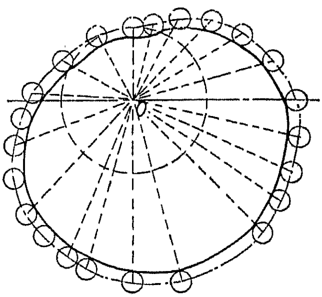


FIG. 227.

Note 1.—Since the follower remains stationary in the rest intervals, there will be no change in the cam radius from beginning to end of the arc, therefore that portion of the curve will be circular.

Note 2.—This cam is laid out to perform in a counter-clockwise direction; *i.e.*, opposite to the layout.

Note 3.—In practical work the curves of the cam outline are made to conform as nearly as possible to circular arcs, so as to facilitate production. This can be done without serious departure from the specifications. Many curves other than circles are used as cam outlines, ellipses, logarithmic spirals, involutes, parabolas, etc.

136. To Make the Outline for a Roller Follower.—The point-contact follower is inefficient, subject to wear and breakage, so the follower is usually provided with a roller. This compels a slight change in the cam design. If the cam just laid out were to operate a roller follower, the cam outline thus obtained becomes the pitch line, and the pitch line is shown by the dot-and-dash line in Fig. 227. The pitch line passes through the center of the roller at each phase of cam and follower. The

true outline is obtained by drawing a series of circles of the diameter of the roller, with their centers on the pitch line. A curve tangent to all these circles will have the correct outline.

In case a grooved cam (face cam) is to be designed, draw curves tangent on the inside and outside of the circles.

137. To Design a Cam B.—The difference between A and B is that the path of the follower does not pass through the cam center in Cam B. To make the outline, extend the path and draw a circle tangent to it. Divide the circle in the same way as before, commencing with the point of tangency. Instead of drawing radii through these points, draw **tangents,** for example XB' and YC'. Set off the points B', C', etc. so that their

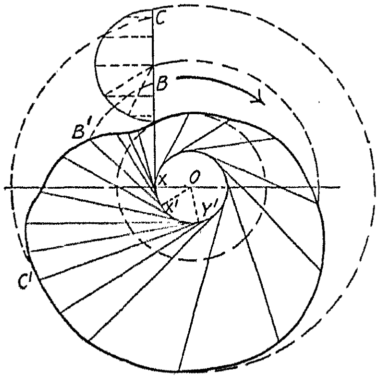


FIG. 228.

distances from the cam center are equal to the follower distances; *e.g.*, make $OB' = OB$, and $OC' = OC$, etc.

Note.—This cam will only run properly when rotated clockwise. The rule is **clockwise** if the follower path is to the **left** of the center, and **counter-clockwise** if to the **right.** Otherwise, the follower would not easily respond to the intention of the cam, and would probably break.

138. To Design Cams C and F.—Both cams, C and F, are drawn by the same method, although the followers are made to take their paths by different modes of constrainment. In this case, the paths do not coincide with either a radius or a tangent. Sometimes the path will be entirely on one side of the radius, but often it will cross the radius, and some points of the path will be to the right and others to the left. This means that some of the points are offset, and the cam outline must conform

to this irregularity of the path. The method of locating these points on the cam outline is:

(1) Draw the various cam radii in the usual way, to give the follower displacements.

(2) Locate the points on the cam outline by giving them the proper offset. Thus, B is a point on the path Bx distant from the zero radius. Lay off B' that distance to the **left** of x' on the corresponding cam radius; $Bx = B'x'$. A point C is Cy distant from the zero radius, to the **right**, and therefore $Cy = C'y'$, and C' is that distance to the **right** of its cam radius. By correctly locating all the points, the curve is drawn.

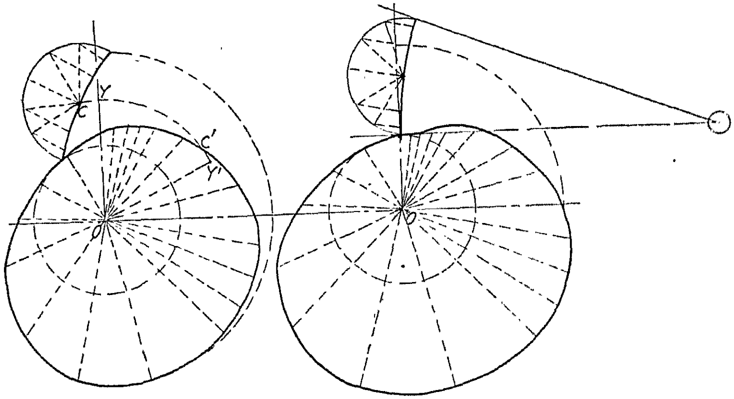


FIG. 229.

139. To Design Cam D.—This follower presents no difficulty. Since the contact is not always on the zero radius OA, this fact must be recognized.

(1) Draw the cam radii to give the necessary displacements.

(2) At the points a, b, c, etc. draw lines perpendicular respectively to the radii Oa, Ob, Oc, etc.

(3) Draw the curve tangent to these lines.

Note.—In this type of cam the surface of the follower is nearly always a plane at right angles to the path. If it is at a different angle, draw the lines through a, b, c, etc., in operation (2), at the same angle with their radii as the actual surface is inclined to the zero radius.

Note.—This cam must have a smooth outline on account of

its tangent surface. The follower cannot enter hollows in the outline, and therefore the action of the follower is limited to

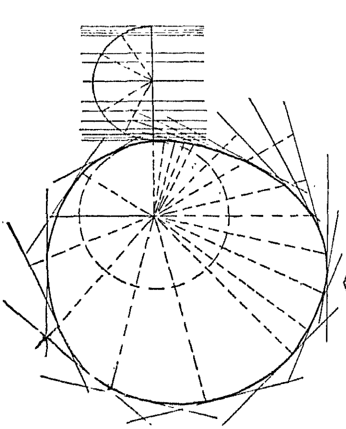


Fig. 230.

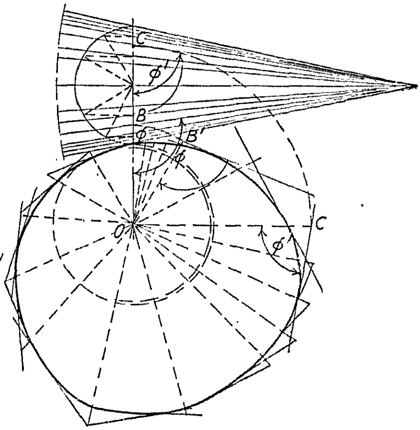


Fig. 231.

simple motions. No sharp changes of velocity or sense of motion can be specified. The same limitations apply to Cam E.

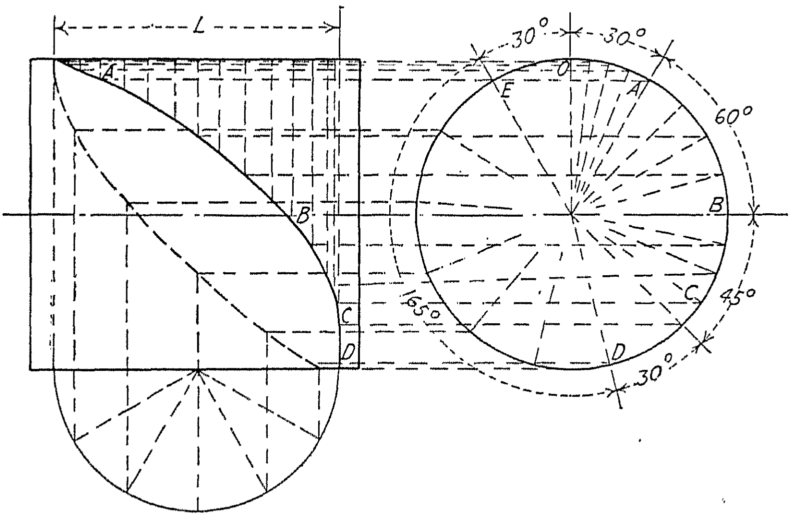


Fig. 232.—Layout of the center line of the groove of a cylindrical cam.

140. To Design Cam E.—In this cam the tangent edge of the follower makes a constantly changing angle with the zero radius. In drawing the tangents through B', C', D', etc., the

angles ϕ, ϕ', etc., must be reproduced at the proper points. The outline is drawn tangent to all these lines.

141. To Design a Cylindrical Cam G.—This cam is a highly useful medium, is easy to manufacture, positive in action, and is perhaps the most adaptable to highly complex requirements.

Let it be required to design such a cam to move a follower a distance L (say 6 in.) right and left at the same time ratios as in the data table heretofore used.

(1) Draw the front and end elevations of the cylinder (say 12-in. diameter) as shown in Fig. 232.

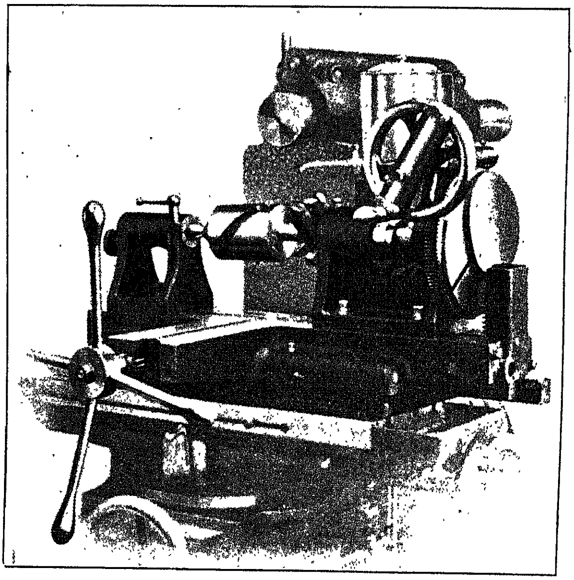


FIG. 233.—Milling a cylindrical cam groove. Brown & Sharpe Mfg. Co.

(2) Divide the end elevation into the main divisions and sub-divisions, as specified in the Time column.

(3) Divide L into its main- and sub-divisions, to conform to the displacement and velocity requirements.

(4) Locate points on the cylinder surface by projecting the path divisions to intersect projections from the corresponding time divisions. These points will be on the center line of the groove that is to be drawn.

Note.—In practical work, this center lines is all that is needed, as the tool will cut the proper outline for the groove, if it follows

the center line. For illustration purposes, the groove is easily **sketched** by any draftsman of ordinary proficiency. The accurate drawing of the groove is a problem in descriptive geometry, and is achieved by developing the center line, and drawing the outlines tangent to the rollers, as shown in Fig. 234. The top curve and the bottom curve are then rolled back on the cylinder. The roller is a cone, whose apex is in the axis of the cylinder.

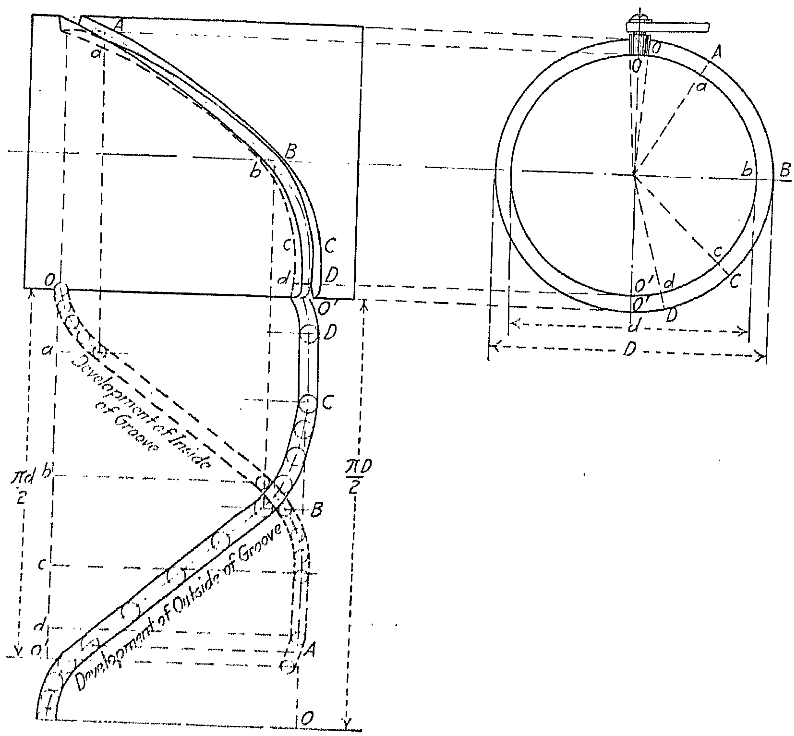


FIG. 234.—The exact method of drawing the groove outlines of a cylindrical cam.

This gives perfect rolling condition. Why? What is the name of the surface of the walls of the groove?

Note.—In many slow-revolving drums, the roller is cylindrical. Hyperboloidal rollers are sometimes used, because the rolling surface is a helicoid, which is a limiting form of hyperboloid.

Note.—A good approximation of the groove can be drawn by laying off parallels to the center line of the cylinder, making

them equal in length to the width of the groove, equal distances on each side of the groove center line. The grooves in Fig. 223 were drawn that way.

142. To Design Cylindrical Cam K.—In this instance the follower swings in an arc, and is usually not on the zero element of the cylinder. This results in a series of offsets of different amounts. In dividing up the end (circular) elevation in accord with the time requirements, the offset must be added or subtracted, in agreement with the advance or retreat of the path

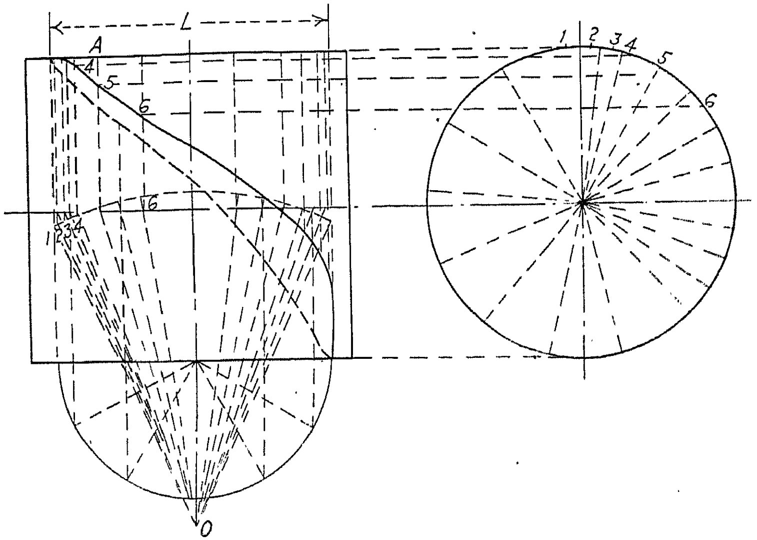


FIG. 235.

from the zero element. This will not change the outline of the path **much,** but it must be right, or the follower motion will not be as intended.

DISPLACEMENT DIAGRAMS

143. Figure 236 shows the displacement of the follower in all its phases. Time values (using any unit) are laid off as abscissae, and follower displacement as ordinates. This diagram makes it easy to study the character of the follower travel, and many engineers lay out their displacement diagram before determining the table of data; that is, they work out the cam from a displacement diagram. The main point to be examined

is whether the speed or direction changes are too violent. This diagram (made from the specifications in the time and velocity table, Art. **135**) shows gradual starting, stopping and reversing, which means smooth action and long life to the machine. A diagram similar to *A* with sudden starts and sharp turns would be **poor design.** In a case like that, where certain displacements and direction changes are necessary, the design can be im-

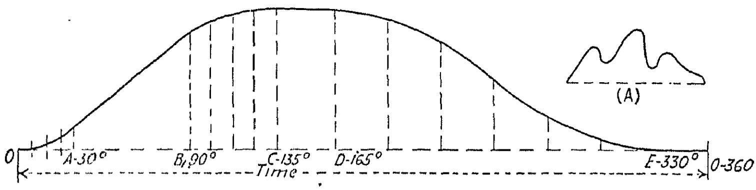


Fig. 236.

proved, and made workable by **enlarging the cam,** thus lengthening out the curves and making them smoother. The possibilities of cam requirements are unlimited, and the displacement diagram can be drawn in infinite variety.

144. Adjustable Cams.—Both plate and cylindrical cams can be made adjustable by the use of straps, such as are shown in Fig. 237. These straps are bolted to holes in the drum or plate,

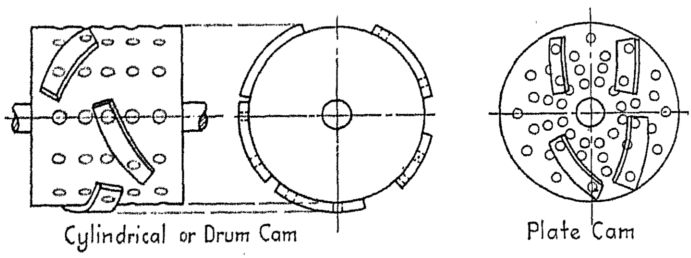


Fig. 237.—Adjustable cams.

and may be changed to give other displacement when occassion arises. The most conspicuous use of this arrangnment is found in the turret control of automatic screw machines. The location of these straps and the amount of the circumference they occupy determines the duration of any operation, and the displacement. Plate cams made this way are not common.

145. Involute and Wiper Cams.—Stamp mills for crushing ore and quartz employ **involute cams** to raise the stamps. As

these machines are made to take heavy blows, there is no necessity for easing off the follower velocity. The involute outline is such that as it is revolved, the follower is raised at a uniform velocity until the cam passes under the follower. The follower

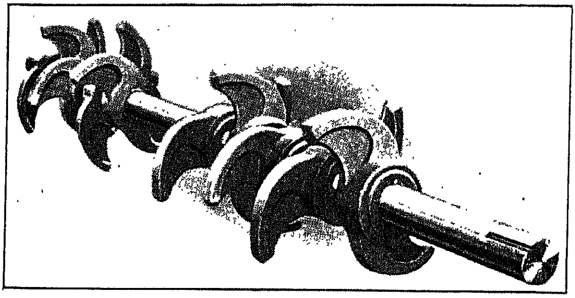


FIG. 238.—A shaft of involute cams to be used in a stamp mill. Worthington Pump & Machinery Co., Milwaukee.

then falls, or is forced down rapidly. The usual practice is to lay out a pair of involute cams in one piece, operating at 180 deg. In stamp mills several cams, single or double are mounted on one shaft.

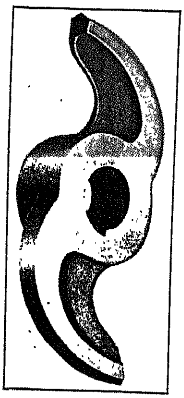


FIG. 238a.—An involute cam. Worthington.

The involute raises the follower on a path tangent to the circle on which the involute is derived (the **evolute**). Each angular unit of cam revolution imparts to the follower an upward movement, equal to the length of the arc on the evolute for this angle. Therefore uniform rotation in the cam produces uniform velocity in the follower. See Fig. 239.

The **toe and wiper cam** is similar, but the path of the follower is radial. This type of cam usually has a reciprocating motion. It is frequently seen in the valve mechanism of marine engines. In both involute and wiper cams there is a large amount of sliding.

Note.—There are many other styles, varieties, and subspecies of cams. The advanced student and the designer of cams will find much information and assistance in their design in the very complete and reliable book "Cams," by Franklin De Ronde Furman.*

* FURMAN, F. DE R., "Elementary and Advanced Cams," J. Wiley & Sons, New York.

Cams Used as Power Transmitters.—It has been said that the most common function of the cam is to take the place of human labor in making the changes of feed, speed, reversing,

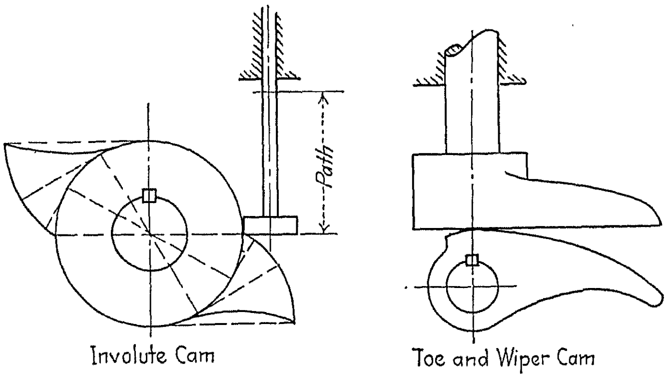


FIG. 239.

etc., required in the quantity production of interchangeable parts or units. An unusual and interesting instance of the use of

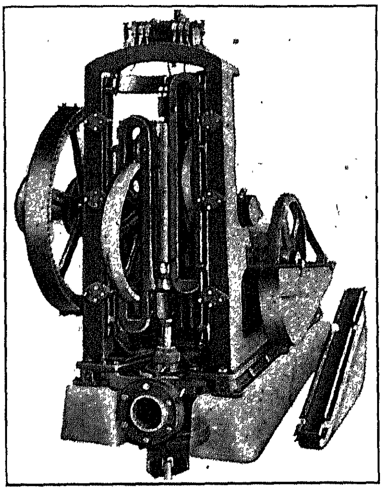


FIG. 240.—The Luitweiler Pumping Engine. Luitweiler Pumping Engine Co., Rochester, N. Y.

cams for power transmission is in the case of the Luitweiler pumping engine. In order to assure a steady flow of water, without the interruption that usually occurs in pumps at the

end of each stoke, the cams, that raise and lower the plungers in this design, are made to raise the follower during a longer period than half the revolution. Thus the upward flow due to one plunger is still functioning after the other plunger has begun its stroke. Cams are used for similar purposes in punch presses, alligator shears, and other machines.

PROBLEMS

In giving the data for the following problems, it is left to the instructor to provide a varied number of **time charts,** such as described in Art. **135.** The time table is capable of a large variety of specifications, with many more movements of the follower than the single forward-and-return motion assigned in that very simple chart. This writer finds it an advantage to have blue prints with a large variety of data in the time tables for individual assignments, to distribute in class and drawing periods. Instead of this form, displacement diagrams may be given as data, but it is preferable to work from the time table and lay out the displacement diagram as part of the problem.

1. Cam A. Base circle 3 in., follower travel 4 in. up and down. Roller ½-in. diameter.

2. Same problem. Base circle ——— in., follower travel ——— in. up and down. Roller ——— in. diameter.

3. Cam B. Base circle 4 in., follower travel 6 in. up and down, offset ¾ in., roller ⅜ in.

4. Cam B. Base circle ——— in., follower travel ——— in. up and down, offset ——— in., roller ——— in.

5. Cam C. Base circle 3½ in., follower travel 5 in. in an arc of 7 in. radius, its center 6 in. to right of cam center, roller ½ in.

6. Same problem. Base circle ——— in., follower travel ——— in. in an arc of ——— in. radius, its center ——— (describe), roller ——— in.

7. Cam D. Base circle 4 in., follower travel 5½ in. up and down, angle of follower edge with the zero radius of the cam 90 deg.

8. Cam D. Base circle ——— in., follower travel ——— in. up and down. Angle of follower edge ——— deg.

9. Cam E. Base circle 5 in., center of follower 3 in. above and 12 in. to right of cam center, arc of oscillation of follower 35 deg.

Note.—This follower motion may also be given in linear units.

10. Cam E. Base circle ——— in., center of follower ——— in. above, and ——— in. to right of cam center. Oscillation of follower ——— in. Edge of follower ——— (straight or curved).

11. Cam F. Base circle 4 in., center of follower 5 in. above and 13 in. to right of cam center. Oscillation of follower 40 deg. Roller ½ in.

12. Same problem. Base circle ——— in., center of follower ——— in. above and ——— in. to right of cam center. Oscillation ——— deg. Roller ——— in.

13. Cam G. Drum diameter 12 in., follower travel 6 in. left and right, conical roller ⅝ in.

14. Same problem. Drum diameter ——— in., follower travel ——— in. left and right, roller ——— (conical or cylindrical) ——— in. diameter.

15. Cam K. Drum diameter 14 in., follower length 24 in. (to oscillating center), angle of follower oscillation 40 deg., roller ¾ in.

16. Same problem. Drum diameter ——— in., follower length ——— in., follower oscillation ——— deg., roller ——— in.

SPECIAL DRAWING BOARD PROBLEMS

17. Design a drum cam G 10-in. diameter, to impart to the piston a travel D of 3 in., according to a given time table. $A = 12$ in., $B = 4\frac{1}{2}$ in., $C = 24$ in. Roller ¾ in. Make a complete working drawing in two views, with complete details, all parts dimensioned, also title and bill of material.

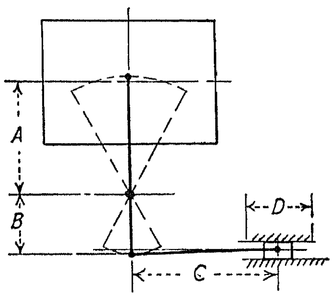


FIG. 241.—Problems 17–18.

18. Same problem. Drum diameter ——— in., $D =$ ——— in., $A =$ ——— in., $C =$ ——— in., B ——— in., roller ——— in.

19. The velocity diagram of the shaper mechanism shown is similar to a. Design some cam or eccentric attachment that will change the diagram to b.

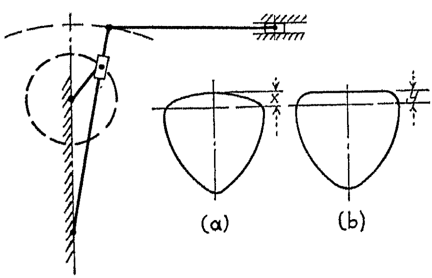


FIG. 242.—Problem 19.

This will give the tool nearly uniform cutting speed, whereas without the attachment it will accelerate to the center and retard toward the end. Let the student undertake the entire design, including all the sizes.

20. Design a mechanism, with or without cams, that will move the belt shifter forward and backward by pulling the rope. Make a complete drawing, assembly and details, with housing.

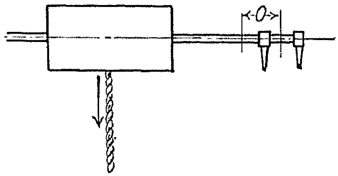


FIG. 243.—Problem 20.

21. A circular eccentric, 8 in. diameter, 2½ in. eccentricity, is used as a cam to raise and lower a radial follower. What is the follower travel? Draw the displacement diagram, and analyze the velocities, making a time table therefrom.

22. Same problem. Diameter ——— in., eccentricity ——— in.

23. Same problem with non-radial follower. Diameter ——— in., eccentricity ——— in., offset ——— in.

24. Same problem with oscillating beam with roller. Diameter ——— in., eccentricity ——— in., length of follower ——— in., arc of oscillation ——— deg., center of follower ——— in. above and ——— in. to right of eccentric center, roller ——— in.

25. Same problem as **24,** with oscillating tangential follower. Diameter ——— in., eccentricity ——— in., center of follower ——— in. above and ——— in. to right of eccentric center.

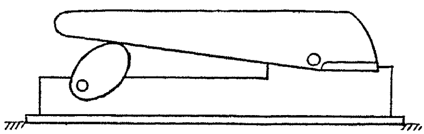


FIG. 244.—Problem 26.

26. Design the cam for an alligator shear of 2½-in. capacity, 12-in. blade. The time of the cutting stroke to be ½ that of the return. Make complete drawing with dimensions and details.

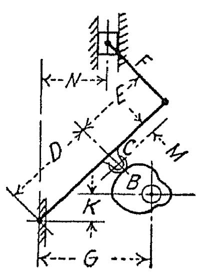


FIG. 245.—Problem 27.

27. Design a cam to give a certain motion to the slider A, data to be supplied by the instructor. Dimensions: Roller C ——— in., D ——— in., E ——— in., F ——— in., G ——— in., K ——— in., base circle ——— in.

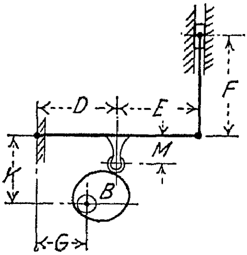


FIG 246.—Problem 28.

28. Same problem, using the mechanism shown in sketch, Prob. 28.

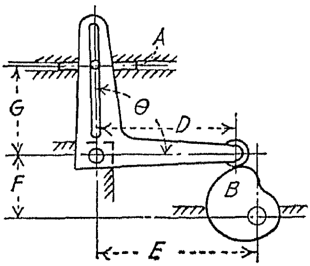


FIG. 247.—Problem 29.

29. Slotted-Rocker Follower Cam. Design the cam to impart a sliding motion to A of ——— in. forward and back (according to time chart supplied by instructor. D = ——— in., E = ——— in., F = ——— in., G = ——— in., base circle = ——— in., θ = ——— deg., roller ——— in.

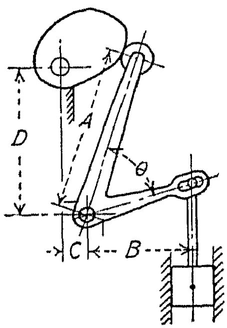


FIG. 248.—Problem 30.

30. Oscillating follower cam. Base circle ——— in., A = ———

in., $B =$ ——— in., $C =$ ——— in., $\theta =$ ——— deg. (60 to 120 deg.), path of plunger ——— in. Time chart supplied by instructor. Make a complete working drawing to scale of all the mechanism, assembly and detail.

31. Design a pair of main-and-return cams (constant diameter) to act in a pump after the manner of the Luitweiler pumping engine. The cams raise and lower plungers with rollers top and bottom, and the upward

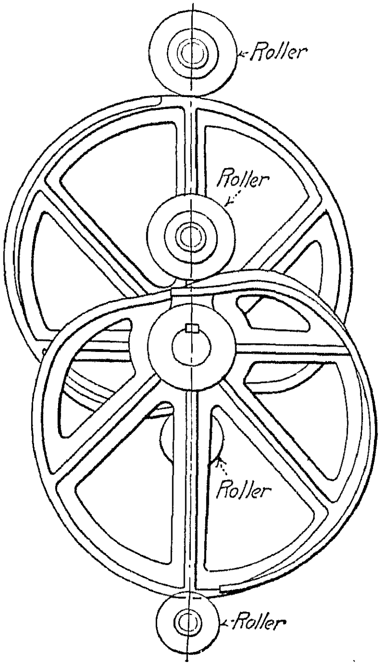
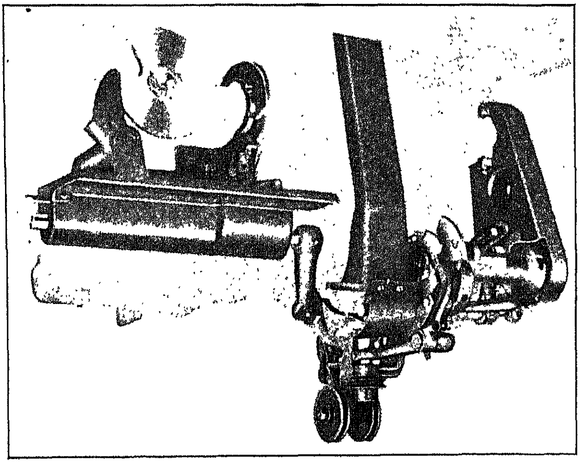


FIG. 249.—Problem 31.

action of one overlaps the upward action of the other. The time of action of each cam is ⅗ of the revolution. Draw the outline of the cam having a base circle of ——— in., the stroke ——— in., roller ——— in. Make the design so that there is the smoothest possible action, with uniform velocity imparted to the follower during ¾ of the upward stroke. Draw a displacement diagram.



Index drilling attachment showing cams. Brown & Sharpe Mfg. Co.

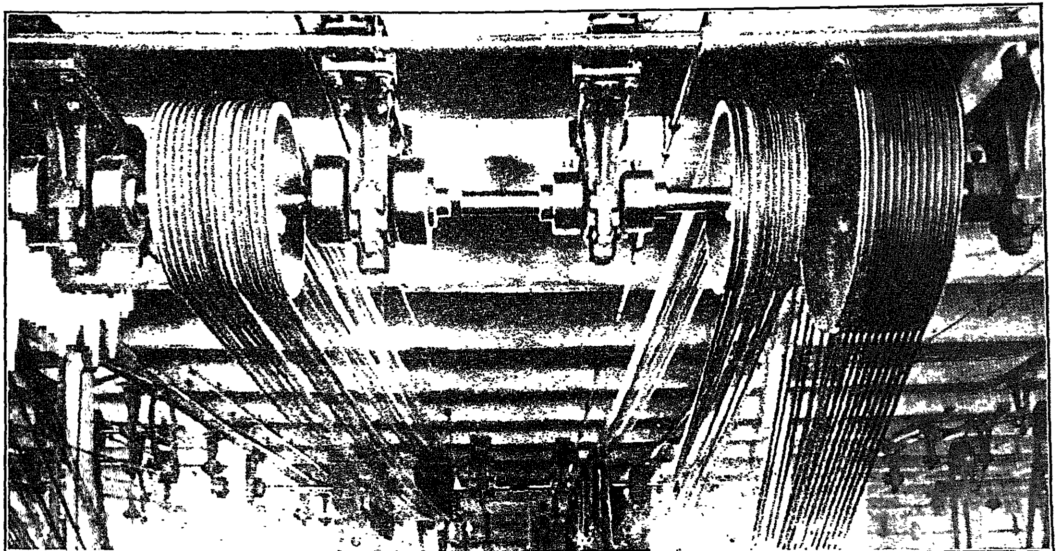


FIG. 250.—Rope drive in a cotton mill in Canada. Transmission Ball Bearing Co. Buffalo.

CHAPTER VIII

BELT, ROPE AND CHAIN TRANSMISSION

146. In every industrial plant there are (1) the prime movers: the engines, motors, etc. called the **power plant**; (2) the **transmission** system, main and secondary, through which the power is distributed to the line shafts in various buildings and floors, and thence to the countershafts for the individual machines, or for groups of machines; and (3) the **machines.** It is true that some machines are driven directly from a main shaft, but most of them have their countershafts. The other departments of a plant, such as the stock rooms, assembly, and shipping rooms, are omitted from this list, as is consideration of **electrical drive,** group or individual, as being outside the subject.

The subject of this chapter is **transmission,** and the choice between **belt, rope,** and **chain** drive is the first thing to claim our attention. This choice is not so much a kinematic problem, as a consideration of installation expense, cost of maintenance, efficiency, durability, and its suitability to the peculiar conditions of the plant, such as dampness, exposure to weather, grit, or chemicals, etc. The kinematics connected with flexible connectors will not be overlooked, but will be given its full consideration.

147. Classes of Flexible Transmission:

(1) **Flat Belting.**—Leather, Rubber, Composition, Fabric.

(2) **Rope.**—Hemp, Cotton, Leather, Wire.

(3) **Chain.**—Roller, Silent, Oblong, Leather.

Note.—It is not the intention to make anything but the most impartial statement in these pages of the merits of the various transmissions. The proponents, the manufacturers and distributors, of the different media have put forth in their literature statements and arguments in favor of their own belting, rope, or chain, and the facts and opinions given in this chapter are taken entirely from the information contained in their catalogues and other literature. The claims have been digested, and in some cases modified, and it is the opinion of this writer that the major portion, perhaps all, is in conformity with fact. The firms and corporations from whom the information has been obtained are progressive and reliable, and their oppor-

tunities for tests covering years, and for actual knowledge of the conditions and necessities in this field, are far greater than could be obtained by any writer of text books.

General Principles.—Flexible transmission divides into two classes, **friction** and **positive.** **Belting** and **Rope** belong to the

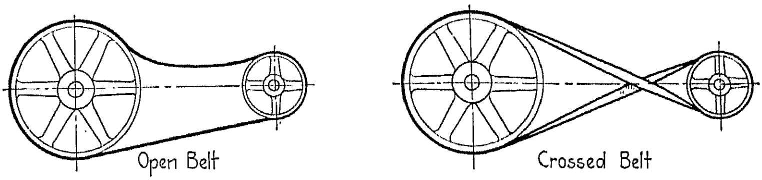


FIG. 251.

friction class, and **Chain** constitutes the positive class. There are two methods of arranging the drive for belts and ropes, the **Open** and the **Crossed.** Open belting drives the follower in the same sense as the driver. Crossed belting drives the follower in the opposite sense. Open belting gives smaller arcs of contact over the pulleys, and its tractive power is therefore less, but it is more efficient and longer lived, because there is considerable transmission loss and belt wear, due to the rubbing and extra bending of the bands as they cross. In rope drive this rubbing and distortion is avoided in cross belting by directing the rope over grooves placed so that the strands travel in separate planes.

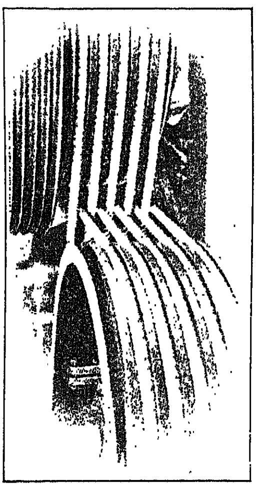


FIG. 252.—Crossed rope drive on very close centers. Weller Mfg. Co., Chicago.

148. Speed Ratios in Flexible Transmission.—Roughly speaking, the speed ratios are the same as those in gearing and friction cylinders; *i.e.*, inversely as the diameters of the pulleys.

$$\frac{\omega^A}{\omega^B} = \frac{D^B}{D^A}.$$

For practical purposes this ratio is accurate enough, because the inaccuracy is more than offset by the fluctuations of the main drive. The writer has had occasion to measure the speed of certain line shafts in large plants, in which the actual speed

varied from the rated speed by over 10 per cent. This, too, in concerns which assigned to the various countershafts speeds calculated to the tenth of a revolution per minute. Needless to say the fractional speeds were not realized.

The first correction in the foregoing equation is that of D. Since the inside and outside of the belt are equal in length before lacing, and the belt is stretched tightly over the pulleys, there is obviously a difference in their lengths when in place. The outside, being the longer, is in tension, and the inside under compression. Somewhere in the interior of the belt is a neutral plane, which has the real speed of the belt. This surface, about in the middle of the belt, is called the **pitch line.** This increases each pulley diameter by the thickness of the belt, and the first correction yields the following equation

$$\frac{\omega^A}{\omega^B} = \frac{D^B + t}{D^A + t}.$$

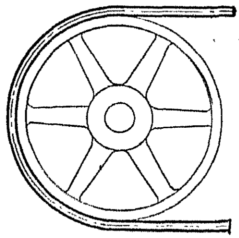


Fig. 253.

A second correction is the result of the **slippage,** which cannot be predetermined, although always present, because its amount varies with conditions of load, speed, surface, and care.

The third correction is what is known as **creep,** a loss resulting from the compressing of the inside of the belt while on the pulley, causing a diminution of the speed, which laboratory experiments have proved to be about 3 per cent. The total loss from slippage and creep can be conservatively estimated at 5 per cent.

The fourth source of error in these calculations is due to centrifugal force. This loss is not great in velocities up to 3,000 to 4,000 ft. per min., but, as it increases with the square of the velocity, it is a great factor beyond that speed, and above 10,000 ft. per min. causes the belt to leave contact with the pulleys. On account of these sources of error, it is evident that really accurate speeds are unattainable in belting. Where **exact** ratios are required, some form of positive transmission must be employed. A reasonable equation for belt speed calculation is the following:

$$\frac{\omega^A}{\omega^B} = \frac{D^B}{D^A} + 5 \text{ per cent},$$

and where the driver velocity is known, and that of the follower is required, it becomes

$$\omega^B = \frac{D^A \omega^A}{D^B} - 5 \text{ per cent.}$$

For rope drive the deduction may be reduced to 2 per cent, and for chain drive to zero.

Example.—A main shaft is running at 250 r.p.m., and it is desired to run a jack shaft at 450 r.p.m., using a 30-in. pulley on the main shaft.

Transpose the equation thus,

$$D^B = \frac{D^A \, \omega^A}{\omega^B + 5 \text{ per cent}} \therefore D^B = \frac{30 \cdot 450}{472.5} = 15.9, \text{ say 16 in.}$$

Note.—This result shows that exactly 450 r.p.m. cannot be expected by using standard pulleys; in fact, the 5 per cent deduction being estimated, no predetermination of the exact speed can be made. Neglecting the deduction altogether would give the jack shaft a pulley of 16 ⅔ in., which is not standard. Therefore a 16-in. pulley is the standard size best suited for this.

BELT DRIVES

149. Leather belting has been standard for mill equipment since the introduction of steam as the motive agent in workshops, and it is still more generally used than any other medium. The main reason for this predominance is its **durability.** Leather belting is probably the longest in life, and requires the least adjustment and repair of any of the flexible transmittors. For certain purposes there are many substitutes. The first consideration here will be the **pulley arrangements,** and the second the **horsepower calculations.**

The length of the open belt can be estimated **roughly:**

$$L = \frac{\pi D^A}{2} + \frac{\pi D^B}{2} + 2K,$$

where K is the distance between centers. The inaccuracy in this is due to the facts that the arc of contact is not 180 deg., and that the length of belt between pulleys is a trifle shorter than twice the center distance, with pulleys of different sizes. This error is offset by the fact that the belt must be stretched to operate. The formula can be used **in ordering; in installing,**

it is customary and proper to measure the distance over the pulleys by stretching a steel tape over the belt distance. It is customary to shorten the length thus determined, by subtracting ¾ in. from every 10 ft. of belt length. In the case of slow, heavy drives, twice that amount is deducted.

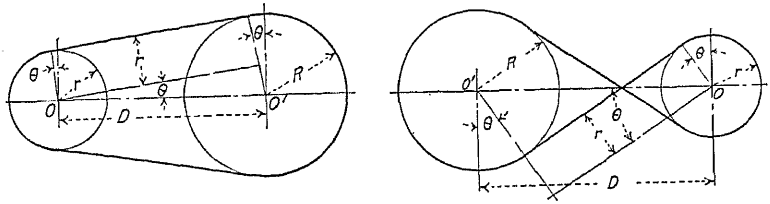


FIG. 254.

150. Exact Belt Length.—An examination of Fig. 254 will show that the mathematical expression for exact length of the open belt (allowing nothing for sag) is:

$$L = 2\sqrt{D^2 - (R - r)^2} + (R + r)\pi + (R - r)\ (2 \sin^{-1} \frac{R - r}{D}) \quad (1)$$

For the crossed belt it becomes:

$$L = 2\sqrt{D^2 - (R + r)^2} + (R + r)\ (\pi + 2 \sin^{-1} \frac{R + r}{D}) \quad (2)$$

These calculations are of little practical use, except in the case of cone pulleys. Let the student make the derivation.

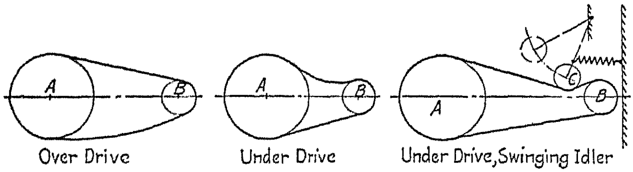


FIG. 255.

151. Belt Tension.—Three open belt installations are shown in Fig. 255, A being the driver in every case. Note that the slack reduces the arc of contact in the first case, and increases it in the underdrive. The idler is used to increase the arc of contact, especially on the smaller pulley, and to produce more tension in the belt. When a **swinging idler** is used, the belt tension can be regulated, and may be released altogether when not necessary.

There must be tension in the belt when idle to produce a drive when starting. This tension is called the **initial tension,** T_2 or **slack tension.** Without it there would be no drive even under no load. When driving under load, additional tension is produced on the **driving side** of the belt, equal to P of the horsepower equation. The sum of T_2 and P equals the **total tension** T_1. Thus $P = T_1 - T_2$. A large value for T_2 imposes heavy pressure on the bearings, and reduces the net amount of horsepower available. Hence it is desirable to have the value of $\frac{T_1}{T_2}$ as large as possible. The mathematical expression for this ratio is, $\frac{T_1}{T_2} = e^{\mu\theta}$, in which e is the logarithmic base ($= 2.718$), μ is the coefficient of friction, and θ the angle of contact in **radians.** This ratio is called the **tractive value** of the transmission.

152. Horsepower of Belting.

$$\text{hp.} = \frac{VP}{33{,}000} = \frac{2\pi RNP}{33{,}000}, \text{ and } P = T_1 - T_2.$$

The total tension T_1 is taken as 350 lb. to the square inch; *i.e.*, 1/10 of the ultimate strength of leather. This is equivalent to 72 lb. per inch of width for the average **single belt.**

Note.—Belts are single, double, triple, etc., in accordance with the number of layers of leather used. Single belts run from 5/32 to 1/4 in., averaging 3/16 in. F. W. Taylor gives as his opinion that single belt is suitable for pulleys up to 12-in., double to 20-in., triple to 30-in., and quadruple for larger than 30. Belting manufacturers do not impose limitations as small.

Example.—Calculate the horsepower possible to transmit through an 8-in. single belt, driving a 12-in. pulley at 600 r.p.m., contact angle 160 deg., and $\mu = .25$. Assume $T_1 = 70$ lb.

$\frac{T_1}{T_2} = e^{\mu\theta} = 2.03$ (solving by logarithms), $\therefore P = \frac{1.03}{2.03} T_1$, $= 35.5$ lb. per in. width, and for an 8-in. belt $= 8 \times 35.5 = 284$ lb.

$$V = \pi \times 600 = 1{,}896 \text{ ft. per min.}$$

$$\therefore \text{ hp.} = \frac{1{,}896 \times 284}{33{,}000} = 16.3.$$

If centrifugal force is taken into account, the tension per square inch =

$$C_1 = \frac{C}{2A} = \frac{12wv^2}{g} = .013v^2$$

(taking w, the weight of a cubic inch of leather $= .035$ lb).

This requires that $.013v^2$ (equal to 12.5 lb. in this example) be deducted

from 350 lb., the safe stress per square inch, in calculating P. This will effect a reduction in the horsepower of $\frac{12.5}{350}$ of the total, giving 15.7 hp. *net.*

From this example it will be evident that the centrifugal force is negligible in ordinary drives, since a factor of safety of 10 for belt strength is arbitrary. In fast drives, 4,000 ft. per min. or more, the value of P will be so reduced, that centrifugal force must be taken into account.

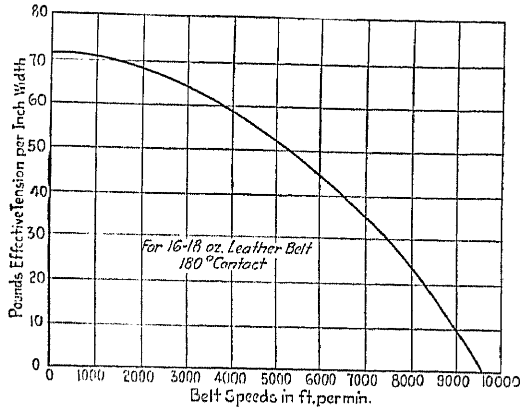


FIG. 256.—Belt tension diagram. From Bulletin 101, Graton & Knight Mfg. Co.

Note.—A fair value for P may be taken as $\frac{3}{5}$ T_1, which can be used in all ordinary calculations, and will take care of the centrifugal loss.

Commercial Calculations.—Belting manufacturers have provided charts like Fig. 256, for the convenience of computers, in which centrifugal loss and slack tension are accounted for. The curve gives the **effective tension** P per inch width that a single (16–18 oz.) belt will carry at all speeds from zero to 9,600 ft. per min., for a belt contact of 180 deg. A set of factors are furnished (given in the following tables), which are used for alterations in the chart values, for different belt thicknesses and arcs of contact.

TABLE X.

Belt weight (oz. per sq. ft.)	Factor (multiply P by)	Belt weight (oz. per sq. ft.	Factor (multiply P by)
12–14	0.8	18–20	1.1
14–16	0.9	Double belt	1.6
16–18	1.0	Triple belt	2.0

TABLE XI.

Arc of contact (deg.)	Factor (multiply P by)	Arc of contact (deg.)	Factor (multiply P by)
130	0.82	160	0.94
140	0.86	170	0.97
150	0.90	180	1.00

153. Arc of Contact.—In open belts, the arc of contact for the smaller pulley =

$$\theta = 180 \text{ deg.} - 2\left(\sin^{-1}\frac{R-r}{L}\right),$$

where R and r are the pulley radii, and L the center distance.

Example 1.—Find the horsepower which can be transmitted over a 19 oz., 8-in. belt, running 4,200 ft. per min. over pulleys 60-in. and 12-in. diameters. Center distance 12 ft.

(1) Find the effective tension for 4,200 on the chart. $P = 58$ lb. per inch width for 16–18 oz. belt.

(2) Multiply by factor for 18–20 in Table X. $58 \times 1.1 = 63.8$ lb. per inch width.

(3) Calculate the arc of contact by the equation. $\theta = 161$ deg., say 160 deg.

(4) Multiply by factor for 160 deg. in Table XI. $63.8 \times 0.94 = 60$ lb. per inch width.

(5) $P = 8 \times 60 = 480$ lb.

(6) Hp. $= \dfrac{4{,}200 \times 480}{33{,}000} = 61$ horsepower.

Example 2.—Determine the width of double belt necessary to transmit 40 hp. from a 48-in. pulley to an 18-in. pulley, running at 180 and 480 r.p.m., center distance 15 ft.

(1) $V = \pi \times 4 \times 180 = 2{,}260$ ft. per min.

(2) From chart, $P = 68$ lb. per inch width.

(3) P (for double belt) $= 68 \times 1.6 = 109$ lb. per inch.

(4) Arc of contact (from equation) $= 180$ deg. $- (2 \times 5$ deg.$) = 170$ deg.

(5) $109 \times .97$ (factor for 170 deg.) $= 106$ lb. per inch width.

(6) Width of belt $= \dfrac{P}{106} = \dfrac{33{,}000 \times 40}{2{,}260 \times 106} = 5.5$ in., say 6 in.

FIG. 257.—Barry quarter-turn belt drive. R. & J. Dick Co., Passaic, N. J.

154. Quarter-turn Drive.—For the greater efficiency, and for the longest life of the belt, the shafting must be as nearly parallel as possible. Nevertheless, it is possible to drive pulleys that are out of alignment, if a certain rule is observed. By this rule it is even possible without guide pulley to drive shafting placed at right angles. This is called the **quarter turn.**

Rule.—The center plane of the receiving pulley must be tangent to the pitch circle of the delivering pulley. The interpretation of the rule is:

(1) The **center plane** is perpendicular to the shaft through the center of the pulley.

(2) The pitch circle is in the center of the belt, both in thickness and width, as it goes over the pulley.

(3) This rule limits the drive to **one direction.** Even a small motion in the opposite direction will cause the belt to leave the pulleys.

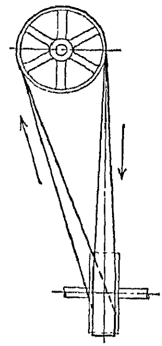


FIG. 258.—Quarter-turn belt drive without idler. Drives one direction only.

The safe system for quarter-turn or any-angled drive is to install idlers which can be arranged in many ways to direct the belts, so that driving either direction will be practical. Many catalogs give illustrations of angle belt drives of numerous varieties, and the matter is quite extensively treated in "Elements of Mechanism," Schwamb, Merrill, and James.*

155. Crowned Pulleys.—To prevent belts from running off the edges of pulleys, the device of crowning them is the universal practice. Flanges on the pulley could be used, and sometimes are for extra safety, but the belt rubs on the flange, causing wear and friction loss.

The principle is this: A belt running over a cone tends to follow the surface of the cone. This means that, as the belt tends to remain in a plane perpendicular to the shaft, it will climb to the highest point of the pulley. Therefore, by making the surface, **double conical,** or **spherical,** the belt will ride at the highest point. Most makers turn them spherically, as in Fig. 259. The amount of crowning is slight, being about ⅛ in. to the foot of pulley face.

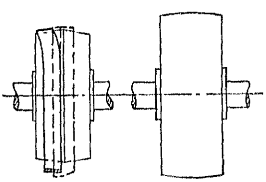


FIG. 259.

Pulleys are made of cast iron, pressed steel, and wood, or with wood, leather, or fiber facing on cast iron frames. For general mill purposes, pulleys are **split;** *i.e.*, made in halves, so that they can be mounted on shafting without disturbing the arrangements. They are made to grip the shaft tightly by bolts through the hub, sometimes using wood bushings over the shafts, and are made fast at the rims by bolts through projecting lugs inside the rim.

*SCHWAMB P., MERRILL, L., and JAMES, W. L., "Elements of Mechanism," J. Wiley & Sons, New York.

The coefficient of friction of leather belting over cast iron pulleys is given by authorities as 0.24, on the average, and over wooden pulleys as 0.47.

The comparative tractive value of wooden and iron pulleys is given in the following table from the Morin-Appleton "Encyclopedia of Mechanical Arts."

TABLE XII.—TRACTION VALUES FOR PULLEYS

Arc of contact (deg.)	Relative value of leather belt over—	
	Iron pulleys	Wood pulleys
72	1.42	1.80
108	1.69	2.43
144	2.02	3.26
180	2.41	4.38
216	2.87	5.88
252	3.43	7.90

156. Tight and Loose Pulleys.—For starting and stopping a machine at will, a favorite device is a countershaft equipped with tight and loose pulleys. Figure 260 shows a countershaft

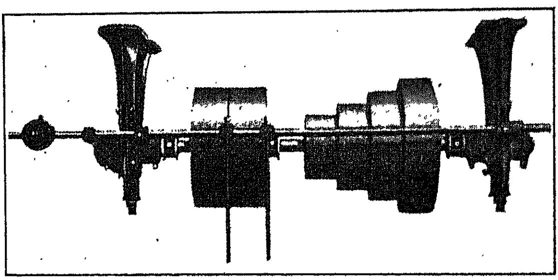


FIG. 260.—Tight and loose pulleys on a countershaft. Also a stepped cone pulley. Dodge Sales & Engineering Co., Mishawaka, Ind.

with the tight and loose pulleys, and a cone pulley to give various speeds. These pulleys are connected to the main drive by a belt running over a wide pulley on the main shaft. One of the pulleys on the countershaft is free to turn, loose, as it is called. When the belt is running over this pulley, there is no drive to the machine. A hand lever is attached to the shifter-

rod, and the latter is supplied with two fingers to guide the belt. On shifting these fingers laterally, the belt is thrown over on the tight pulley (keyed to the countershaft), and the machine will then be driven. Both tight and loose pulleys are usually crowned, and the main shaft pulley is flat-faced to allow shifting. The loose pulley is usually designed with a longer hub for greater wear.

Cone Pulleys.—This is a colloquial term for Stepped Pulleys, but it is in univeral use. Genuine cone pulleys, such as are shown in the illustration, are not so commonly used, but are of great value when an unlimited number of speed changes are desired. The cone pulley of two to five steps is paired with

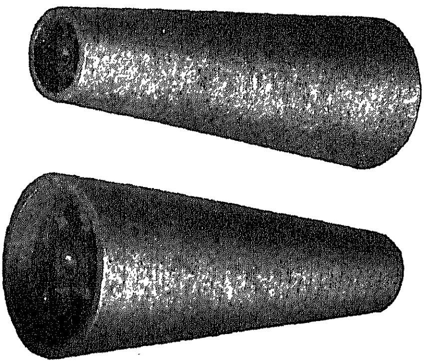


FIG. 261.—Genuine cone pulleys. Dodge.

another having the same number of steps, on the spindle of the machine and gives that number of speeds. The largest step is paired with the smallest on the mating pulley, and the intermediate steps are graduated to the other end, so that each pair will have the same belt length. It is essential that this belt length be exactly the same for every pair, since the tension must be approximately the same for any pair of steps. In designing these steps therefore, Eq. (1) in Art. **150** must be observed. The fact that the contact angle will vary for the different pairs proves that the sum of the diameters of each pair will differ from that of its neighbor (unless they are reciprocal). This can be solved mathematically, since the steps on one pulley may be decided arbitrarily, and thus each step can be calculated by Eq. (1). Most designers prefer one of the two graphical solutions given here.

C. A. Smith's Approximation.—Given the distance between centers as L, two steps, A and B, on the driver (or as many as wanted), and one step, C, on the driven, to pair with A.

At M, the center of L, erect a perpendicular, $MN, = \frac{\pi}{10} L$. From N as a center, draw a circle T, tangent to the belt (common

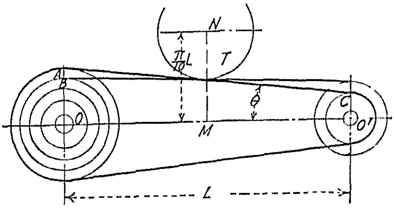


FIG. 262.—Smith's approximate method of determining diameters of cone pulley steps.

tangent to A and C). Draw the common tangent to B and T. A circle at O' tangent to this line will have the correct diameter for the required step.

Note.—This method may be used for as many steps as are desired, provided the center distance is long enough to keep the belt angle θ with the center line, less than 18 deg.

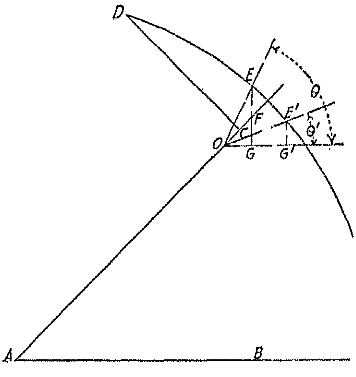


FIG. 263.—The Burmester method.

Burmester's Method. Given L, the center distance, and the radii of the first step of each of the cone pulleys.

(1) Draw $AC = L$, 45 deg. from AB.

(2) Draw $CD = \frac{1}{2}L$, 90 deg. from AC.

(3) From A as center, draw an arc of a circle through D

(4) Locate E on this arc and F on AC, so that EF shall be equal to $R - r$, the given radii.

(5) Extend EF to G, so that $FG = r$. Then $EG = R$. Why?

(6) Draw through G a horizontal line OG. Then $OG = FG$. Why?

From this construction, it is evident that $\tan \theta = \frac{EG}{OG} = \frac{R}{r}$, the velocity ratio of the first pair of steps.

For any desired velocity ratio, lay off $\theta' =$ the angle whose tangent is that velocity ratio. From which $\frac{E'G'}{OG'} = \tan \theta'$, and $E'G'$ and OG' will be the radii for that velocity.

SUBSTITUTES FOR LEATHER BELTING

157. Fabric Belts can be used on light drives, as they are cheap and have fair tractive power. They are stiff, and do not conform to the pulleys, and do not take up the load so quickly as more pliable belts. They are easily ruined by dampness, oil, or the breaking of a thread.

Rubber-Canvas Belts.—They are waterproof, short-lived, and low in power transmission.

Balata Belts are made of layers, or plies, of cotton duck (three- to eight-ply or higher), impregnated and covered with balata gum, combined with rubber and treated chemically and physically to produce a strong, pliable article. They can be furnished in any width, and will drive up to 4,000 ft. per min. Experience has proved them excellent for all transmission, except where they are likely to be soaked with oil, or where the temperature exceeds 120° Fahrenheit. They have tractive power and durability.

Motor Drive.—In regard to substituting motor drive for belt drive, the main advantage is in its convenience. It is economical where water power can be used, and where the machinery is widely scattered in the building. The cost of individual motor drive is high, and its maintenance for repair parts, wiring repairs, switchboards, etc., is much more expensive than for belting.

158. Rope Drive.—The use of hemp rope for transmission in the industries is becoming more general and more widespread

with the passing of the years. While there are advantages in belting and electrical transmission, which are undoubted, there are many places where the millwright will decide to adopt the rope drive. Properly installed transmission of this type will be found to possess the following advantages, which will be apparent and valid even to the casual reader.

(1) **Distance.**—Satisfactory installations are in operation without carrying pulleys between shafts 175 ft. apart. Using carrying pulleys, this distance may be increased very greatly.

(2) **Power.**—Many large steel and tin plate mills employ rope drives which deliver 3,000 to 4,000 horsepower.

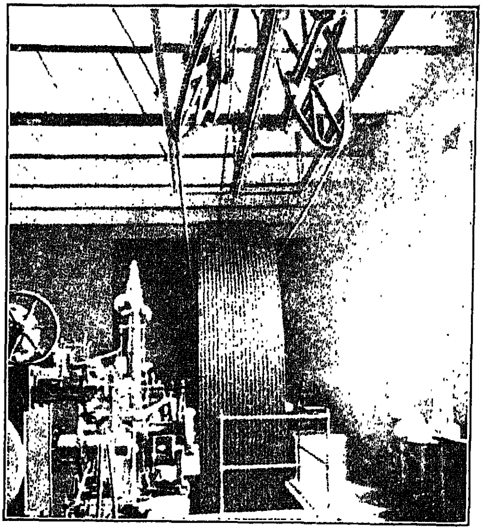


FIG. 264.—Rope drive. Main drive, 500 horsepower. Weller Mfg. Co., Chicago.

(3) **Economy.**—In drives of 30 ft. or over, carrying 200 or more horsepower, there is an undoubted economy in first cost and maintenance.

(4) **Close Bearings.**—The width of rope drive, compared with flat belting, is less per horsepower, and this enables the supports to be set closer, a marked advantage in heavy work.

(5) **No Electrical Disturbance.**—This fact is of value particularly in textile mills.

(6) **Free Alignment.**—Belts, chains, and gears require accurate setting of the shafts to ensure good running, whereas ropes

accommodate themselves easily to shafts out of parallel, whether intentional, or on account of poor installation or maintenance.

(7) **Slipping.**—The rope is wedged into its groove, so that the slip and creep are negligible in calculating speed ratios.

(8) **Shock Absorbing.**—The large sheaves, acting as flywheels, and the elasticity of the rope combine to neutralize the effects of shock in rope drive. This is important in heavy presses, shears, punches, etc.

(9) **Distribution.**—From one flywheel the power needed for the various floors and buildings may be distributed directly.

159. Rope Transmission Systems.—**The American system** employs one continuous rope traveling around each pulley, using a tension carriage to take up the slack.

The English System consists in running a series of independent ropes side by side in the grooves of the sheaves.

160. Advantages of the Systems.—The American system is especially adapted to vertical and quarter-turn drives, and in drives exposed to the weather, and in complicated transmission of any kind. The rope travels from the outside groove on the driving sheave around the driven sheave, returns around the second groove, and then out again. There may be several grooves in the pulleys, so that, after reaching the outside groove on the opposite side, it must return to the original groove. This is made possible by the tension carriage, which directs the return, and takes up the slack due to stretch and changes in load.

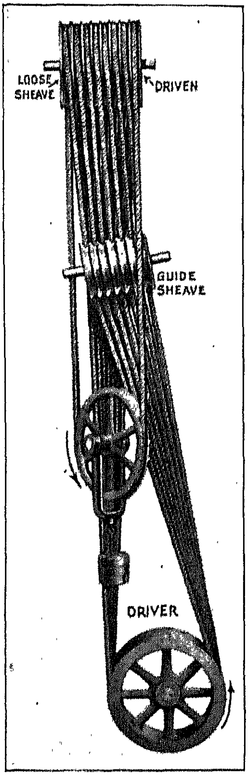


FIG. 205.—Quarter-turn rope drive. American system, showing tension carriage. Note that vertical drives are quite practical by this system. Weller Mfg. Co.

The English system is best suited to heavy drives. The breakage of one rope does not cause a shut down, because it is designed to carry a big overload, and the remaining ropes will carry more than their ordinary assignment, if necessary. One objection made against it is that it requires frequent tightenings to prevent swaying and

uneven tension. This defect is not serious, because the value of $\frac{T_1}{T_2}$ is large, and the slack tension correspondingly small; therefore the ropes can carry nearly equal loads, even though they may not be equally taut. This system requires that the shafts be nearly parallel, 50 ft. or more between centers, and protected from the weather, for best results. It is not safe to operate a mill where the entire transmission depends on a single strand, which makes the English system the preferable installation for the power supply in large factories.

Transmission rope differs considerably from ordinary hauling and binding rope. In order to resist the wear of the grooves, the constant bending and straightening, and the internal wear on the fibers from grinding on each other, it must be made from the finest quality of hemp, thoroughly lubricated, and must have a special core of lubricated hemp.

The most economical speed for rope drive has been found to be between 4,000 and 4,500 ft. per min. The greatest horsepower can be transmitted at 5,200, but at this speed the wear is very great, and the life is diminished.

161. Horsepower of Ropes.—The capacities of rope transmission, both English and American systems, are given in the following tables, taken from the calculations of the Dodge Sales and Engineering Company, Mishawaka, Indiana. The values are based on arcs of contact of 180 deg., and corrections

TABLE XIII.—AMERICAN SYSTEM.

Rope diam.	Rope speed in feet per minute										
	500	1,000	1,500	2,000	2,500	3,000	3,500	4,000	4,500	5,000	5,500
¾	1.5	3.0	4.5	5.8	7.1	8.1	9.0	9.7	10.2	10.4	10.3
⅞	2.1	4.1	6.1	8.0	9.7	11.3	12.6	13.7	14.5	15.1	15.2
1	2.7	5.4	8.0	10.5	12.8	14.9	16.8	18.4	19.7	20.6	21.1
1⅛	3.4	6.8	10.2	13.3	16.3	19.1	21.6	23.8	25.6	27.0	28.0
1¼	4.3	8.5	12.6	16.5	20.3	23.8	27.0	29.8	32.3	34.3	35.8
1½	5.2	10.2	15.2	20.0	24.6	29.0	33.0	36.6	39.7	42.4	44.6
1⅝	6.1	12.2	18.1	23.9	29.4	34.6	39.5	43.9	47.9	51.3	54.1
1¾	8.3	16.6	24.7	32.7	40.3	47.6	54.5	60.8	66.7	71.9	76.4

TABLE XIV.—ENGLISH SYSTEM.

Rope diam.	Rope speed in feet per minute							
	1,500	2,000	2,500	3,000	3,500	4,000	4,500	5,000
¾	3.3	4.3	5.2	5.8	6.7	7.2	7.7	7.7
⅞	4.5	5.9	7.0	8.2	9.1	9.8	10.8	10.7
1	5.8	7.7	9.2	10.7	11.9	12.8	13.6	13.7
1¼	9.2	12.1	14.3	16.8	18.6	20.0	21.2	21.4
1½	13.1	17.4	20.7	23.1	26.8	28.8	30.6	30.8
1¾	18.0	23.7	28.2	32.8	36.4	39.2	41.5	41.8
2	23.1	30.8	36.8	42.8	47.6	51.2	54.4	54.8

for different arcs may be made in the same manner as in flat belting.

Rope Pulleys.—The sheave, as it is usually called, is made in the three shapes shown in Fig. 266. Different values are

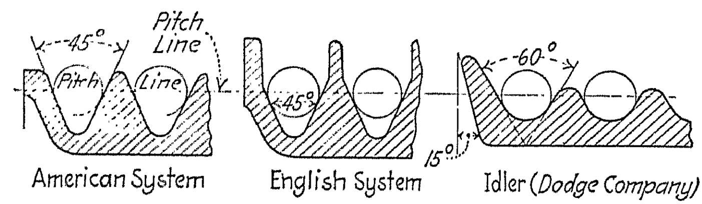


FIG. 266.—Grooves for hemp rope transmission pulleys.

used by different manufacturers for the angle of the groove. For example, the Dodge Company and others make the grooves for the American system 60 deg. both for the working and idler pulleys, and the Weller Company makes them 45 deg., while all companies make the English system grooves 45 deg. Rope drives require sheaves of large diameter on account of the strain on the rope in being bent to a small radius. The sheave diameters must not be less than forty rope diameters at slow speeds, and larger than that at speeds over 4,250 ft. per min. At 5,500 they must be fifty diameters. There is no objection to the use of still larger sheaves; these values are the minimum.

162. Wire Rope Transmission.—For power transmission the field of wire rope is limited and becoming more so. Probably

the only locations adapted to it are remote regions where the nature of the business demands a certain amount of long span transmission, over a valley or a river. It is even debatable whether electric transmission could not be substituted successfully for it in such a place. The minimum distance between shafts is about 60 ft., and the limit in a horizontal direction is about 600 ft. It is not well adapted to factory transmission on account of its weight, rigidity, and rapid destruction due to bending and

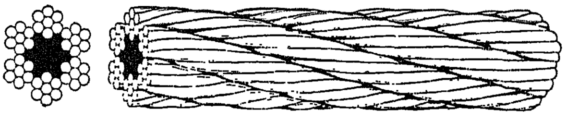


FIG. 267.—Transmission, haulage, or hoisting rope. Six strands of 7 wires each, with hemp core. American Steel & Wire Company, New York.

straightening. The great usefulness of wire rope is in haulage and hoisting, where it has a very large field to itself, but comparatively few industries are now using it for transmission only.

Wire rope is made in six grades or strengths, as follows: Iron, Mild Steel, Crucible Cast Steel, Extra Strong Crucible Cast Steel, Plow Steel and Monitor Plow Steel, increasing in strength in the order named.

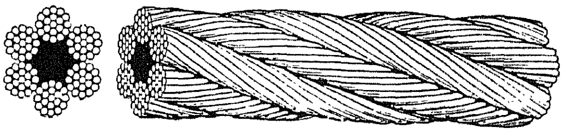


FIG. 268.—Hoisting rope. Six strands of 10 wires each, with hemp core. American Steel & Wire Co.

The American Steel and Wire Company gives the following values for **hoisting wire rope** for 1-in. rope.

	Pounds, safe strength		Pounds, safe strength
Iron..................	5,800	Extra strong crucible cast steel	13,500
Mild steel.............	9,600	Plow steel	15,000
Crucible cast steel......	12,000	Monitor plow steel	18,000

For diameters other than 1-in., multiply these values by the square of the diameter.

Example (1).—What is the safe strength of Crucible Cast Steel Rope of $\frac{5}{8}$-in. diameter?

Ans. $(\frac{5}{8})^2 \times 12{,}000 = 4{,}700$ lb. (nearly).

Example (2).—What diameter of plow steel wire rope should be used on a 5-ton hoist? Single rope.

Ans. $D = \sqrt{\frac{10,000}{15,000}} = .817$ in., say $1\frac{3}{16}$ in.

Flexibility increases with the fineness of the wires, and therefore with the number of wires to the strand. Thus, referring to Fig. 268, the hoisting rope, which must usually be wound on a drum, is made of strands of nineteen wires, whereas the haulage rope, which must resist abrasive wear from dragging along the ground or over rollers, is made of coarser wires, seven to the strand, and is therefore stronger. For very flexible ropes, thirty-seven wires are allotted to the strand.

163. Wire Rope Sheaves.—The shape of the grooves for the two purposes is shown in the sketch, Fig. 269. Wire rope will not stand the wedging that is possible with hemp rope, so

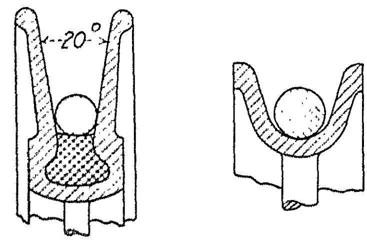


FIG. 269.—Wire rope sheaves.

the tractive power is provided by inserting a soft core in the groove for transmission purposes. This core is usually made of alternate blocks of leather and gutta percha, short pieces shaped to the groove and pressed into place, until the entire circumference is filled. The side walls are made high to prevent jumping out of the cable.

164. Chain Transmission.—For short spans on machinery, or between machines, where positive drive is necessary, or at least desirable, and where high linear velocity is not needed, there is nothing superior to chain drive. The small arc of contact, which is a serious objection in belt drives of short span and different sizes of pulleys, is no objection in a chain drive.

For the highest grade of work, such as motor to machine, and from one shaft of a machine to another in the same machine, the so-called **silent** chain is preferred. Where slower speeds and

work of less importance, light loads, or out-of-doors locations are involved, **roller** chains find a considerable field.

165. Roller Chains are particularly good at speeds of 600 to 800 ft. per min., but can be run up to 1,200. They are composed of steel side links, hardened rollers and pins. The greatest wear is on the pins, consequently these must be made of a high grade of steel, such as nickel steel, heat-treated for toughness. Their worst defect is in stretching (so-called), which results from wear on the pins. In a 5 ft. chain of 1-in. pitch, if each pin is worn .001 in., that means an added length of 1/16 in., which is enough to cause rattling, lost motion, and accelerated wear. Take-ups must be provided to care for this. Chains run over **sprockets,** designed to give perfect roll and easy release.

166. The Silent Chain.—This chain, designed by Renold, and later improved by American engineers, is essentially a steel belt, and can be furnished in any desired width and span. Installations of 5,000 horsepower and upwards are giving satisfaction, some of them delivering at 2,000 ft. per min., and at high efficiencies, 98 to 99 per cent. The links are made in their peculiar shape, so that as the sprocket teeth are worn, the chain will accommodate itself to the increased groove in the wheel. Two varieties are shown in Fig. 271, the original Renold, and the rocker-joint. The greatest usefulness of the silent chain is over short spans, too short for belts, and too long for gears.

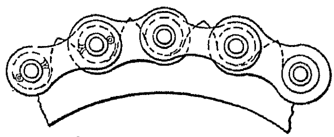


FIG. 270.—Transmission chain and sprocket.

Besides transmission duty, chains find very extensive employment in a variety of forms for conveyors of all kinds. This field is too extensive to be taken up in a small text book, but is thoroughly treated in engineering catalogues, which show a bewildering variety of links and attachments for every conceivable purpose in the handling of commodities. The student will find a partial list of these catalogues in the bibliography at the end of this text.

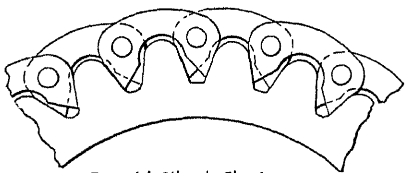
The old-fashioned oblong-linked chain has many uses, and is

extensively employed in the ordinary block-and-tackle, and in differential pulleys.

167. Summary.—Rating the various flexible transmissions according to the speeds to which they are best adapted, they stand in about the following order:

600 to 800 ft. per min.

Roller Chains.—Noisy, positive drive, traction almost perfect.



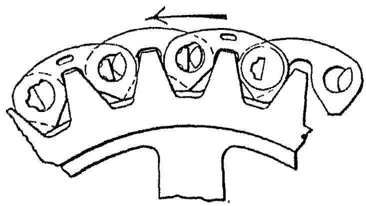


FIG. 271.—Silent chain and sprockets.

Up to 2,000 ft. per min.

Silent Chains.—Quiet, efficient, positive in either direction, give heavy service.

Up to 5,000 ft. per min.

Band Belts.—Not positive, pulleys must not be placed on very short spans, or spans greater than 30 ft. Should not be used in exposed, oily, or steamy locations. $\frac{T_1}{T_2} = 2.5$ on the average. Good alignment necessary. Develops most bearing pressure of any transmission.

Hemp Rope.—Nearly positive, $\frac{T_1}{T_2} = 6$ or more, can be used on spans up to 175 ft., perhaps more, lends itself to convenient distribution, alignment need not be perfect, takes up small space laterally.

Wire Rope.—For long spans, stands exposure and heat, transmits heavy loads. Especially adapted to remote regions. Now being largely superseded by electric power.

In regard to bearing pressure, it should be borne in mind that the larger the initial tension is in proportion to the tractive power, the harder the transmission is on the bearings. Thus, belting imposes more wear on the bearings than rope drive, and rope drive more than chain, and chain more than gearing. Herringbone gears the least of all.

PROBLEMS

Note.—In the following problems, the effect of centrifugal force may be disregarded on all drives slower than 4,000 ft. per min., unless called for in the problem, or as directed by the instructor.

1. The main driving pulley of a punch press is 16 in., and turns at 300 r.p.m. If the pulley is driven by an open belt from a 15 hp. motor, what is the effective belt tension when under full load? If $\frac{T_1}{T_2} = 2.5$, what width of single belt should be used?

2. Same problem. Pulley diam. ——— in., r.p.m. ———, hp. ———, $\frac{T_1}{T_2} =$ ———.

3. An electric motor is driven at 1,600 r.p.m. by a 250 h.p. stationary engine, through a double belt over a 66-in. flywheel, at 110 r.p.m. Assume $\mu = 0.30$, and center distance $= 18$ ft.

Required.—(a) Linear belt speed.
(b) Effective belt tension.
(c) Diameter of motor pulley, taking slip and creep into account.
(d) Angles of contact and belt length.
(e) Belt width.
(f) Centrifugal force.
(g) Taking centrifugal force into account calculate the belt width.

4. Same problem. Hp. ——— (up to 1,000), engine flywheel ——— r.p.m. (50 to 250), motor ——— r.p.m. (800 to 3,000, depending considerably on hp.), flywheel diam. ——— in. (36 to 200 in., depending on size of engine) μ ——— (0.20 to 0.50), pulley centers (12 to 30 ft.), belt thickness ——— (single to quadruple, depending on engine capacity).

5. A steam turbine, running at 600 r.p.m., carries a 32 in. drive pulley, wood faced, delivering its power through a 10 in. single 18 oz. open belt, to a motor running at 2,000 r.p.m. Center distance 22 ft., and $\mu = 0.45$.

Required.—(a) Linear belt speed.
(b) Effective belt tension.
(c) Size of motor pulley, allowing for slip and creep.
(d) Angles of contact and true belt length.

(e) Ratio of total to initial tension.
(f) Horsepower possible.
(g) Centrifugal force.
(h) Taking centrifugal force into account, what diminution in effective tension and horsepower?

6. Same problem. R.p.m. ———, drive pulley ———, belt ——— in., ——— (single to quadruple), r.p.m. of motor ———. μ ———, center distance ——— ft.

7. The drive pulley of a bulldozer rotates at 200 r.p.m., requiring 10 hp. The open belt is 8-in., double, and runs from a countershaft turning at 350. Center distance 10 ft.

Required.—(a) Size of both pulleys, if effective tension is 80 lb. per in. width, allowing for slip and creep.
(b) Arcs of contact and true belt length.
(c) If the bulldozer pulley were 20 × 6 in. (taking a 5½-in. belt), what would be the effective tension and belt thickness?

8. Same problem. R.p.m. ———, hp. ———, belt ——— in., ——— (single, double, or triple), countershaft speed ——— r.p.m., center distance ——— ft., effective tension ——— lb.

9. An open balata belt drives a 350 hp. motor, having a ——— in. cast iron pulley, at 1,800 r.p.m., from the 70-in. flywheel of an engine. The belt is put on at 50 lb. tension per in. width, and is rated to carry 105 lb. per in. width (no more). Center distance 24 ft., and $\mu = 0.40$.

Required.—(a) R.p.m. of engine, allowing for creep, slip, etc.
(b) Width of belt, neglecting centrifugal force.
(c) Arcs of contact.
(d) Centrifugal force.
(e) Width of belt, taking centrifugal force into account.

10. Same problem. Hp. ———, motor pulley ——— in., motor r.p.m. ———, flywheel ——— in., initial tension ——— lb. per in. width, rated tension ——— lb., μ ———, center distance ——— ft.

11. Calculate the centrifugal force of an 8-in. double belt running at speeds up to 10,000 ft. per min., for each 1,000 ft. per min., and make a chart showing what horsepower the belt will carry at any speed. Assume $\theta = 150$ deg., and $\mu = 0.30$.

12. Same problem for ——— in. belt, ——— (single to quadruple), assuming $\theta =$ ——— deg. (120 to 180) and $\mu =$ ———.

13. Make a table of values of $\frac{T_1}{T_2}$, using $\mu = 0.20$ up to 0.50, and θ from 100 to 180 deg. inclusive.

14. Design a pair of stepped pulleys, one on a countershaft running 180 r.p.m., driving a lathe spindle at 360, 225, 144, and 90 r.p.m. Center distance 9 ft. 6 in. Belt 2½ in., single, open, and $\mu = 0.30$.

Required.—(a) Diameters of all steps on both pulleys, no diameter to be larger than 14 in.
(b) Maximum hp.
(c) Length of the belt.

(d) Check the diameters determined graphically by mathematical calculations.

15. Same problem. Countershaft r.p.m. ———, Spindle r.p.m. ———, ———, ———, ———, center distance ——— ft., belt width ——— in., μ ———.

16. Same problem as **14.** Use a crossed belt.

17. Same problem as **15.** Use a crossed belt.

18. A sensitive drill press is to have three speeds. The driving cone pulley runs at 325, and the driven cone at 325, 450, and 650. Smallest step on driver 6 in. Open belt. **Calculate** the diameters of the steps to two decimal places. Allow 5 per cent for loss through slip and creep. Center distance 12 ft. Check the results graphically.

19. Same problem. Four speeds. Driving pulley ——— r.p.m. driven ———, ———, ———, ——— r.p.m. Smallest step on driver ——— in. Center distance ——— ft.

20. Same problem. Five speeds. Driving pulley ——— r.p.m., driven ———, ———, ———, ———, ——— r.p.m. Smallest step on driver ——— in. Center distance ——— ft.

21. A five-step pulley, 16½, 14, 11½, 9, and 6½ in., running 225 r.p.m., drives another at a distance of 16 ft., through a 3 in. crossed single belt. The slowest speed of the driven is 160 r.p.m. Determine the step diameters of the driven cone to two decimal places, allowing 5 per cent for transmission loss. What horsepower can be transmitted at the five speeds, if $\frac{T_1}{T_2} = 2.42$?

22. Same problem. Pulley diameters ———, ———, ———, ———, ——— in., r.p.m. ———. Center distance ——— ft., ——— in. belt, ——— (single or double), slowest speed of follower ——— r.p.m. $\frac{T_1}{T_2} =$ ———.

23. Two equal pulleys having curved faces, resembling frusta of cones, are 30 in. long, and mounted 3 ft. apart. The driver has a speed of 140 r.p.m., and the largest and smallest diameters are 4 and 12 in. The speed changes of the driven pulley are in arithmetical progression as the belt is moved from end to end.

(a) Determine the diameters of the two cones at every 6-in. progression.

(b) Determine the speeds at these points allowing for slip and creep.

(c) Make a drawing of the cone outline. Scale 3 in. = 1 ft.

24. Same problem. Length of cones ——— in., center distance ——— ft., driver speed ——— r.p.m., end diameters ——— and ——— in.

25. A 2,000-hp. engine in a steel mill delivers its power over a 10-ft. flywheel sheave by means of the English rope drive, to two sheaves, respectively 5 ft. and 6 ft. in diameter. The first sheave takes 800 hp. and the second 1,200. Size of rope used is 1½-in. Flywheel revolves 110 r.p.m Center distance 40 ft.

Required.—(a) How many ropes are needed for each sheave?

(b) All arcs of contact.

(c) Length of rope necessary for the entire drive.

(d) $\frac{T_1}{T_2} = 6$. Driving tension and total tension on each rope of each sheave.

26. Same problem. Hp. ———, flywheel ——— ft., r.p.m. ———, rope ——— in. Sheaves ——— ft. and ——— ft., hp. ———, and ———, center distance ——— ft.

27. In a tinplate mill, using the English rope drive, a 14-ft. flywheel, 95 r.p.m., drives a 10 ft. sheave through 60 2-in. ropes. The center distance is 42 ft.

Required.—(a) Arcs of contact.

(b) Length of ropes.

(c) If $\frac{T_1}{T_2} = 6.5$, what is the driving and total tension on each rope?

(d) Hp. transmitted.

28. Same problem. Flywheel ——— ft., ——— ropes ——— in. diam., r.p.m. ———; driven sheave ——— ft., center distance ——— ft.

29. An American rope drive is installed in a cotton mill, distributing 500 hp. from the engine to various shafts over 40 turns of 1¼-in. rope. The engine speed is 130 r.p.m.

Required.—(a) The sheave diameter.

(b) If $\frac{T_1}{T_2} = 5.5$, what are the driving and total tensions in the rope?

(c) How much weight should be placed on the tension carriage?

30. Same problem. Hp. ———, ——— ropes, ——— in., ——— r.p.m.

31. A brick manufacturing plant is installing a rope drive, American system. The engine is 200 hp. at 225 r.p.m., and the sheave is 6 ft. in diameter with 15 grooves for carrying the rope.

Required.—(a) Rope diameter. Is the sheave safe for this diameter?

(b) If $\frac{T_1}{T_2} = 5.8$, what are the driving and total tensions in the rope?

(c) How much weight should be placed on the tension carriage to assure this hp?

32. Same problem. Hp. ———, r.p.m. ———, flywheel sheave ——— ft. diameter, ——— grooves. $\frac{T_1}{T_2} =$ ———.

33. A watch factory, having an engine delivering 500 hp. over a 10-ft. flywheel at 125 r.p.m., decides to convert its plant to American rope drive by installing a hardwood lagging over the flywheel, increasing its diameter to 10 ft. 8 in. The number of grooves is to be 30.

Required.—(a) Largest ropes advisable.

(b) How much horsepower could be delivered with safety?

(c) If $\frac{T_1}{T_2} = 6.1$, what are the driving and total tensions?

(d) How much weight on the tension carriage?

34. Same problem. Hp. ———, flywheel ——— ft. diameter, increased to ——— ft., r.p.m. ———, grooves ———.

35. The chain drive in a certain installation transmits 50 hp. from a motor over a 4-in. sprocket. The sprocket revolves at 1,600 r.p.m. What is its effective tension? What pitch would be advisable and what style of chain? If the steel in the side links had a safe tensile strength of 15,000 lb. to the square in., what would be a good size for the minimum cross-section? If the pin steel were of 20,000 lb. safe shearing strength, what diameter should they be made at the smallest point?

36. Same problem. Hp. ———, sprocket ——— in., r.p.m. ———.

BIBLIOGRAPHY

The Graton & Knight Co. Worcester, Mass. Catalogue.

The Dodge Sales & Engineering Co. Mishawaka, Ind. Catalogue C-16, Rope Drive Section.

Diamond Chain Company, Indianapolis, Ind. "Chains. Charts, Formulae, and Simple Rules."

The Morse Chain Company, Ithaca, N. Y. Catalogue.

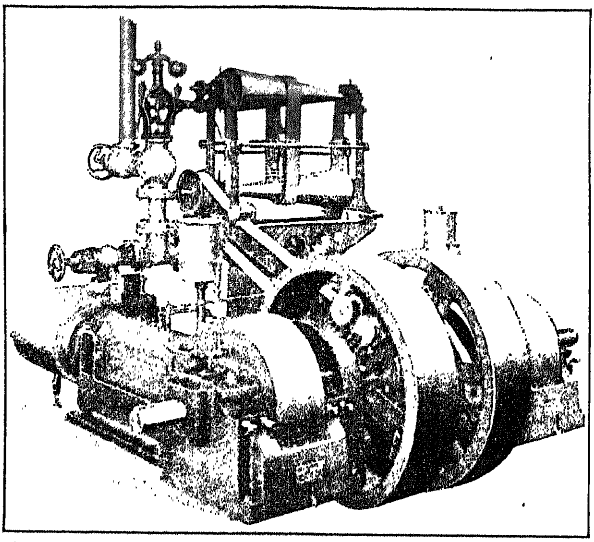


FIG. 272.—Variable speed engine, utilizing unstepped cones as speed transformer. The Chandler & Taylor Company, Indianapolis.

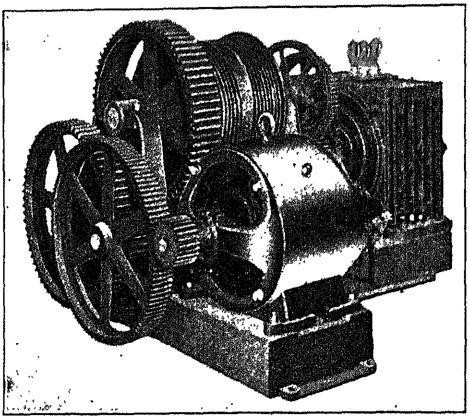


Fig. 273.—Power hoist with triple-reduction gear train. Northern Engineering Works, Detroit.

CHAPTER IX

TRAINS OF MECHANISM

168. Single pairs, from prime mover to product, are seldom found in anything but hand-operated machines, such as grindstones, hand pumps, hoists, and the like. Practically all mechanisms are made up of combinations of units and pairs, working in trains, and are called **trains of mechanism.** These trains may consist of a variety of constituents, bands, gears, frictions, cams, linkages, any or all. The objects of these trains are: (1) To connect the prime mover to the tool, (2) to provide mechanical advantage, (3) to give flexibility to the machine, by making changes of speed and direction easy, (4) compactness, and (5) a wide range of definite speed ratios. Flexibility finds expression in many machines, lathes, drill presses, automobiles, planers, shapers, and milling machines, where reversals of motion and swift changes of speed are conveniently possible to suit the requirements of the work. Mechanical advantage is necessary in all heavy machinery, hoists, ore crushers, conveyors, broaching machines, large presses, shears, and lift bridges, in order that a small, speedy motor, with comparatively low torque, can impart very powerful action, with correspondingly low velocity, to the machine. Compactness is nearly always desirable in mechanism. Exact speed ratios of numerous values are necessary in screw cutting lathes, gear cutters, stock feeders, and all precision tools. Some machines require all these advantages, and others only a part of them.

169. Gear Trains.—A simple gear train, often called commercially a speed reducer, is that shown in Fig. 274. Four gears are mounted on three shafts, the **drive shaft,** the **intermediate** and the **driven.** From drive to intermediate, the velocity reduction is $\frac{A}{B}$, as the number of teeth in the two gears. Gear C is keyed to the intermediate shaft, and therefore C has the same angular velocity as B, and the reduction from intermediate to driven is $\frac{C}{D}$. It will be evident from the teeth in

this train that the intermediate will have $\frac{1}{4}$ the velocity of the driver, and the driven $\frac{1}{5}$ of that of the intermediate. The final velocity ratio, then, of driven to driver will be the product of these reductions, or $\frac{1}{20}$.

Formula.—To calculate the speed ratio of any gear train, divide the product of all the driven gears (their diameter or number of teeth) by the product of all the driver gears. This will give the ratio between the first and last shafts.

$$\text{Thus: } \frac{\omega^A}{\omega^B} = \frac{N^2}{N^1} \times \frac{N^4}{N^3} \times \frac{N^6}{N^5}, \text{ etc.}$$

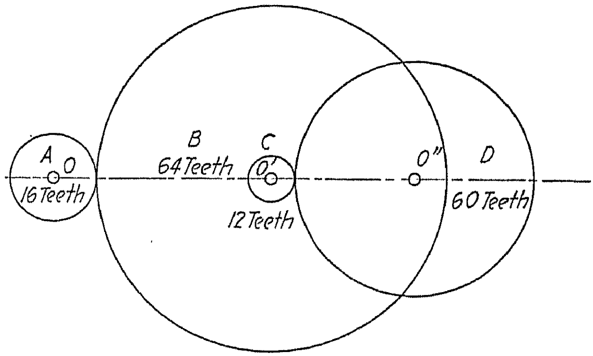


FIG. 274.—Layout of a double-reduction gear train.

Note.—Figure 274 shows what is called a double reduction gear. Adding another shaft, and another pair of reducing gears, would make it triple reduction, etc.

The **compactness** of this arrangement is evident, when it is considered what would be the size of a single pair to produce this reduction. A 12-toothed gear would have to be mated with a 240-toothed gear, to give the same reduction. If the pitch of the gears is taken as 3, the diameter of the 240-toothed gear would be 6 ft. 8 in., whereas in the train the large gears would be 20 and $21\frac{1}{3}$ in. Moreover, in the single pair, the pinion gets 20 times the tooth wear of the large, and this would soon result in poor transmission. Further, the faster pair of the train may be designed to have a smaller pitch than the slower pair, which makes additional compactness and ease of running. Still further compactness may be obtained by placing the first and last shafts on the same center line, as is shown in the picture furnished by the Brown Hoisting Machinery Company. Com-

paring this arrangement with Fig. 274, A and D run on shafts having the same center line. The center lines of A and D may have any relative position desired.

As was stated, all varieties of transmission units may be joined in train, and this will make possible any desired result as to the **plane** of the ultimate motion, its **variety** (rotary, reciprocating, spherical, helical, etc.), its **velocity** (constant, intermittent, irregular, etc.), and its **sense** of direction.

For example.—The prime mover in the automobile; the gas explosions impel the pistons in rectilinear reciprocation, causing the crankshaft to rotate (colloquially spoken of as turning the engine over). The rotary motion is slightly changed in direction through the universal joints to the differential, where it is changed into rotary motion in planes perpendicular to the plane

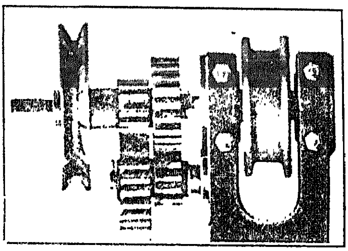


FIG. 275.—A reverted train, double reduction. Brown Hoisting Machinery Company, Cleveland.

of crankshaft rotation. It will be noted that there are several trains involved in this operation and several trains,—the valve train, oiling train, cooling train,—as auxiliaries. The various trains are considered both as units, and as the complete train. Thus, the motor linkwork, the clutch, the gear box, the universal joints, the differential, and in some cars more pairs or trains, and in one, at least, fewer pairs or trains, are assembled in one coordination, under almost perfect control through **other trains** and **pairs** by the driver.

The metal-turning lathe affords another good example of the use of trains, and is typical of the requirements of all machine tools. First, there is the **back gearing** for providing a second spindle speed for each belt speed, and, second, the **screw cutting train,** sometimes called the change gears, which transmits certain definite speeds to the lead screw carrying the tool, so that threads of any standard pitch may be cut, either right hand or

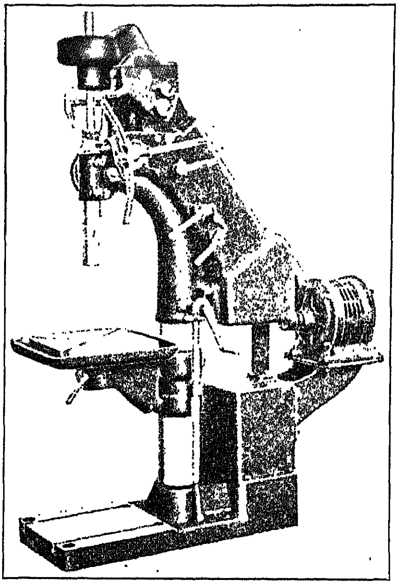


FIG. 276.—All-geared drill press. Barnes Drill Company, Rockford, Ill.

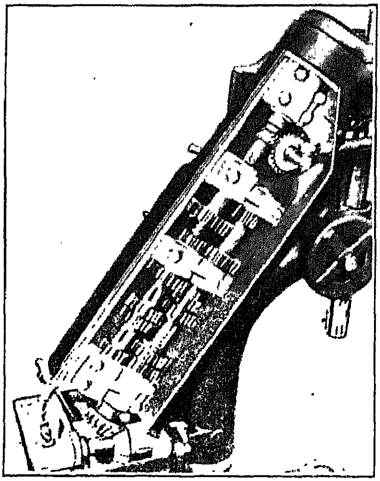


FIG. 277.—Sectional view of gear train of Barnes drill.

left hand. It is evident that the transmission for this purpose must be positive. The trains used in the automobile and screw cutting lathe are typical, and operate under the same kinematic laws as those in other machines, so they will suffice in these columns as representative trains for all machinery.

IDLERS

170. When gears operate in a single pair (except in the case of an internal gear), the rotation of each gear is opposite in sense to the other. Therefore, in the train illustrated in Fig. 274, the first and last gears rotate in the **same** sense. If the train consisted of **four** shafts, the rotation of first and last would be **opposite,** and so on. The conclusion is then that where there is an **odd** number of geared shafts, the sense of rotation of first and last is the **same,** and, where it is **even,** the sense is **opposite.** This conclusion is not of great importance, because so many trains are complicated by open or crossed belts, internal gears, etc., in which connected pairs rotate in the same sense, that any one trying to go by a rule is bewildered.

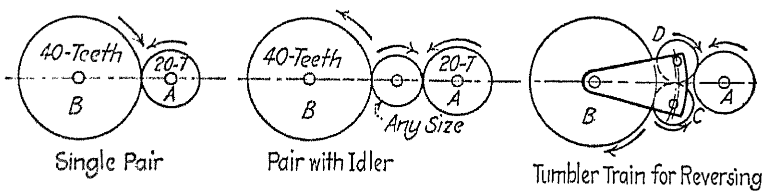


FIG. 278.

The introduction of an **idler** in a train serves to change the direction of rotation, but not the speed ratio. An idler is a gear on an intermediate shaft that transmits motion **directly** from the driver to follower. Note the difference in the arrangement of gears in the train Fig. 274 and in either of the arrangements with idlers in Fig. 278.

Take the case of the pair with idler in the latter figure, in which the idler has any number of teeth, say 30. In the case of the single pair, the ratio is

$$40 : 20, \text{ or } \frac{\omega_A}{\omega_B} = \frac{2}{1}.$$

Insert the idler. The ratio will be

$$= \frac{40}{30} \times \frac{30}{20} = \frac{2}{1}.$$

The inference is clear that the insertion of the idler does not change the speed ratio of driver to follower. It does change the **sense of rotation,** and that is its function. Only rarely is it ever used for any other purpose. Sometimes, to bridge a gap where gears are mounted too far apart for their size, an idler is inserted. If the change of rotation does not interfere with the design, such a device may be used.

A very convenient reversing device, that is used in the screw feed of lathes and in many other places, is the **tumbler,** shown in Fig. 278. Two idlers are mounted on a swinging arm. Gears C and D are always in mesh, but at different center distances from B. In the position shown, the train runs $A—D—C—B$, and A and B rotate in opposite directions. By swinging the arm through a small angle, C is brought into mesh with A, and D is out of engagement with A. The train now runs $A—C—B$, and A and B rotate in the same direction.

EXAMPLES OF GEAR TRAINS

171. The Screw Cutting Lathe Back Gears.—The spindle of an **ungeared** lathe, driven by belts from a countershaft to the cone pulley on the lathe, may have three, four, or five speeds according to the number of steps in the cone pulleys. By the addition of **back gears,** the number of speeds is doubled. A four-step cone pulley for a lathe-head is shown in Fig. 279, with the back gears added. The countershaft, suspended from the ceiling above the machine, is driven by a belt from the main-shaft over a tight-and-loose pulley at a uniform speed. When the belt is shifted to the tight pulley, the countershaft turns, carrying with it a cone pulley of the same number of steps as on the lathe; see Fig. 260.

Example.—The countershaft cone pulley has four steps, $8\frac{13}{16}$, $7\frac{3}{4}$, 6 and $4\frac{1}{2}$ in., running at 275 r.p.m. The lathe steps are $3\frac{1}{4}$, $4\frac{1}{2}$, 6, and $7\frac{3}{4}$ in. The gears are:

$$A = 28 \text{ teeth},$$
$$B = 84 \text{ teeth},$$
$$C = 28 \text{ teeth},$$
$$D = 84 \text{ teeth}.$$

The value of the back gear train is therefore $\frac{84}{28} \times \frac{84}{28} = 9$; *i.e.,* there is a reduction from the cone pulley speed to that of the spindle through these gears, of 9:1.

The cone pulley speeds are calculated as follows, for **direct drive.**

(1) $8\frac{13}{16} \div 3\frac{1}{4} \times 275$ r.p.m. = 675 r.p.m.
(2) $7\frac{3}{4} \div 4\frac{1}{2} \times 275$ r.p.m. = 474 r.p.m.
(3) $6 \div 6 \times 275$ r.p.m. = 275 r.p.m.
(4) $4\frac{1}{2} \div 7\frac{3}{4} \times 275$ r.p.m. = 160 r.p.m.

When the back gears are connected, the spindle speeds are calculated as follows:

(5) $675 \div 9 = 75$ r.p.m.
(6) $474 \div 9 = 53$ r.p.m.
(7) $275 \div 9 = 31$ r.p.m.
(8) $160 \div 9 = 18$ r.p.m.

Fractions are omitted in the foregoing calculations, since an absolute driving speed of 275 r.p.m. is never maintained, and the uncertainty of transmission, due to belt thickness, slipping, and creeping, is considerable, so that great accuracy in calculations is a useless refinement.

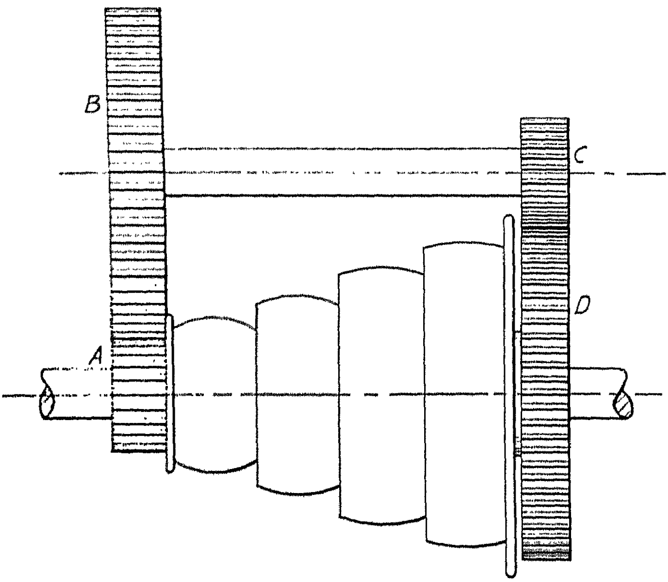


Fig. 279.—Cone pulley with back gearing.

The back gears may be disengaged and connected by the operator, by disconnecting a catch allowing *A* and *D* to revolve independently, and throwing the gear shaft out of its driving center to disengage *B* and *C* from *A* and *D*. When the back gears are out, the gear *D* is coupled to the spindle by a simple catch, and when the back gears are in, the gear *D* must be disconnected from the spindle.

In some of the higher types of lathes **double back-geared spindle drives** are employed. As this means 3 speeds for each cone step, there are 9 speeds obtainable in most lathes having this equipment. The two ratios are effected by sliding the gear pair at the left in or out of contact, in much the same manner as gears are shifted in an automobile.

A still higher type is the **all-geared** head, in which the cone pulley is replaced by a single pulley, and the various spindle speeds are effected by a series of gear trains, as shown in the accompanying view, Fig. 280.

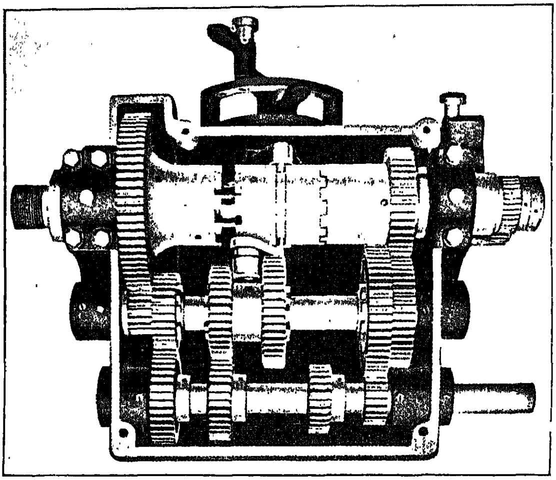


Fig. 280.—All-geared lathe head stock. This set of trains replaces the cone pulley and back gearing in ordinary lathes. Monarch Machine Tool Company, Sidney, Ohio.

172. Change Gears for the Lathe.—The change gears, as the screw-cutting train is usually called, are the gears between the lathe-spindle and the **lead screw** which drives the tool carrier. In order to cut screw threads of a definite pitch, there must be a **positive relation** between the speed of the spindle, which revolves the piece to be threaded, and the advance of the thread-cutting tool. For example; it is desired to cut a thread in a rod, 8 threads to the inch ($\frac{1}{8}$ in. lead). That means that the point of the tool must move horizontally $\frac{1}{8}$ in. for every turn of the rod. If the pitch of the lead screw is $\frac{1}{6}$ in., it must make $\frac{3}{4}$ of a revolution to move the tool $\frac{1}{8}$ in. It must therefore be geared to the spindle in the ratio of 3:4 in order to do this.

Finer or coarser threads require corresponding speed ratios to the lead screw. This adaptability requires a gear train in which the first and last members may be changed at will, and also a reversing device, so that left hand threads may be cut.

173. Standard Threads.—The standard requirements for commercial purposes include the following sizes of threads (number per inch): 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 11½, 12, 13, 14, 15, 18, 20, 22, 24, 26, 28, and 32. The 11½ thread is the standard for **pipe** of 1 in., 1¼ in., 1½ in., and 2 in. nominal inside diameter. The Society of Automobile Engineers has adopted special threads, finer than standard threads for bolts, because

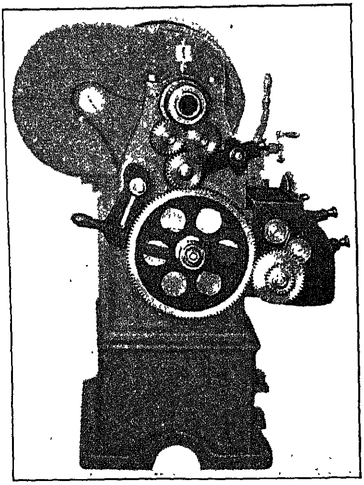


FIG. 281.—Screw cutting train of the Monarch lathe.

they specify steel of better grade (elastic limit not less than 60,000 lb. per square inch). The automobile threads range from 28 for the ¼-in. screw to 12 for the 1½-in. Practically all lathes are equipped to cut all the sizes mentioned, although some small lathes can only cut the finer sizes. There are many odd sizes of threads in the U. S. standards, but they are for the uncommon sizes of bolts, and for the very large ones up to 6-in. diameter. These odd sizes include such threads as 2¼, 2⅜, 2½, etc., and many of the heavier lathes are equipped to cut such sizes. All mechanical engineering handbooks contain the full list of threads for all standards, American, foreign, and automobile.

The picture of the screw cutting train of the Monarch lathe,

shown in Fig. 281, illustrates the usual train employed on most lathes. It will be noted that the train from stud to lead screw is made up of idlers. This is to allow a tumbling train to operate for reversing, and a swinging idler to bridge varying center distances when the change is made at the lead screw. That is, the intervening gears between spindle and stud have no effect on the speed ratio, but this is a direct ratio between the spindle and the lead screw gear.

Example.—If a 24-toothed gear is used on the spindle, and a 40-toothed gear on the lead screw, the ratio is $^{10}/_{6}$ from spindle to lead. If the thread on the lead screw were 6 to the inch, this would mean that 10 threads to the inch could be cut by this combination.

By changing the gear on the lead screw, the various sizes of threads can be cut.

The screw-cutting train, however, is not always composed of idlers. Many up-to-date lathes employ compound trains in which gear changes can be made on intermediate gears. In such a case the speed ratio becomes

$$\frac{\text{Spindle speed}}{\text{Lead screw speed}} = \frac{B}{A} \times \frac{D}{C} \times \frac{F}{E}, \text{ etc.}$$

This ratio is simplified by making some of the pairs of equal size. Idler gears are of course excluded from this equation.

Example.—Take a train in which D, C, F, and E are all unequal. To cut 18 threads to the inch on a lathe equipped with a 6-thread lead screw.

$$\frac{\text{Spindle speed}}{\text{Lead screw speed}} = \frac{18}{6} = \frac{3}{1} = \frac{D}{C} \times \frac{F}{E}.$$

Many combinations could be thought of to satisfy this ratio, one of which might be $\frac{3}{2} \times \frac{2}{1}$ $\therefore$ the following gears could be used: $C = 32$, $D = 48$, $E = 20$, and $F = 40$.

The student will be struck with the fact that this could become confusing to an operator having slight education, consequently many modern lathes are made with automatic change gears, in which the changes are made in a manner similar to the gear-shift of automobiles. The operator need be no better informed on the trains and their working than is the average driver in regard to his gear box. The arrangement calls for three sets of trains from spindle to lead screw, giving the equivalent of three different gears on the stud. A set of gears is mounted on the lead screw having from 30 to 100 teeth, and any of these may be

connected with any of the three trains mentioned. The complete train requires no fewer than 45 gears, and by moving a handle into one of three positions, the back train of that number is placed in engagement, and the other thrown out. By this means the intermediate shaft may have three different speeds. On this intermediate shaft is a **sliding gear** with a **rocking idler** to make connection with the lead screw gear. This sliding gear and the idler are moved horizontally on their respective axes, and engagement is made with any of the ten gears on the lead screw. With the three speeds of the intermediate shaft and ten speeds for each on the lead screw, 30 different ratios between spindle and lead screw are available, which means that 30 threads may be cut on this lathe. Any of these threads may be cut at varying cutting speeds, since these trains give certain ratios, and the spindle may be run at several speeds for certain purposes. The setting for any desired thread is made "fool proof" by a plate on the lathe giving instructions to the operator as to where to insert the handles for that thread.

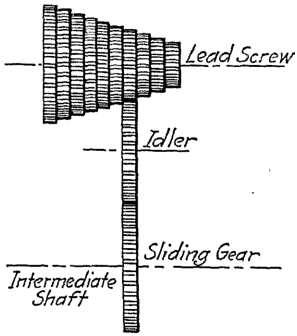


Fig. 282.—Layout of a nest of ten gears, showing arrangement of connecting the lead screw with the change gear trains of a modern screw cutting lathe.

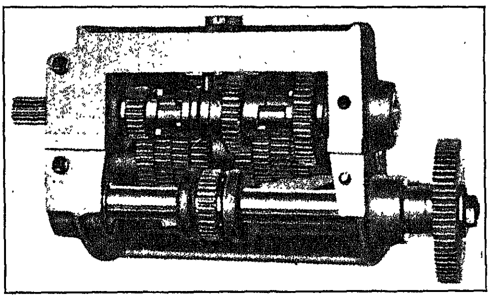


Fig. 283.—Gear box of Monarch lathe.

174. The Automobile Gear Box.—Most of the American automobile designers have come to agreement on a standard gear shift, giving three forward speeds to the drive shaft, and one reverse. The gear box shown in Fig. 284 is a typical example.

The power from the motor is delivered through a friction clutch (not shown) to the shaft S. This shaft carries a gear A, which is in constant engagement with a gear B on an auxiliary shaft X. The **drive shaft** T, which connects through universal joints to the differential on the rear axle, is driven by the crankshaft in one of two ways; (1) **direct,** connecting T and S through a positive clutch (CC'), or (2) **in train** through one of the three gear trains.

The gears B, E, G, and L are permanently fixed to the auxiliary shaft X, and cannot be moved along the shaft. The gears

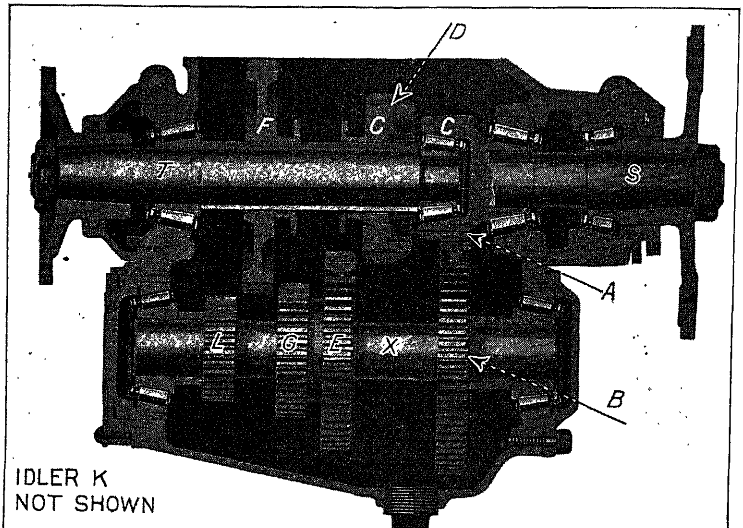


Fig. 284.—Automobile transmission gear box. Timken Bearing Co., Canton, O.

D and F slide on the splined shaft T, so as to engage the gears on X, or be thrown out of engagement. There is an idler K, not shown, which is in constant engagement with L. The positive clutch, CC', is the connecting member which makes T integral with S. C' is a part of the gear D, and may be either an internal gear, or a jaw clutch. The gears in Fig. 284 are "in neutral."

High Gear.—Slide C' into engagement with C, and the result is **direct drive.** T goes at engine speed. X revolves but transmits no power.

Intermediate Gear.—Slide C' out of engagement, and D into engagement with E. The train is $ABED$.

Low Gear.—Slide D out, and F into engagement with G. The train is $ABGF$.

Reverse.—Slide F into engagement with K, which always engages L. The train is $ABLKF$, four shafts, which causes T to take the opposite sense.

The idler K runs on a separate shaft not in the plane of T and X, and has no effect on the speed ratio of its train. When the clutch CC' is thrown in, all gears are out except A and B. The friction clutch operates between S and the crankshaft of the motor, and is needed to give smoothness to the starting, and to the disengaging of the motor while shifting gears. This clutch must be "out" when gears are shifted, to prevent trouble which always results when one fast shaft is connected positively with another running at a different speed, or not running at all. Such gears are known as "clashing gears," and must be rounded, or chamfered, on the ends to allow easy engagement. The gears and shafts must be made very strong, of special steel, heat treated, to stand the heavy demands on them.

A typical arrangement is the following: $A = 16$ teeth, $B = 29$, $D = 22$, $E = 23$, $G = 15$, $F = 30$, and $L = 12$. The idler has 15, but does not influence the ratio.

High Gear.—Direct drive, ratio $= 1:1$.

Intermediate.—$^{29}\!/_{16} \times {}^{23}\!/_{22} = 1.89$; *i.e.*, the engine makes 189 turns to 100 of the power shaft.

Low.—$^{29}\!/_{16} \times {}^{30}\!/_{15} = 3.625$; 363 turns to 100.

Reverse.—$^{29}\!/_{16} \times {}^{30}\!/_{12} = 4.53$; 453 turns to 100, and power shaft turns in the opposite direction.

EPICYCLICAL GEARS

175. A curious, interesting, and useful train is that called the **"epicyclic,"** and usually in commercial circles the "planetary gear." Wherever high speed ratio at high efficiency is desired there is a use for this mechanism. It has been employed in automobile transmission (Ford), hoists, tapping machines, high speed drills, pulley blocks, and many other devices, and is deserving of more attention by machine designers than it has had up to this time.

The epicyclic train has for its basis a stationary gear with another (or several) rotating about it, hence the appellation "planetary." Let A, Fig. 285, be a stationary gear, of 48 teeth

for example, and B, one of 24 teeth in mesh with A, but impelled to rotate around A by a crank C, or other motive agent.

First.—Consider A and B as cylinders not in contact. If C were rotated about its center O, and carried B through a revolution **without rolling on A,** B would make a revolution about O and about O', as is shown by the successive positions of the point K in Fig. 285.

Second.—Consider the gears in mesh. In addition to the revolution about O' shown in the first consideration, B would revolve n times ($n = 48/24 = 2$ in this case) about O' by reason

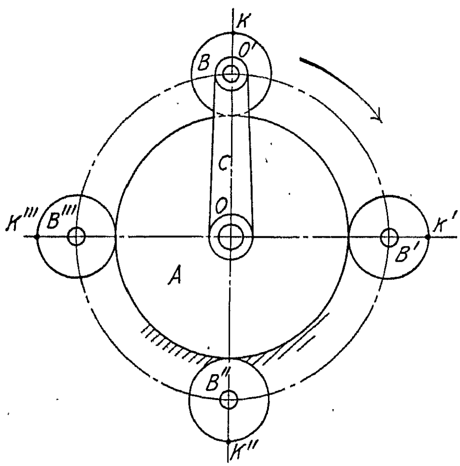


FIG. 285.

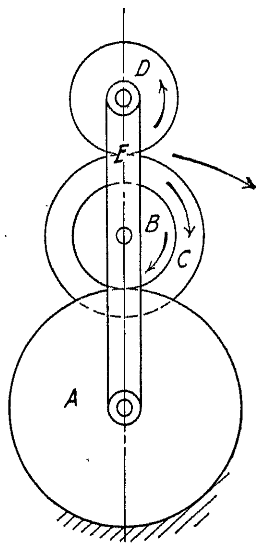


FIG. 286.

of its engagement with A, making a total of $1 + n$ revolutions in the direction of the crank motion.

Third.—Consider a similar train of four gears, as shown in Fig. 286. Here the direction of the gear D would be **opposite** to that of C, and therefore to that of E. Obviously then D would rotate about its crank $1 - n$ revolutions to each crank revolution. **Note.**—N is the value of the train. If n were greater than 1, D would rotate in the opposite sense to that of E, and if it were less than 1, D would rotate **with** E, but more slowly.

It is of no importance whether the arm E carries O, O', and O'' in a straight line, or bent. It can be so arranged as to carry O'' back to coincide with O, making what is called a reverted

train. This is the customary arrangement, and is shown in Fig. 287. The Foote speed reducer is of this design.

Speed Reductions.—By making the value of n very near to unity, the value of $1 - n$ becomes very small. Take this possi-

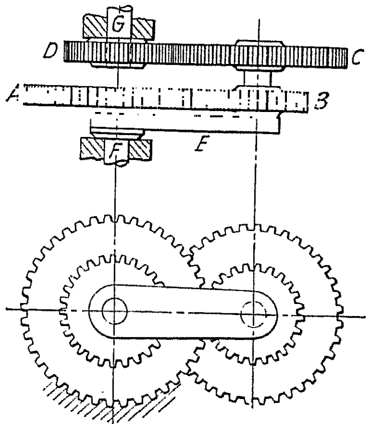


FIG. 287.—Reverted epicyclic train.

bility: $A = 79$, $B = 80$, $C = 80$, and $D = 80$, giving a value to $n = {}^{79}\!/_{80}$. This means a speed reduction of $1 - {}^{79}\!/_{80} = {}^{1}\!/_{80}$. Other trains could be designed to give about the same reduction, with smaller numbers of teeth; *e.g.*, $A = 37$, $B = 40$, $C = 43$, and

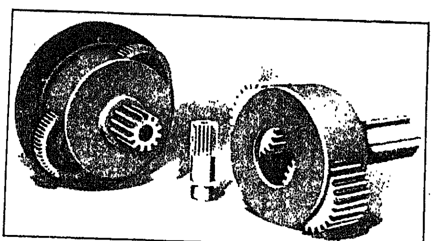


FIG. 288.—Epicyclic gear parts of a Shepard hoist. Shepard Electric Crane & Hoist Co., Montour Falls, N. Y.

$D = 42$. It will be noticed that, although the center distance of A—B and C—D are equal, the sum of the teeth in each pair is not necessarily the same. This means that they will have to be made of different pitches, and it necessitates special cutters for one of the pairs. Barr cites the following train as an example

of the speed reducing possibilities of this mechanism: $A = 99$, $B = 100$, $C = 101$, $D = 100$. From this $n = \frac{99 \times 101}{100 \times 100} = \frac{9,999}{10,000}$; *i.e.*, the shaft of B will only turn once while the crank E turns 10,000 times.

Other arrangements of this train are frequently used. The train shown in the illustration, Fig. 288, is that of the **Internal Gear Epicyclic Train,** sometimes called **Simple Epicyclic Gearing.**

There is no difference in the principle involved, nor in the calculation of the speed ratio, from that composed entirely of external gears. There are no fewer than seven arrangements, or inversions, possible in this train, and the various combinations with the speed ratios of each are given in the following table taken from Machinery's "Handbook:"

TABLE XV.—GEAR TRAIN ARRANGEMENT.

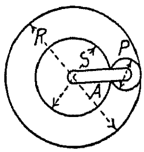


FIG. 289.

S = Sun pinion.
P = Planet pinion.
R = Internal gear.
A = Arm carrying P.

Table gives the relative speeds of each element for one revolution of driving member.
Letters in formulas denote P. D. or N.

Stationary member	Driving member	Driven member	Revolutions of S	Revolutions of A	Revolutions of P around its own center	Revolutions of R
A	S	R	I	O	$\frac{S}{P}$	$\frac{S}{R}$
A	R	S	$\frac{R}{S}$	O	$\frac{R}{P}$	I
R	A	S	$\frac{R+S}{S}$	I	$\frac{R}{P}$	O
S	A	R	O	I	$\frac{S}{P}$	$\frac{R+S}{R}$
R	S	A	I	$\frac{S}{R+S}$	$\frac{S}{R+S} \times \frac{R}{P}$	O
S	R	A	O	$\frac{R}{R+S}$	$\frac{R}{R+S} \times \frac{S}{P}$	I
P	A	S and R	I	I	O	I

In the picture of the Shepard Hoist, the gears shown are S and P. R is omitted in the picture. Three speeds are possible in this hoist.

FORD EPICYCLIC TRANSMISSION

176. The transmission of the Ford car is interesting because it is the only well-known automobile utilizing the epicyclic train. The flywheel of the Ford engine is made to serve a triple

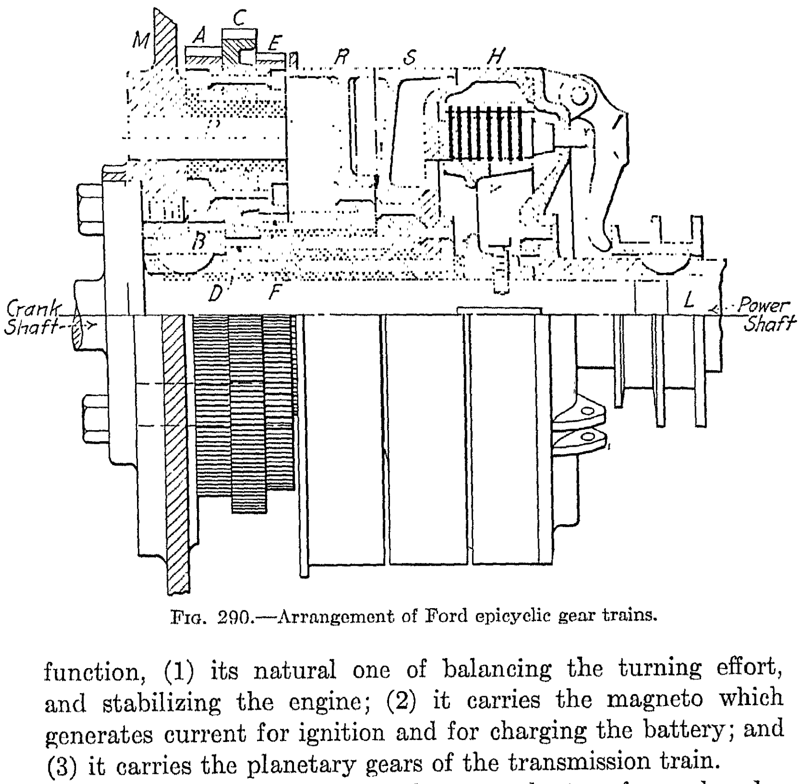


Fig. 290.—Arrangement of Ford epicyclic gear trains.

function, (1) its natural one of balancing the turning effort, and stabilizing the engine; (2) it carries the magneto which generates current for ignition and for charging the battery; and (3) it carries the planetary gears of the transmission train.

This transmission develops three speeds, two forward and a reverse. They are shifted by foot levers, which tighten band brakes on the drums R, S, and H. R is the drum which travels with the reverse, S is the slow speed, and H the high speed, and also the brake.

The planetary gears A, C, and E are carried on the pin P, which rotates with the flywheel M. They are loose on the pin,

and are fastened to each other by rivets, shown in the sketch, and so revolve as one rigid body. Each of these gears engages a gear that is concentric with the engine shaft; *i.e.*, A with B, C with D, and E with F.

By pressing the left foot lever forward, the drum S is tightened, and the gear train A—B—C—D is thrown into action, connecting the motor shaft and power shaft. The teeth in order are 27, 27, 33, and 21, therefore the value of the trains is $N = {}^{27}\!/\!_{27} \times {}^{21}\!/\!_{33} = 0.636$. Since the speed $= 1 - N$, the transmission ratio $= 0.364$, $\therefore$ the power shaft revolves 36 times to 100 of the crankshaft. This is **low gear forward.**

By allowing the pedal to move backward as far as possible, the engagement is made between H and the clutch K, which gives **direct** connection between motor and power shafts. This is **high gear** and the ratio is **unity.**

Reverse.—The central pedal is pressed forward, while the left pedal is held in neutral position. This causes the band on R to tighten, and engages the gears in this order: A—B—F—E. As E has 24 teeth and F has 30, the value of the train $= {}^{27}\!/\!_{27} \times {}^{30}\!/\!_{24} = {}^{5}\!/\!_{4}$. The transmission then $= 1 - {}^{5}\!/\!_{4} = -{}^{1}\!/\!_{4}$, that is, 25 revolutions of the drive shaft to 100 of the engine shaft and **in the opposite sense.**

THE DIFFERENTIAL GEAR TRAIN

177. The cut of a Timken rear axle, Fig. 291, gives the arrangement of the Bevel Differential gear that is commonly used in the rear drive of automobiles, trucks, tractors, etc. The object of the differential is to drive both wheels, and to turn corners with both wheels driven at their proper speed. This means that the outside wheel is turning faster about its axis than the inside, when turning corners. To accomplish this, the rear axle must be in two parts, one for each wheel. This requires the seeming impossibility of a single gear train giving two different angular velocities to two axles, and it is accomplished through the arrangement called the **differential.**

Study Fig. 291. It has been shown that the engine of an automobile transmits its power at different speeds through the gear box to the drive shaft connected by universal joints, and then to the rear axle through the differential. Where the level of the crankshaft is different from that of the rear axle, the

change in level is usually taken care of by the universal joints, but in many trucks this can be accomplished without the universals, by using a worm gear, and in one pleasure car it is done by using a friction disc drive. In all cases, however, the differential must be called into operation. The bevel gear type is here illustrated, and this will be considered here.

The figure shows the rear axle in two parts, *R* and *L*, driving the right and left wheels independently. Mounted rigidly on *R* and *L* are the two bevel gears *D* and *E*. The power shaft *P* carries a bevel pinion *A*, which engages a bevel gear *B*. This bevel gear rotates about the axis of the axles, but is not directly

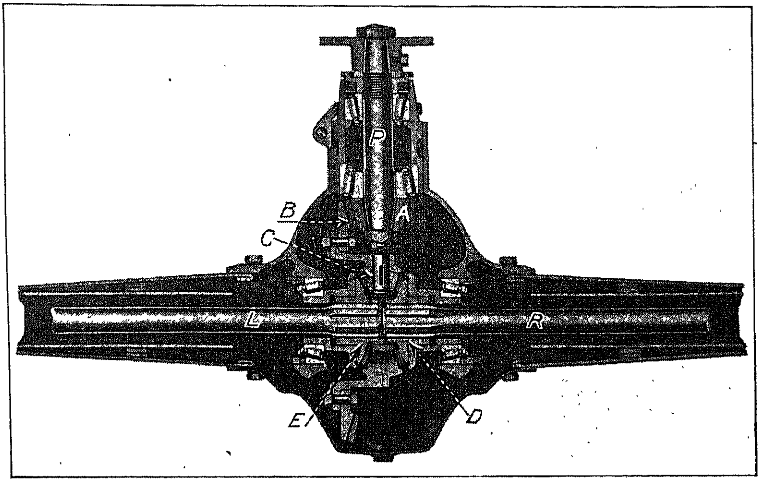


FIG. 291.—The differential gear train in a Timken rear axle.

connected to them. In the web of *B* are mounted some pinions *C*. (Sometimes there are 2, and sometimes 3 of these pinions.) These pinions are in engagement with both *D* and *E*, and are free to turn in their sockets in the web of *B*. When they drive *D* and *E* **at the same speed** (when the car travels straight), they do not turn in their sockets, but travel around the rear axles, and so drive *D* and *E* in the same direction, and at the same speed. When the turn is made and one wheel must travel faster than the other, the shafts of the pinions *C* turn in their sockets slowly, so that the relative travel of the gears *D* and *E* will be automatically changed to suit the requirements of the situation.

PULLEY BLOCKS

178. For hoisting by hand there are a number of arrangements for increasing the mechanical advantage, so that fairly heavy

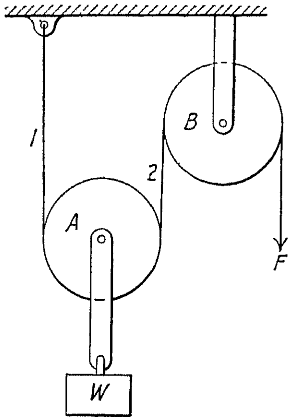


Fig. 292.

loads can be handled by one or two men. A mechanism, which compels the man's hands to travel faster and farther than the load travels, will yield a proportional mechanical advantage, provided the mechanism itself does not consume the difference in friction and lost motion. For such purposes there have been devised a variety of pulley arrangements, which are of great assistance where power operation is not available, convenient, or necessary.

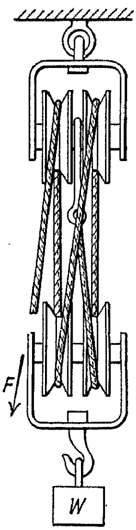


Fig. 293.

Figure 292 shows the simplest compound arrangement of pulleys. A rope is fastened to a support at C, passes under a pulley A, which carries a hook for supporting the load W, and then passes over a pulley which is supported from an overhead beam or ceiling. It is evident that pulling the rope downward at F will raise W. If W is raised one foot, both sections 1 and 2 must be shortened a foot each. This means that **two feet** of rope must pass over B. The distance traveled by the force F is therefore twice that traveled by the load. Therefore the force F need be only half the weight of the load to be raised, neglecting the friction and the rope-bending resistance.

By multiplying the pulleys, making the alternate ones fast and free, it is possible to multiply the mechanical advantage. If the number of portions of the rope between the point of support and the last stationary pulley be 4, as in Fig. 293, the mechanical advantage is 4: 1. This sketch shows the common arrangement of block and tackle, two free pulleys and two fast pulleys. The slack must be taken up in four places, hence the hand must travel four times as far as the load.

THE DIFFERENTIAL PULLEY

179. The Weston Differential Pulley, shown in Fig. 294, is an ingenious arrangement where the difference in size of the pulleys A and B gives the mechanical advantage. In this arrangement a **chain** must be used instead of the rope, because it is endless, and slack when in operation. A rope would slip. In an ordinary block and tackle, one end is fast, and the rope has no chance to slip, since the tension is applied throughout its length. The pulleys in this block are sprockets, and the chain drives positively over the sprocket teeth, and slipping would destroy the performance. The chain used is of the ordinary oblong type. The sprocket A (diameter = D) and the sprocket B (diameter = d) are both free to rotate on their supporting axle. The sprocket C is also free to rotate on its axle, but its size does not affect the mechanical advantage. However, to insure parallelism in the portions of the chain D and E, it is usual to make C a mean between the diameters of A and B.

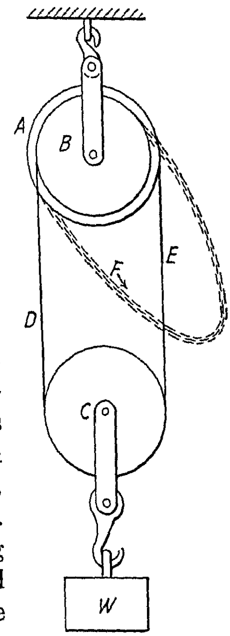


FIG. 294.—Differential pulleys.

The mechanical advantage is arrived at in this manner: the force F is applied in the direction of the arrow so as to give A a certain velocity, say πD. This would raise C at a velocity equal to $\frac{\pi D}{2}$. But the chain, instead of being stationary on the other end, as in the ordinary block, is run over the smaller pulley B at a slower velocity = πd. It is therefore **lowering**

the weight at a velocity $\frac{\pi d}{2}$. Then the actual upward velocity of the load will be half the difference in velocities in the chain sections E and D.

$$\therefore V^C = \frac{\pi D - \pi d}{2}. \quad \text{Since } V^F = \pi D,$$

$$\frac{V^F}{V^C} = \frac{2\pi D}{\pi(D-d)} = \frac{2D}{D-d} = \text{mechanical advantage.}$$

Since the number of sprocket teeth in any two pulleys of equal pitch is proportional to their diameters, N can be substituted for D,

$$\therefore \text{mechanical advantage} = \frac{2N}{N-n}.$$

Example.—A differential block has sprockets on A, B, and C of 24, 22, and 23 teeth. What is the mechanical advantage?

Ans. $M.A. = \frac{2 \times 24}{24-22} = 24$. So, to lift 240 lb. a pull of 10 lb. will be needed.

If the student is desirous of pursuing further the applications of Kinematics in a more detailed manner, especially along the line of Mechanics of Machinery, he is advised to consult the splendid treatment of the subject given in "The Theory of Machines," by Professor Robert W. Angus, B.A. Sc. (McGraw-Hill Book Company, New York.)

PROBLEMS

The following list is made up of problems of genuine engineering value, and those from 17 to 26 may be made the bases of extensive drafting exercises. If the student's time will permit, they may be assigned to be drawn in full, assembly and details, with bill of material, and all necessary directions for manufacture. If the course is too short to allow such elaborate work, they may be cut down to include only the gear trains.

1. A steam engine in a factory carries an 8-ft. flywheel, driving the main belt at 4,000 r.p.m., over a pulley on the main floor, turning at 250 r.p.m. A lathe is driven from the main shaft through a countershaft, the tight and loose pulleys of which are 15 in. in diameter, and turn at 325 r.p.m. The cone pulley of the countershaft has steps of 12, 10, 8, and 6 in., and the largest step on the cone is also 12 in., center distance 12 ft. Draw a layout of this equipment, with all pulley sizes, and determine the spindle speeds of the lathe. Allow for loss of speed by slip and creep.

2. Same problem. Flywheel ——— ft., belt speed ——— ft. per min., main shaft ——— r.p.m., countershaft ——— r.p.m., its pulleys ——— in., cone pulley ——— in., ——— in., ——— in., ———

in., ——— in., center distance ——— ft. Lathe cone, largest step ——— in.

3. Design an automobile transmission of standard type to give approximately the following ratios between engine and power shaft; 1: 1, 1: 0.45, 1: 0.3, and reverse 1: 0.225. Use stub tooth gears 6/8 pitch. Calculate the horsepower that could be safely transmitted at 2,000 r.p.m. of the engine, if the gears were 3½ per cent nickel steel, heat treated, with an elastic limit of 150,000 lb. Factor of safety 10. What are the car speeds at this engine speed, if the wheels are 32 in.? Differential reduction 3:1.

4. Same problem. Ratios forward ———, ———, and ———, reverse ———. Pitch ———, engine speed ——— r.p.m. Wheels ——— in.

5. Design the cone pulleys and back gearing for a 4-step cone lathe. The countershaft runs at 375 r.p.m., and carries a cone pulley, whose largest step is 13½ in. Center distance 10 ft. The desired speeds for the lathe spindle are 750, 525, 375, 225, 100, 70, 50, and 30. Make a working drawing of the cone pulley and back gearing, with a design for a clamp for the direct drives. Use the Burmester method of determining step diameters. Allow for slip and creep.

6. Same problem. Number of steps ———. Countershaft speed ——— r.p.m., largest cone step ——— in., center distance ——— ft., lathe spindle speeds ———, ———, ———, ———, ———, ———, ———, ———, ———, ———.

7. Make up a list for the change gears of a lathe to cut all standard threads. Lead screw is 4 threads per in., single. Center distance from spindle to lead screw, 16 in. What should be the pitch of these gears? Draw the train for 9 threads per in.

8. Same problem. Lead screw ——— threads per in. Center distance ——— in.

9. Design a four-gear epicyclic train (Fig. 287) to give a speed ratio between driving crank and driven shaft of 125: 1. If the center distance is 8 in., determine the pitches of the gears.

10. Same problem. Speed ratio ———, center distance ——— in.

11. Design a planetary automobile transmission (Ford type) to give 1: 0.225 on low, 1: 0.175 on reverse. Determine all circular pitches, and make a drawing of the gears in arrangement. Center distance 7½ in.

12. Same problem. Low gear ratio ———, reverse ———, center distance ——— in.

13. Engine in Prob. **11**. If the engine r.p.m. are 2,000, the differential ratio 3: 1, and the wheels 30-in., calculate the rate of travel in miles per hr. for all three speeds.

14. Same problem. Engine r.p.m. ———, differential ratio ———, wheels ——— in. Use either the engine of Prob. **3** or **11**. At the instructor's option, this may be added to Prob. **4** or Prob. **12**.

15. Design a simple internal epicyclical train for a motor hoist, to give a speed reduction of 10: 1. The internal gear is stationary and 12 in. in diameter. Gears A and A are loose and are driven by a crank. They drive B. Make no gear less than 15-teeth. If the motor rotates at 300 r.p.m., calculate the hp. for stub teeth, s = 12,000 lb.

16. Same problem. Reduction————, minimum gear teeth————, motor speed ————, internal gear ———— in.

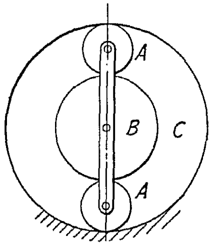


FIG. 295.—Problems 15–16.

17. Design the gear train for a two-man hand winch, like sketch. Double reduction. Handles 15-in. throw, drum 10 in. diameter. Capacity: To lift 3,000 lb. 60 ft. Each man can exert 35 lb. throughout the circle. Hemp rope safe strength, $W = 700\ d^2$. The frame, gears, and drum are of cast iron, and the shafts machine steel.

Required: all gear dimensions, s(for cast iron) = 3,500 lb., all shaft diameters, s = 6,000 lb. At 20 r.p.m. of the cranks, how fast can a load be lifted 60 ft.?

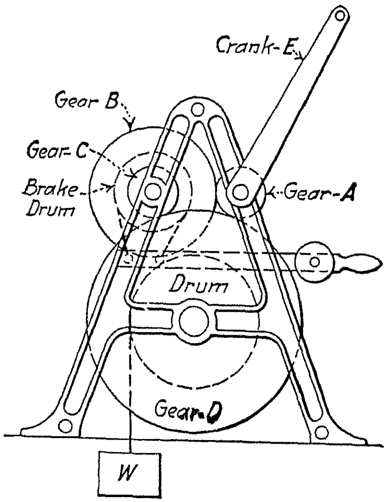


FIG. 296.—Problems 17–18.

Make a complete working drawing, either assembly alone, or assembly and details at the instructor's option, to scale. If the instructor wishes, a self-acting brake (see Appendix) may be designed to be mounted on the intermediate shaft. Three turns of rope should be left on the drum when paid out to full depth.

18. Same problem. Handles ——— in., drum ——— in., load ——— lb., height of lift ——— ft.

19. A one-man hand winch is provided with spur and worm gear train. The handle is 16-in. throw. Design a gear train that will most efficiently raise 2,500 lb. a distance of 75 ft. Use cast iron for the gears, except the worm, using $s = 3{,}500$ lb. per square in. Use machine steel for shafts, including the worm, having a value of $s = 6{,}000$ lb. per square in. Wire hoisting rope of cheapest quality. Drum 12-in. diameter, grooved for the wires. Make a complete drawing of the winch, without housing. As this is probably self-locking, no brake need be designed for it.

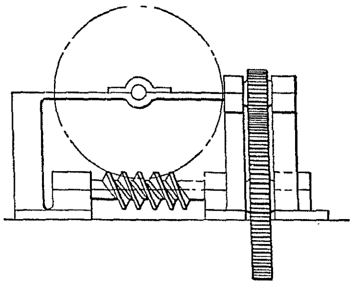


FIG. 297.—Problems 19–20.

20. Same problem. Capacity ——— lb., ——— ft. lift, drum diam. ——— in., crank throw ——— in.

Note.—In the problems 21–26, do not draw the motor, which is supposed to be direct-connected to the drive shaft.

21. A motor driven winch is run by motor at 2,000 r.p.m. Its drum is 16-in. diameter, can lift 5 tons, at 30 ft. per min., at an efficiency of 75 per cent. Design a double-reduction train (single set or double set) of gears, automatic brake, and frame. Determine the proper shaft diameters, $s = 8{,}000$ lb. per sq. in., and make the gears of cast iron, $s = 3{,}500$ lb. per per sq. in. Calculate the horsepower required to run it, and employ wire rope for the drum. Make a complete drawing.

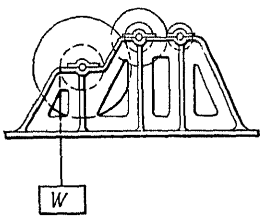


FIG. 298.—Problems 21–26.

22. Same problem. Motor runs ——— r.p.m., capacity of hoist ——— lb. at ——— ft. per min. Efficiency ——— per cent.

23. Same problem. Motor runs ——— r.p.m., capacity of hoist

——— lb. at ——— ft. per min. Efficiency ——— per cent. Use machine steel for the gears, $s = 8{,}000$ lb.

24. Same problem. Triple reduction train. Load 30,000 lb. raised at 10 ft. per min. Motor runs 1,600 r.p.m. Drum 20 in., automatic brake. Steel gears, $s = 8{,}000$ lb. per square in. Efficiency 80 per cent.

25. Same as Prob. **24.** Load ——— lb., at ——— ft. per min. Motor runs ——— r.p.m. Drum ——— in., efficiency ——— per cent.

26. Design a motor-winch similar to the foregoing, inserting a worm gear pair, instead of one of the spur gear pairs. Load ——— lb., at ——— ft. per min. Motor runs ——— r.p.m. Drum ——— in. No brake. Efficiency ——— per cent.

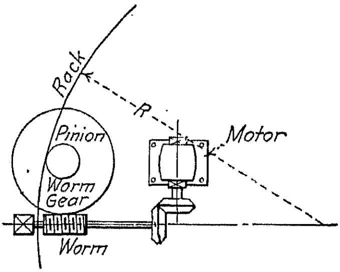


FIG. 299.—Problems 27–28.

27. A draw bridge is driven by a motor, 1,500 r.p.m., acting through a gear train composed of bevel gears 16 and 40 teeth, single thread worm, 36-toothed worm gear, a pinion 18 teeth, and a rack. The radius of the rack is 8 ft. 6 in. and the circular pitch of rack and pinion is 1½ in. (1) How long will it take the bridge to open 90 degrees?

(2) Design a simple reversing device to work with this train as the motor does not reverse.

28. Same problem. Motor runs ——— r.p.m., bevel gears have ——— and ——— teeth, worm ——— thread, worm gear ——— teeth, pinion ——— teeth, rack radius ——— ft. ——— in.; c.p. of rack and pinion ——— in.

29. In a differential pulley, the smaller diameter of the upper pulley is 12 in. It is found necessary to haul 7 ft. of chain over the pulley to raise the weight 6 in. What is the other diameter of the upper sheave? Neglecting friction, how much would a 40 lb. pull on the chain raise?

30. Same problem. Larger pulley on upper sheave ——— in., ——— ft. of chain pulled raises weight ——— in. How much would a ——— lb. pull raise?

31. A differential pulley has a chain of 1½-in. pitch. The sprockets on the upper sheave have 15 and 16 teeth respectively. How much will a pull of 60 lb. lift? How much chain must be hauled over to raise the weight 28 in.? What are the pulley diameters?

32. Same problem. Chain ——— in. pitch, sprockets ——— and ——— teeth, pull ——— lb. Weight to be raised ——— in.

A ⅜-in. cable is wound on a 16-in. drum (E). A man turning a crank, 18-in. throw, carrying a 14-toothed cast iron gear, engaging a 60-toothed gear (F), both 1-in. c.p., to drive the drum. The cable is run over a differential pulley, whose various sizes are, A = 18 in., B = 16 in., C = 16 in., D = 18 in. Calculate the weight that a man can raise, if he exerts a continuous pressure of 40 lb., and the efficiency of the machine is 80 per cent. How fast can he raise this weight if he turns the crank at the rate of 15 r.p.m.? Is the cable strong enough for this load? Is the gear pitch correct?

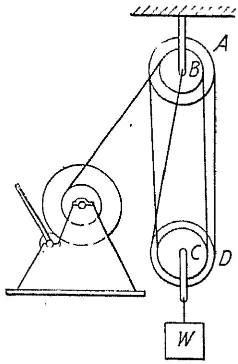


FIG. 300.—Problems 33–34.

34. Same problem. Cable ———— in., Drum ———— in., Crank ———— in. throw, Gear G ———— teeth, F ———— teeth, Pulleys A ———— in., B ———— in., C ———— in., D ———— in. Man exerts ———— lb. pressure, ———— r.p.m., and machine efficiency is ———— per cent.

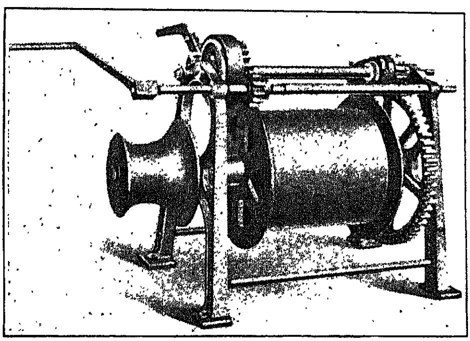


FIG. 301.—Hand winch, or crab. Double reduction gear train and automatic ratchet brake. Brown Hoisting Machinery Company, Cleveland.

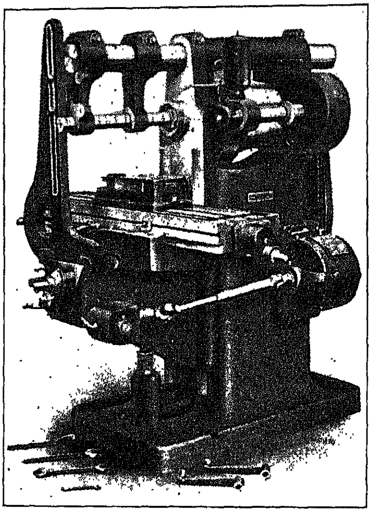


FIG. 302.—Milling machine, showing universal joint. Kempsmith Manufacturing Co., Milwaukee.

APPENDIX

A LIST OF DEVICES, UNITS, AND ATTACHMENTS TO SUPPLEMENT THE STANDARD TRANSMISSIONS

In addition to the standard transmission agencies that have been studied in the regular chapters of this work, there are many ingenious devices for controlling, regulating, and adapting them, which should be familiar to all engaged in the design of machinery, or in production. The number of such items and their combinations is so vast that a mere catalogue of them would be of no value, but there are a few items which have wide application, and are of prime necessity to every engineer. It should be evident that this does not call for much detailed information here, and that it could not include consideration of such special subjects as pumps, blowers, conveying machinery, cranes and air compressors. The student working along any of these lines will find volumes on each of them, filled with detail and engineering information, written by men who have devoted their lifetimes to their particular field of endeavor.

There are, nevertheless, a few units which can be utilized to supplement the linkages, cams, gear trains, belts and pulleys already studied, so as to make complete mechanisms and machines, which will control the stopping, connecting, or safety of these machines, or even take the place of some of them. These units will be considered in the following order:

(1) Couplings and Clutches.
(2) Intermittent Transmitters.
(3) Substitutes for human energy.
(4) Brakes.
(5) Springs.
(6) Anti-friction Bearings.
(7) Conveyors.
(8) Automatic Pick-ups and Hand-lifters.

180. Couplings and Clutches.—Couplings are chiefly **permanent** connections between two lengths of shafting. Shafting may be bought in any desired length from 10 to 30 feet, and it

is frequently necessary in line shafting to connect lengths so as to make a solid shaft 100 feet or more in length, sometimes, if good design is observed, of different diameters. There are numerous designs of couplings, of two classes, **rigid** and **flexible.** The illustration shows a **flanged** coupling, probably the most common type of the rigid coupling. It is made in two parts, and each part is fastened rigidly to the end of a piece of the shafting to be connected. Placing the flanges face to face, they are bolted through and thus the shafts are made one solid shaft with a single axis. The flanges are made with an overhanging lip for protection to the workers from projecting bolt heads or nuts. Where two shafts of different diameters are to be joined, a **reducing** coupling is used. There are many other designs of rigid couplings in use, but it is not necessary to illustrate or discuss them here.

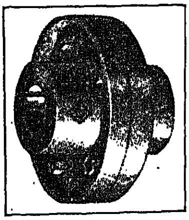


FIG. 303.—Flanged shaft coupling. Dodge Sales and Engineering Company, Mishawaka, Ind.

181. Flexible Couplings.—The Oldham Coupling is a compensating device, in which there is an intermediate disc placed between the flanges. This disc is designed to slide in grooves on the flanged ends, which are rigidly attached to the shaft ends. If the shaft center lines are parallel, and a **very short distance** apart, or if they are slightly out of line, this coupling will conform to the irregularity. In **worm gearing,** for example, it is of utmost importance that the center line of the worm shall be in the center plane of the gear, and where the worm

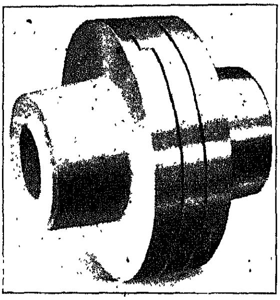


FIG. 304.—Oldham coupling, sometimes called semi-flexible coupling. Weller Mfg. Co., Chicago.

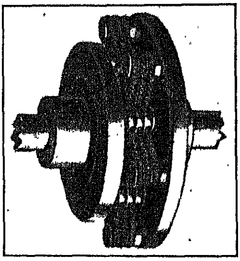


FIG. 305.—Flexible coupling. Link-Belt Co., Chicago.

is driven directly from a motor, it is necessary to use this coupling, or one of the other type illustrated. This will correct any lack of alignment that may occur, temporarily or permanently, from the motor shaft getting out of line with the worm.

Angular Couplings.—When the necessity arises for driving from one shaft to another whose axis makes an angle with the first, some sort of **angle drive** is necessary. In many cases bevel gears are used. If the angle of the shaft axes is greater than 165 deg., the **universal joint** (Hooke's Coupling) is often used. The usual design of this connection is that illustrated from the catalogue of the Boston Gear Company. It consists of two forked ends, one on the end of each shaft, and a block pinned to each fork. The fork is free to turn on the pin, and by this arrangement adjusts its position while driving. The milling machine at the head of this appendix shows a universal joint, in which one of the shafts slides in a sleeve, driving by means of a feather (sliding) key. Being in the sleeve, the distance between centers may be changed, as

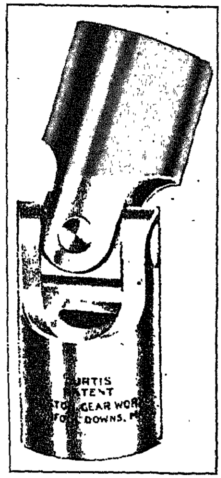


Fig. 306.—Universal joint. Boston Gear Works.

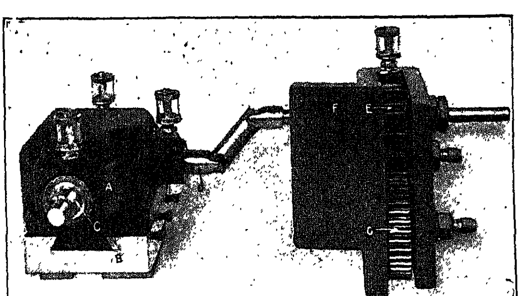


Fig. 307.—Driving drills through universal joints. The National Acme Co., Cleveland.

the table is lowered or raised. The universal joint must not be allowed to swing through a greater angle than 30 deg., as the compensating action of the block will be lost, and the trans-

mission will be **locked.** For larger angles, a **double** universal joint is used, but bevel gears are apt to have the preference. By using a series of knuckles, a flexible shaft may be obtained capable of delivering power in any desired direction. The flexible shaft in dentists' drills is an example of this.

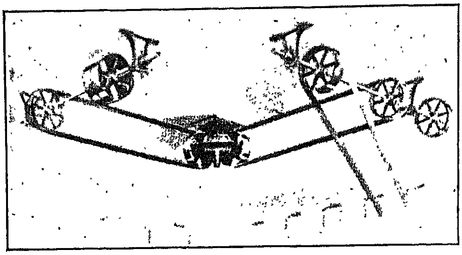


FIG. 308.—Almond right angled drive in place. The Almond Company, Ashburnham, Mass.

182. Right Angled Drive.—For shafts permanently set at right angles (or other angles), it is common to use bevel gears, or flexible band (rope or belt) drive. However, a special device

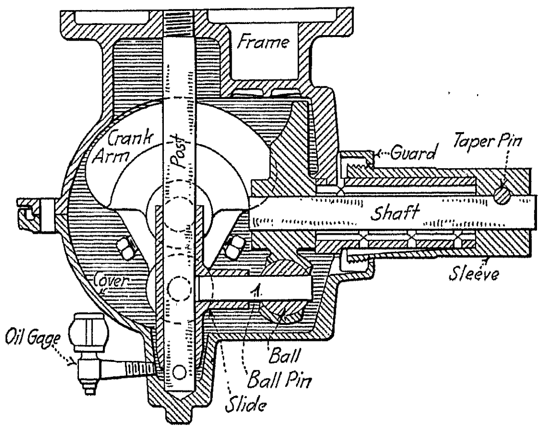


FIG. 309.—Sectional view of Almond right angled drive.

for right angled drive has found considerable adoption in large industries, in which a principle is used differing from the usual practice. This is the Almond drive shown in the illustration. The medium of transmission is pure crank motion, transmitting rotation from one plane to another at right angles. A study of the sectional view will show that the driving shaft, as it turns,

carries a pin on a spherical bearing. The pin is attached to a sleeve, to which another pin is attached at right angles, and the second pin drives the follower shaft by being mounted on a spherical bearing. The motion of the pins is helical and reciprocating. The efficiency of the drive is claimed by its makers to be 90 to 93 per cent, and therefore better than bevel gearing and considerably better than quarter-turn belt drives and bevel frictions.

183. **Clutches.**—A clutch is a temporary coupling, to be thrown out of engagement when transmission is not wanted. Clutches are of two main classes, **positive,** and **friction.** The positive clutches are shown in the illustration. The one shown in Fig. 311 is positive **both ways,** and must never be thrown in when the shafts are in motion, unless that motion be **very** slow. The others are **one-way** clutches, and only drive in the direction of their respective names. They may be thrown in when the

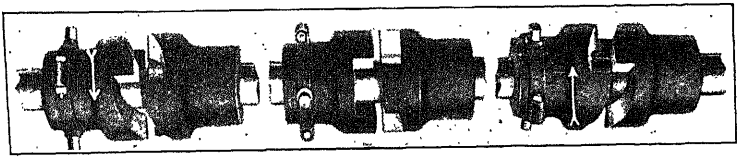


Fig. 310. Fig. 311. Fig. 312.
Positive jaw clutches. Weller Mfg. Co., Chicago.

drive shaft is moving. There is bound to be a jerk to the machine when positive clutches are thrown in, while the shaft is turning.

Friction Clutches are numerous in their designs, and are dependent on pressure to provide their gripping property. Friction clutches are of three main classes: (1) Those that expand **inside** a drum; (2) those that contract **outside** the drum; and (3) those that are pressed together **axially.**

The **expansion clutch** has for its main member a drum integral with the driven shaft. A split ring on the driving shaft runs close inside the drum, so that a slight expansion of its diameter (by opening the gap) will cause it to grip the drum and drive the other shaft length. The expansion of the ring is effected in various ways, the most positive being the right- and left-hand screw. The screw is operated by a lever, connected to a toggle-joint linkage, which is thrown into position by pushing a cone-

pointed shifter between two expanding jaws. This design of clutch, having its grips and levers all on the inside of the drum, presents a neat appearance on the shaft.

The **compression clutch** is well illustrated by the accompanying illustration of the Medart clutch. This is typical of many

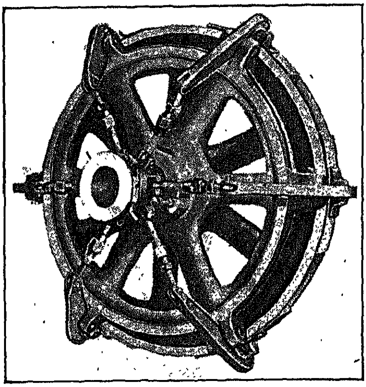


FIG. 313.—Compression clutch. The Medart Company, St. Louis.

designs, in some of which the jaws press into conical grooves similar to this, and in others on flat, cylindrical surfaces.

Double-grip Clutches are often found in which the grip is both on the inside and outside of the drum.

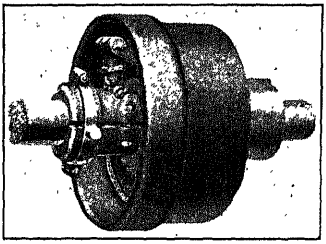


FIG. 314.—Weller internal expansion clutch.

Axial Thrust Clutches.—The two principal varieties in this class are the **disc,** or plate, and the **cone clutch.** The disc clutch depends, for its grip, on the friction between two flat plates having a high coefficient of friction. The plates are pressed together with considerable force, and there is considerable heat developed before the pressure is sufficient to provide the

driving grip. The reason for this is that one plate is rotating rapidly, when the second one is stationary. On this account, the soft plate must be made of some heat-resisting and durable material, like asbestos. In order to distribute the pressure over

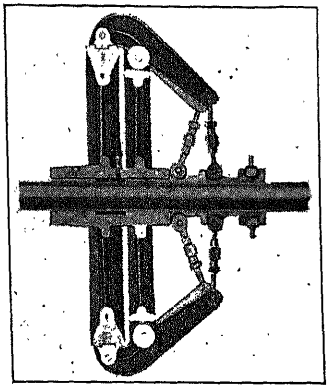


FIG. 315.—Sectional view of Medart clutch.

a considerable area, most disc clutches are **multiple**; *i.e.*, they are composed of two or more pairs of plates which receive the pressure simultaneously.

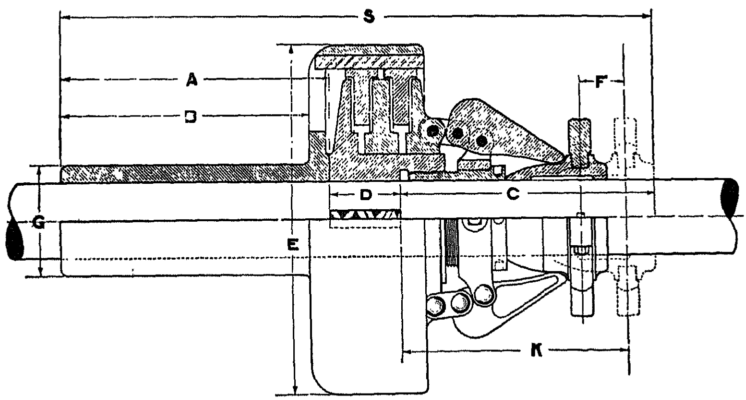


FIG. 316.—Sectional view of Dodge disc clutch.

184. Cone Clutches.—The cone clutch has one immense advantage, its great gripping power. The same increase in pressure is brought about through the deflection of the applied force in the same way as that obtained in the case of grooved

frictions and rope drives. In the case of the clutch, however, this advantage may prove a danger, because it cannot always be released when wanted. If the elemental angle of the cone with the axis is less than 5 deg., this difficulty is pretty certain to be present. The main reason that cone clutches are not used on automobiles is that they have given trouble in releasing, so that it has sometimes been necessary to kill the engine to stop the car. The clutch shown in the illustration is double, and connects with either forward or reverse drive by shifting to right or left. There is a certain make of slotting machine, in which a quick return motion is obtained by the use of a reversing cone clutch of this type. The reversing train is geared to give twice the

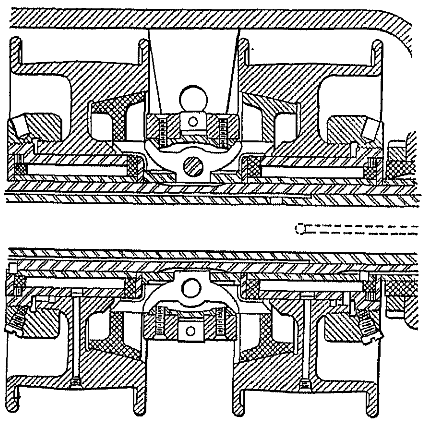


FIG. 317.—Sectional view of Brown & Sharpe reversing cone clutch.

velocity to the carrier as does the forward feed. In the case of this slotter, the clutch is automatically shifted, and is practically instantaneous.

185. Variable and Intermittent Transmitters.—Among the numerous devices for giving an unlimited number of speed changes, such as the disc frictions and Evans cones, shown in Chapter IV, and the belted cones in Chapter VIII, the transmission illustrated here has attained wide employment in heavy work. A *V*-shaped belt is run between two pairs of cones, set with their apices toward each other. By changing the distance between the surfaces, the belt is shifted up or down in the groove. The driver runs at a constant speed, and when the cones are at their widest, the belt settles to its lowest position of contact.

When the cones of the driver are at their closest, the belt is high and the belt speed is high. The driven pulley runs fastest when the belt is down in the notch, and slowest when the belt runs high. There are two rocking levers attached to the frame, which accomplish a harmonic action of the four cones, so that

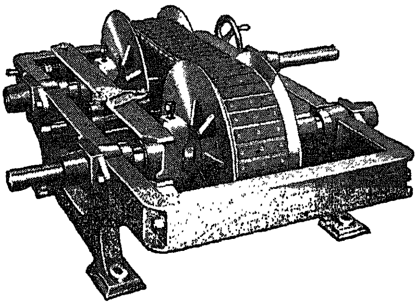


FIG. 318.—Variable speed transmission. Reeves Pulley Company, Columbus, O.

if one pair is brought together, the other pair is separated. The belt may then be shifted to any position, so that any speed within the limits may be imparted to the follower. The usual maximum and minimum range is about 10:1 in follower velocities. That is, a shaft running 200 r.p.m. can drive through this

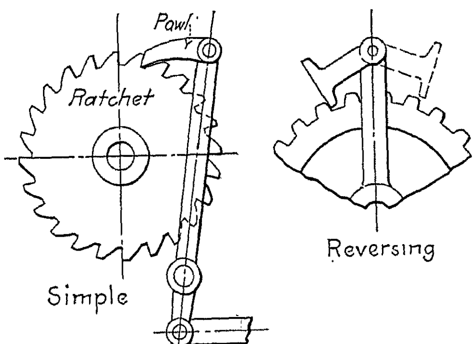


FIG. 319.—Ratchets and pawls.

transmission the follower at any speed between 60 and 600. This device has been extensively applied to automatic stokers, so that a thermostat can very easily regulate the stoking feed.

186. **Ratchets and Pawls.**—For regular intermittent transmission there is nothing superior to the ratchet and pawl. On

the forward stroke, the pawl catches in the corner of the tooth of the ratchet, and advances it, but on the return, the pawl slips over the teeth without engaging them. It is evident then that a reciprocating motion is usually given a pawl when it is used for driving. This motion and force may be derived from the mechanism of the machine of which it is a part, and thus it is easy to synchronize the action. Or it may be operated from the outside. Many machine feeds, such as rivet makers, bolt headers, eye-bolt machines, punch presses, and others used in quantity production, employ the ratchet and pawl to move the stock into position for the work to be performed. Many hand drills, screw drivers, jacks, and other implements for manual use are equipped with pawl and ratchet. A form of reversing pawl is shown in the accompanying sketch. Many designs of brakes are able to function only by being equipped with ratchet and pawl.

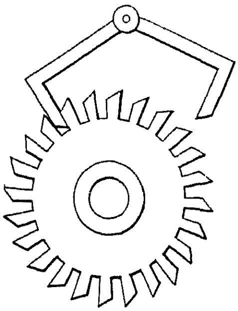


Fig. 320.—Escapement.

In clockwork a form of ratchet and pawl, called the **escapement,** is used to control the motion of the gear train. To the pendulum are attached the two arms shown in the figure, and as it swings, they oscillate concurrently, and so allow a certain

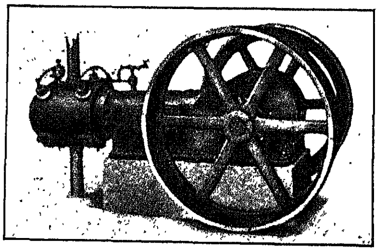


Fig. 321.—Air compressor. Chicago Pneumatic Tool Company, New York.

motion of the toothed wheel to each stroke of the pendulum. The wheel is actuated by the main spring, or, in old-fashioned clocks, by weights suspended by strings which are wound on a drum.

187. Substitutes for Human Energy.—The student has been shown how cams can take the place of human energy and watch-

fulness, and the object here is to instance a few items wherein the cam is either replaced or reinforced in order to make the human substitution.

Compressed Air.—In addition to supplying the energy for many pneumatic tools, such as drills and riveting hammers, compressed air is found to be a most convenient supply of intermittent energy in many factories. Besides other duties **air hoists** are often employed to lift heavy pieces into place in lathes and other machines, and to remove these pieces when the machine work is done on them. The air hoist is operated by opening a valve (sometimes by pulling a chain) which allows the air to enter the cylinder and press the piston forward in the same manner as in the steam engine. In automatic operation of machines, an air hoist controlled by a **solenoid** accomplishes wonderful results. A **solenoid** is a very simple type of an electromagnet. It consists of a cylindrical coil of circular or rectangular section, inside of which fits an iron plunger. When the coil is excited, the plunger is attracted; when the current is stopped, the plunger falls (if in a vertical position). If a **contact cam** (similar to the electric sign mechanism) is installed so as to make and break contact at certain intervals and for certain periods, the solenoid can be made to admit the air into the cylinder of the air hoist for a certain period at any desired interval. The action of the solenoid is almost instantaneous. Where work is to be inserted and withdrawn at definite intervals, an air hoist operated as just described, can do the work of a man, and do it with a precision and regularity difficult of attainment by a man. Many large industries have air pipes running in all parts of the plant, so that attachments may be made wherever needed. The usual pipe pressure for industrial supply is 80 to 100 lb. per square inch. Practically any desired pressure can be supplied for any purpose from 10 to 2,000 lb. per square inch. Compressed air is also used for cushioning purposes, and for air brakes on railroad trains. Air is furnished for

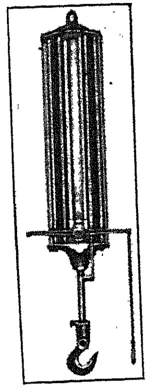


FIG. 322.—Air hoist. Northern Engineering Works, Detroit.

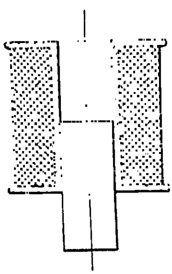


FIG. 323.—A solenoid.

oxidizing blasts for furnaces, for forges and for spraying, drying, and cleaning purposes, and there are many designs of blowers for such work.

Electricity and **Hydraulics** are substitutes for human energy, and could properly be discussed in connection with this paragraph. The subjects and their applications are so vast in extent, that they form the bases of several distinct and important branches of engineering, and it is no exaggeration to say that a thorough understanding of either can only come from a life-long study.

188. Brakes.—Most brakes depend on friction for their operation. Like clutches, their designs are numerous, but since there is no great difference in their action, or even in the manner of applying the force, the intention here is only to illustrate two instances, an automatic band brake for a certain design of hand

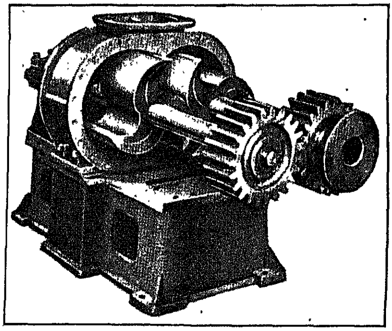


Fig. 324.—Blower, impellor type. Goulds Mfg. Co., Seneca Falls, N. Y.

hoist, or **winch,** and an **electric brake.** This winch is shown in Fig. 301.

Automatic Brake for Winch, Its Operation.—In this machine the weight is raised by one or two men each turning a crank. The crank shaft is connected by an ordinary double reduction gear train to the drum shaft. The cable bearing the weight is wound on the drum. In some of these designs it is possible to shift the pinion A so as to engage D instead of B. By doing this, the men are able to raise light loads in 1/3 to 1/4 the time, by driving through a single pair. In other designs, the cranks may be changed to the intermediate shaft, effecting the same object.

The brake drum is mounted **loose** on the intermediate shaft, and a pawl is pinned to the drum. A ratchet is keyed to the

intermediate shaft and as long as it turns in the **lifting direction,** it turns past the pawl without resistance. When the weight is lowered, accidentally or intentionally, the pawl slips into the

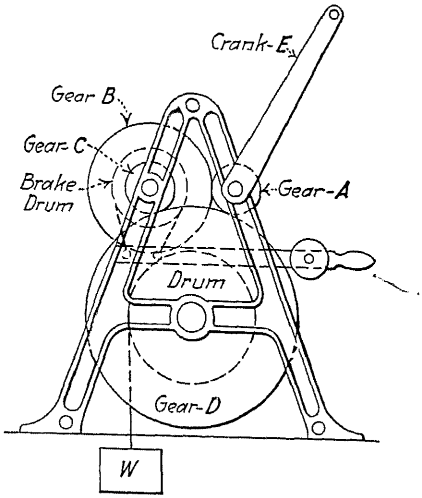


FIG. 325.—Layout of gear train and ratchet brake for hand winch.

notch of the ratchet to prevent its turning. This prevention is accomplished by the friction of a steel band bearing on the surface of the drum. The pressure between the drum and band

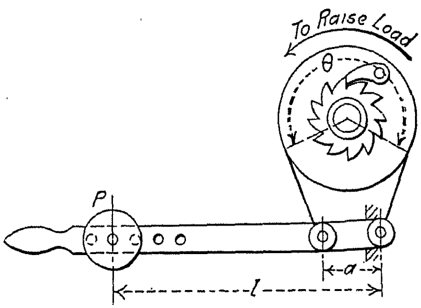


FIG. 326.—Detail of automatic ratchet brake.

surfaces depends on the load, and whether it is desired to hold it stationary, or allow it to descend slowly. If the weight P is just enough to balance the downward pull of the load, the latter will be allowed to descend gently.

To determine the balance weight P for a given load and pair of gears, the following factors are involved: W, the weight of the load, μ, the coefficient of friction, D, the diameter of the rope drum, d, the diameter of the brake drum, θ, the contact angle of the band, the distances a and l, and the number of teeth in the gear pair, N and n.

The mechanical advantage $= \frac{n}{N} \times \frac{d}{D} \therefore F$ (the force on the brake drum surface) $= W \frac{n}{N} \times \frac{d}{D}$.

$$P = F \frac{e^{\mu\theta}}{e^{\mu\theta} - 1} \cdot \frac{l}{a}.$$

Since

$$F = \frac{Wnd}{ND} \therefore P = \frac{Wnd\, e^{\mu\theta}\, l}{ND\,(e^{\mu\theta} - 1)\, a}.$$

Note.—θ is taken in radians.

The following values of $e^{\mu\theta}$ are taken from Nachman's "Elements of Machine Design." Professor Nachman uses a value of 0.18 for μ, and this will be found conservative:

TABLE XVI.

Angle of band contact =	36°	72°	108°	144°	180°	216°	252°	288°	324°
$e^{\mu\theta} =$	1.12	1.25	1.40	1.57	1.76	1.97	2.21	2.47	2.77

189. Electric Brakes are much in favor for cranes, trolley cars, etc., because of their instant application. Various designs are in use, and a typical instance is the Cutter-Hammer brake shown in the illustration. The brake shoes are mounted on arms and are tightly pressed against the drum surface by heavy springs. When the brake is not needed, the shoe pressure is released by attracting them from the drum, or the wheels, by exciting a powerful magnet. Thus, the machine or car is free while the current is on, and locked when the current is off. The current in the brake magnet is controlled independently of the motive or lifting mechanism.

190.—Springs.—Springs serve four purposes:

(1) To maintain certain relative conditions between two members of a machine, until some external force changes them.

Examples: Valves, controllers, governors, clutches, gates, etc.

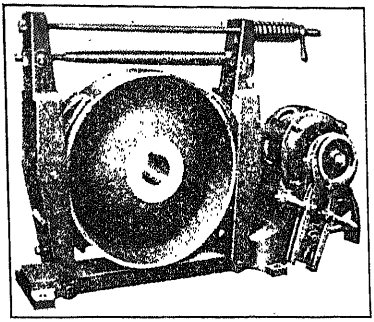


FIG. 327.—Magnetic brake. Cutler-Hammer Mfg. Co., Milwaukee.

(2) To absorb the energy due to shocks, sudden applications of force.

Examples: All vehicles, railway trains, etc.

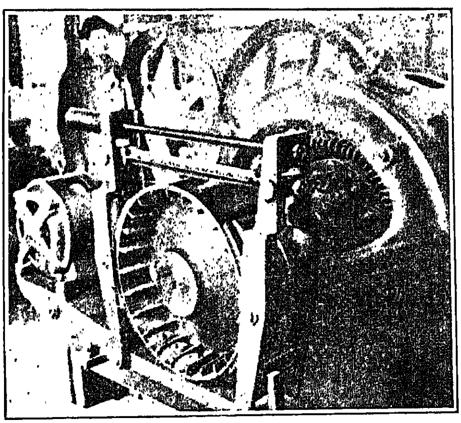


FIG. 328.—Mine hoist equipped with Cutler-Hammer magnetic brake.

(3) To store energy and give it out. Clockwork.

(4) To measure forces.

Examples: Steam indicators, spring scales, gauges.

Springs depend for their usefulness on their elasticity and their **lack** of rigidity. Unlike most machine members, springs

perform their duties through large deformations. These peculiar properties of springs require steels of special chemical composition and heat treatment. The most common of the springs in use are shown in the sketch.

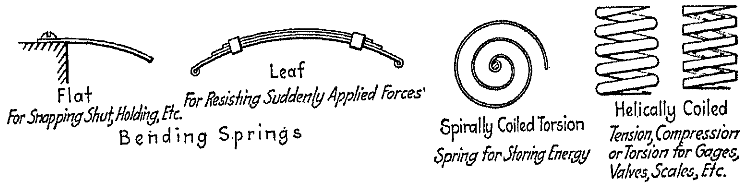


FIG. 329.

191. Anti-Friction Bearings.—The elimination of waste in materials is one of the great problems and inspirations of the chemists, and the elimination of waste of **power,** which means waste of coal, oil, and other fuel, is one of the great problems of the industrial engineer. The worst cause of waste of power in transmission is **friction.** Every rotating or sliding member of a transmission or machine is constrained by **bearings,** and the problem of the efficiency of that transmission or machine is chiefly dependent on the excellence of the bearings. If the contact between the rotating member and its sleeve is one continuous surface, there is bound to be more frictional loss than if that contact is made up of minute rolling areas, quite small in the aggregate. There is no definite amount that can be established as the loss in any bearing. This amount is variable, depending on speed, load, lubrication, and the character of the bearing surfaces. If the lubrication of a surface bearing is neglected, the disastrous effect of a hot box is familiar to all.

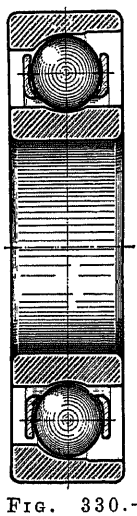


FIG. 330.—Radial bearing. Will also take a considerable thrust load. The Norma Company of America, New York.

The development of the bicycle, which began about 1880, demanded improvement in the bearings in wheels, pedals, crank axle, and steering head. The effort to propel a bicycle with rubbing bearings, and the universal desire for more speed, soon compelled bicycle makers to develop a better bearing, and this was the **Ball Bearing.** While ball bearings may have had earlier applications, the

bicycle was the compelling agency that made them practical and standard. Steel tubing and the invention of the pneumatic tire (about 1888) completed the solution of the problem of the best means of self-propelled transportation ever designed. The automobile has continued the development of ball and roller bearings, so that now it is almost criminal waste to design expensive machinery with surface bearings. Many engineering brains have been busy studying the design, load effects, materials, and applications of these bearings. How well they have succeeded is best illustrated by the automobile, where ball and roller bearings are subject to the most severe duty imaginable, even on good roads, whereas in driving over rough roads, the side pressures, bumps, and twists that the bearings must stand are almost unbelievably severe, yet repairs on them are quite infrequent.

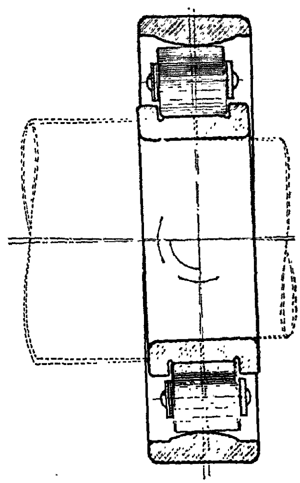


FIG. 330a.—The Norma roller bearing.

Ball and roller bearing design comes into two main divisions: (1) To withstand and transmit **radial** pressures, those that are exerted in a plane perpendicular to the rotating shaft; and (2)

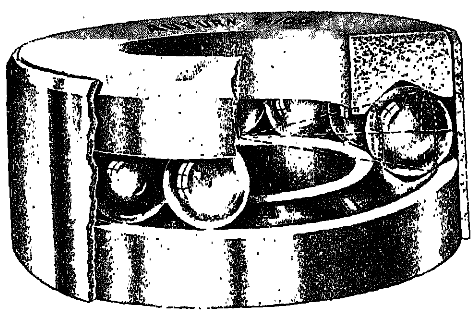


FIG. 331.—Thrust bearing. Auburn Ball Bearing Company, Rochester.

to withstand **thrust**, in which the pressure is exerted in the direction of the axis. Most, if not all, radial bearings must be subject to some thrust, more or less, and so most of them are designed to withstand at least 10 per cent of thrust in their

load. There is also a type of bearing called the **Radio-Thrust** in which there is resistance to a larger amount of thrust.

The thrust bearing designed by the Auburn Ball Bearing Company engineers is a four-point rolling contact in which the

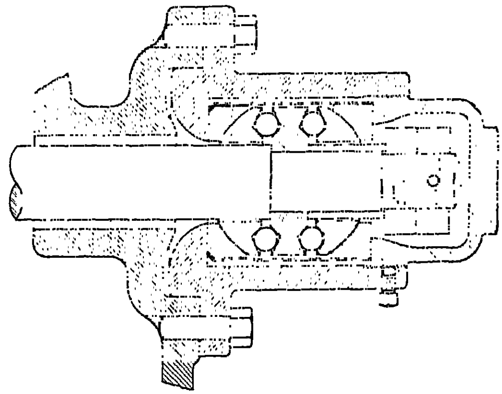


Fig. 332.—Thrust bearing, spherically seated for self-alignment. Auburn Ball Bearing Co.

four points of contact between the ball and the raceways lie on the surfaces (imaginary) of cones having a common apex, ensuring pure rolling. A self-aligning bearing also is shown in the accompanying sketch, in which the raceways are provided with

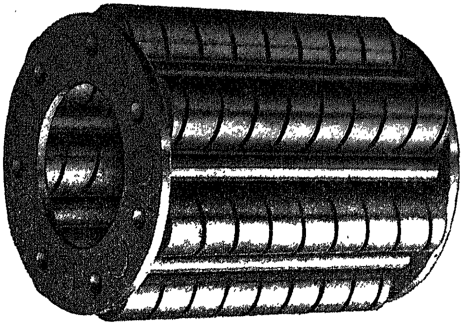


Fig. 333.—The Hyatt roller bearing.

spherical seats. Thrust bearings are numerous in their applications, the most conspicuous being worm gears and jib cranes.

192. Roller Bearings.—The two best known designs of roller bearings are the Hyatt and the Timken. The Hyatt bearing is a flexible cylinder, made of flat steel wound helically into a

cylinder, then heat treated, and ground. The flexibility, which is not very noticeable, is said to give true line contact, which a solid cylinder could not be expected to give, throughout its length. A bearing of this type cannot be expected to take much thrust, because a cylindrical bearing makes no constrainment in the axial direction. These bearings find a wide adoption in all sorts of machinery, in transmission, and in certain of the automobile bearings where there is little or no end thrust.

The Timken Bearing is a conical roller acting between two conical raceways. Since these three conical surfaces all have a common apex, the condition of perfect rolling is assured. This bearing is seen illustrated in several places in this work, and is probably the best known of any of the autcmobile bearings on account of the prominence of Timken axles.

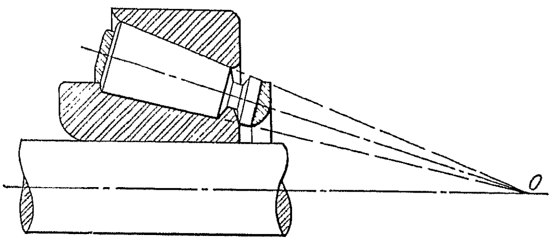


FIG. 334.—The Timken roller bearing.

Ball Bearing Loading is a complex subject, depending on many variables. After making an analysis of the work required of the machine, and all its probable performances and duties, the following points will give the information that a ball bearing engineer would require for the commencement of his calculations.

(1) Is the load all radial? all thrust? or partly both? If the latter, what proportion?

(2) Is the load constant, variable, or intermittent?

(3) Are the load fluctuations gradual, or are there likely to be heavy shocks?

(4) What are the speed conditions? Low, moderate, high, constant, variable, or intermittent? What is its average service?

(5) Will high efficiency be demanded?

(6) Or will a moderate approximation to real efficiency serve the purpose?

With the answers to the above questions well considered, the best thing for a designer, not well schooled in the calculation of

ball bearings, is to send his specifications to a well-rated ball or roller bearing manufacturer and get his estimates. There has been a vast amount of research by the engineers of these com-

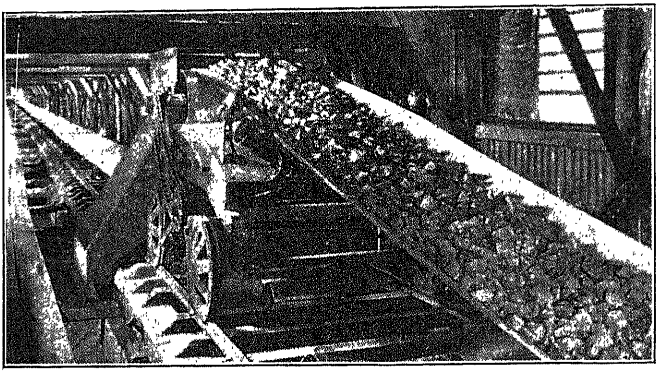


FIG. 335.—Belt conveyor. Jeffrey Mfg. Co.

panies, and much valuable literature has been issued by them. A very good guide for the ordinary machine designer to use in

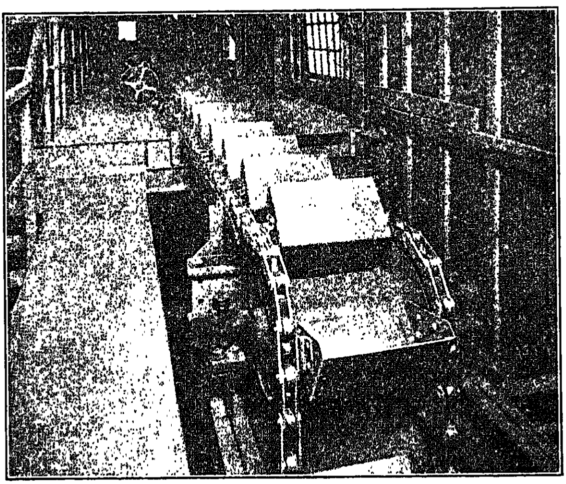


FIG. 336.—Chain conveyor. Jeffrey Mfg. Co.

his calculations, is "Calculating Bearing Loads," issued by the United States Ball Bearing Manufacturing Company, Chicago.

193. Conveyors.—The fifty years just passed have witnessed the most spectacular improvements in the handling of bulky

commodities, both in the raw and in the finished state, and in the process of manufacture. Less than fifty years ago, when an ore steamer from Lake Superior arrived at a dock in Cleveland, Buffalo, or Toledo, the wharf swarmed with laborers and wheelbarrows. From 50 to 100 men in endless file would be kept busy back and forth over the gang planks for hours, sometimes days, removing the cargo by hand, and then repeating the process from the coal pile to the hold. Visit one of these terminals today, and you will see an array of cranes and other conveyors, which do the work of hundreds of men in much less

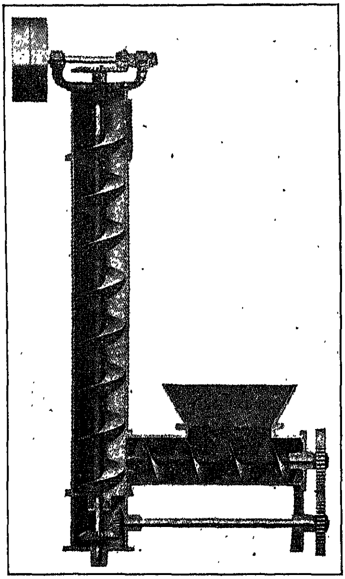


FIG. 337.—Helical conveyor. Link Belt Co.

time, and with less confusion, so that the ship is sent off with her return cargo a few hours after making port. This saves her owners days of carrying time, making the cost to the consumer much less.

Not only is this true at the wharves, but in the mine and in the mill and warehouse both bulky materials and small articles and packages are sent from place to place, as is necessary in their course of production and shipment, by many ingenious and efficient transporting devices. The automobile industry has been stabilized by such methods, as is evidenced by a trip, not

only through the Ford plant, but through any first-class establishment.

The catalogues of all the large engineering concerns are copiously illustrated with examples and applications of every conceivable conveyor, and the student interested in production is advised to examine these catalogues. No text book can do justice to the subject for two reasons, the material is so vast in extent, and designs are continually changed and improved. The illustrations shown here are merely examples of some of the favorite methods of handling the bulky materials, such as ore, crushed stone, cement, grains, flour, sand, and the like. But there are many other labor-saving conveyors, cranes, elevators, trippers, and attachments for keeping things moving, and the devices for routing manufactured articles and packages are too numerous to even list. A very good book on one department of this subject is "Belt Conveyors and Belt Elevators," by Frederick V. Hetzel (1922).

194. Automatic Pickups and Hand Lifters.—The Pritchard Automatic Pickup is one of those ingenious devices that are worked out when the designer is faced with the necessity of adapting something to his needs, or losing a large part of the labor-saving he had contemplated. This was devised by R. W. Pritchard, at the time an engineer for the Pullman Company, as an attachment for a monorail crane, to be used in picking up containers in one part of the plant, and depositing them in another without the assistance of a helper to make the fastening and release. It can be of much more universal application, as it can be made an attachment to large cranes and other conveyors, especially in factories.

The pickup is attached to the lifting cable, and, as it is lowered, the curled "fingers" pass over the handle of the container or package to be lifted. As the fingers descend into contact with the handle, the curvature of the finger surface, as it presses against the handle, compels the pickup to turn in its socket through an angle of about 90 deg. When the pickup is lifted, the fingers are firmly gripping the handle. When the container is set down, on the floor, on a freight car, or wherever it is going, the pickup is again turned 90 deg. by letting it down on the handle, bringing it into such a position that it can be lifted clear and free from the handle.

In handling large quantities of steel and iron, and especially

scrap, the best device is the **magnet** used as a pickup, and handled by a crane. In large steel mills cranes are employed equipped with lifting magnets of 50 tons capacity. These cranes pick up several rails, or structural pieces, sheets, or large quantities of miscellaneous scrap at each load, and transport them to cars, furnaces, or stock rooms, as they may require, in a few minutes. All steel works use a vast amount of scrap, which is hard to

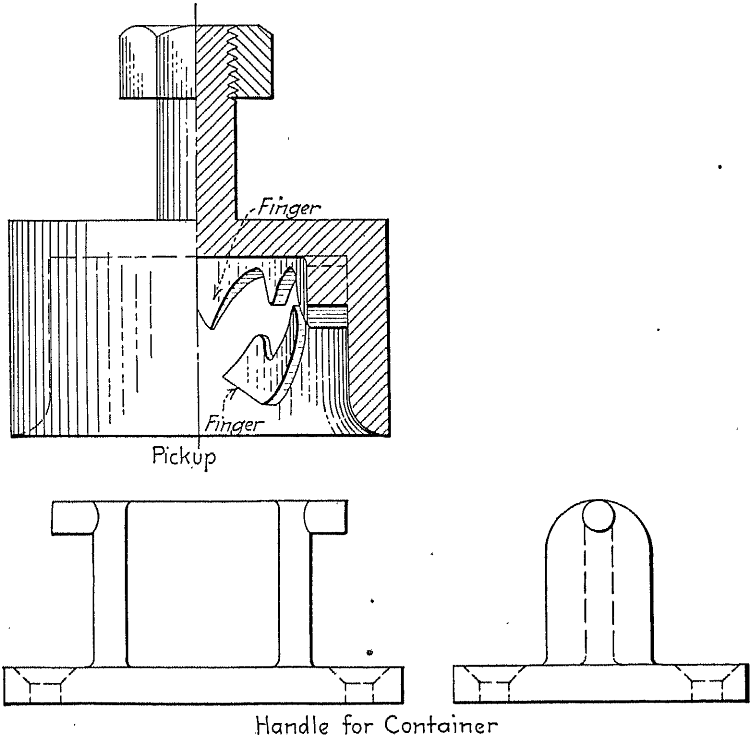


FIG. 338.—The Pritchard automatic pickup.

handle, but with one of these magnet cranes the problem becomes a very simple one.

Jacks.—These indispensable adjuncts to every automobile come in a variety of designs and for a variety of purposes. The main object is always the same, that of lifting heavy objects by human power. Probably the most widely used jack for automobiles is the ratchet type, similar to the one shown in the picture. This type finds favor, because of its quick action.

For railroad purposes and for other very heavy work, the geared jack like the one in the accompanying illustration is employed. Notice the train of gears, bevel and spur, notice the screw thread on the lifting screw, and notice the triple ring of ball bearings. The capacity of this jack is 150,000 pounds, and this immense load can be raised by a man using only his own power. At the pivot of the hand lever is a ratchet and pawl, not shown in section like the rest of the jack, which enables the operator to pump continuously, exerting all his pressure on the down stroke, the thread holding against back-sliding. The gear train gives a double reduction of 6.25 : 1.0, and the screw thread is double-thread buttress, advancing about ¼ in. with each turn

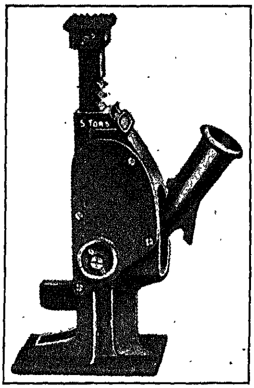


FIG. 339.—Ratchet Jack. The Buda Company, Harvey, Ill.

of the last gear in the train. The **buttress** thread is a combination of square thread, which is easy-running, and efficient in transmission, and the V-thread which is strongest in design. The buttress thread is designed to drive in one direction only, and its driving surface is flat. The root of the thread is as thick as that of a V-thread, hence the buttress is the ideal thread for the purpose, possessing strength for a heavy load, and smoothness in transmission. Like the worm gear of low pitch angle it is self-locking, which prevents the heaviest loads from causing any back pressure on the lever.

The **mechanical advantage** lies (1) in the lead of the thread, (2) the gear train ratio, and (3) the length of the hand lever. In the last analysis it is the ratio of the travel of the hand to the advance of the screw in one stroke of the lever. The screw

moves upward about $\frac{1}{200}$ of an inch, as the hand on the lever travels about 20 inches, which is a mechanical advantage of about 4,000 to 1. That would raise (neglecting friction) the extreme load (150,000 pounds) by the exertion of about 40 pounds on the handle.

Other jacks have been designed with worm gear and epicyclic trains but they are not common. There is no reason, however, why they should not be used for this purpose.

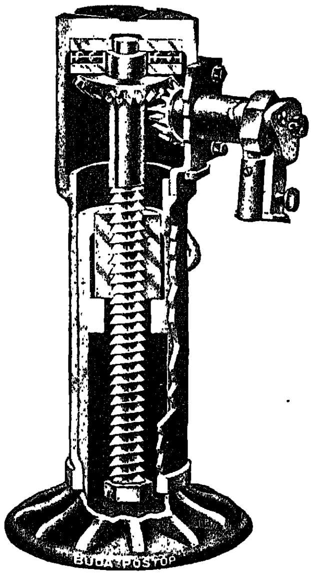


FIG. 340.—Buda railroad jack.

BIBLIOGRAPHY

"Cyclopedia of Engineering" (7 Vols.), American Technical Society, Chicago.

Conveyors

Cat. No. 35, Weller Mfg. Co., Chicago. Cat. No. 400, The Link-Belt Co., Chicago. General Catalogue. Brown Hoisting Machinery Co., Cleveland, Ohio. General Catalogue. Jeffrey Mfg. Co., Columbus, Ohio. Catalogue D-19—Dodge Sales & Engineering Co., Mishawaka, Ind.

Ball Bearings

Auburn Ball Bearing Co., Rochester, N. Y. "Ball Bearing Engineering." Norma Company of America, New York. "Norma Precision Bearings."

United States Ball Bearing Company, Chicago. "Calculating Bearing Loads." "Lubrication of Ball Bearings." New Departure Mfg. Co., Bristol, Conn. "New Departure Ball Bearings (Automotive Edition)." Transmission Ball Bearing Co., Buffalo, N. Y. "Chapman Type Ball Bearings for Transmission."

Roller Bearings

Hyatt Roller Bearing Co., New York. "Hyatt Roller Bearings." Timken Roller Bearing Co., Canton, Ohio. "Timken Roller Bearings."

Note.—The above list and the authorities previously mentioned are selected as representative of an enormous and varied literature, going into all branches of the subject. In following up this line of research, the student will discover a rich and fascinating field both in the literature of our vast industrial organizations and in the multitude of books by noted writers on all the subjects mentioned in this small and necessarily brief text book.

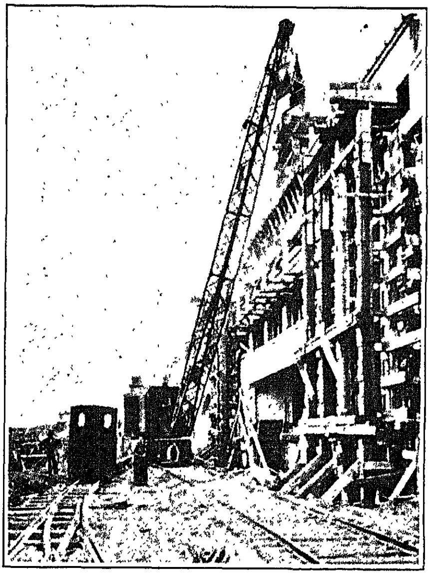


Fig. 341.—Locomotive crane at work on the New York Elevated Railroad structure. Brown Hoisting Machinery Company, Cleveland.

INDEX

A

Acceleration, 6, 41
  definition, 6
  determination of, 42
  diagrams, 41
  scale, 43
Addendum, 110
  definition, 110
  determination, 118, 119
Almond right-angled drive, 255
Angle gears, 141
Automatic handling, 16, 274
Automobile gear box, 235

B

Balata belts, 209
Ball bearings, 268
Base circle, 114
  cam, 178
  involute, 114
Bell crank, 69
Belt drive, 197
  exact length, 201
  horsepower of, 202
  leather, 197
    substitutes, 209
  speed ratio, 198
  tension in, 201
    effective, 202
    slack, 202
    total, 202
Bevel gears, 140
  design, 142
  manufacture, 148
  spiral, 141, 152
  Tredgold's approximation, 147
Brakes, 264

C

Cams, 173
Centro, 30
Centrode, 44
Chain transmission, 215
  roller, 216
  silent, 216
Circular pitch, 109
Clutches, 256
Compressed air, 262
Conveyors, 272
Couplings, 254
Crown gears, 141
Cycloid, 107, 108
  design, 112
  tooth outline, 111, 124
Cylindrical cam, 176, 184

D

Dead points, 55
Dedendum of gears, 110
Differential gear train, 242
Differential pulley, 245
Direct contact pairs, 79
  angular velocity ratio, 80, 81
  line contact, 79
  positive drive, 80
  pure rolling, 80
  surface contact, 79

E

Eccentric, 65
  reversing mechanism, 66
  swinging, 67
Ellipsograph, 56
Elliptical chuck, 57
  gears, 135
Epicyclical gear train, 237
  Ford, 241

F

Flywheel, 11
  governor, 68
Ford transmission, 241

Friction transmission, 88
  cone, 93
  cylinder, 91
  disc, 96
  Evans cones, 95
  grooved, 93
  pressure required, 89

G

Gears, 105
  angle, 141
  bevel, 140
  crown, 141
  cycloidal, 108
  helical, 155
  herringbone, 123
  hyperboloidal, 169
  internal, 107, 121
  involute, 114
  Maag, 117
  mitre, 141
  modern development, 116
  pitch, 109, 110
  Simmons' hobbed, 117, 135
  spiral bevel, 141, 152
  spur, 107
  stub toothed, 116
  terminology, 107
  theory, 106
  tooth dimensions, 109
    outlines, 107, 124
  twisted, 107, 122
Governor, 67
  flywheel, 68
  fly ball, 4

H

Helical gears, 155
  design, 156
    graphical method, 158
Herringbone gears, 123
Horsepower, 7
  calculation, 8
  developed, 12
  indicated, 8
Hyatt roller bearings, 270
Hyperboloidal gears, 169
Hypocycloid, 108

I

Instant radii, 31
Instantaneous center, 29
  definition, 30
  permanent, 32
  to locate for four-link mechanism, 32
  to locate for six-link mechanism, 34
Internal gears, 107, 121
  used as clutch, 121
Involute, 107
  comparison with cycloid, 115
  to draw, 127
  tooth outline, 114, 127

J

Jacks, 275

K

Kennedy's theorem, 33

L

Lathe gear trains, 230
Lewis formula for gear teeth, 146
Link work, 46
  chains, 46
  multiple vibrating, 62
  slider-crank chain, 54
    inversions, 54
Logarithmic spiral, 87

M

Machine design, 14, 143
Manufacture of gears, 130
  bevel, 148
  elliptical, 136
  spur, 131
  worm, 166
Mechanical advantage, 15
Mitre gears, 141
Module, 118
Motion, 2
  constrained, 2

Motion, definition, 2
  Newton's laws, 7
  varieties of, 3

O

Octoid tooth outline, 151
Odontograph, 125
  cycloidal, 126
  involute, 128
Oldham coupling, 254

P

Pantograph, 72
Pawl, 261
Pinion, 107
Pitch of gears, 109
  circular, 109, 145
  diametral, 110
Pitch circle, 108
Plate cams, 176
Positive return cams, 177
Pritchard automatic pickup, 274
Pulleys, 205
  cone, 207
  crowned, 205
  differential, 245
  tight and loose, 206
  trains of, 244

Q

Quick return mechanisms, 58
  MacCord's, 62
  oscillating beam, 59
  reverse anti-parallel cranks, 62
  rolling ellipses, 62, 86
  single-slide swinging, 61
  Whitworth, 58

R

Rack, 107
Racket, 261
  jack, 275
Rocker arms, 66, 68
Roller bearings, 270
  Hyatt, 270
  Timken, 271
Rolling curves, 83
  ellipses, 84
Rope transmission, 209
  American system, 211
  English system, 211
  horsepower of, 212
  pulleys for, 213
  wire, 213
    pulleys for, 215

S

Score card for centros, 35
  for cams, 179
Scotch yoke, 65
Skew gears, 169
Sliding pairs, 28
Solenoid, 263
Speed reducer, 246
Spiral bevel gears, 141, 152
Springs, 267
Standard screw threads, 233
Steam engine, 3, 9
  indicator, 71
  piston displacement, 63
Straight line motions, 70
Stub tooth gears, 116
  dimensions, 117
  layout of, 129

T

Time table for cams, 179
Timken roller bearings, 271
Trains of mechanism, 225
  automobile transmission, 235
  differential, 242
  epicyclic, 237
  Ford, 241
  idlers for, 229
  lathe back gear, 230
  lathe screw cutting, 232
  reverted, 226, 227, 239
  tumbler reversing, 230
Turning pair, 27
Twisted gears, 122

U

Universal joint, 255

V

Variable speed transmission, 95, 260
  belted cones, 207, 223
  disc friction, 96
  Evans cones, 95
  Reeves, 260
  stepped pulleys, 207
Velocity, 4, 31
  angular, 4
  determination of, 36, 38, 39
  diagrams, 39
    polar, 40
    space, 40
    time, 40
Velocity, linear, 4
  units of, 5

W

Work, 7
  measurement of, 7
Worm gears, 159
  design, 164
  efficiency, 161
  helical angle, 162, 165
  layout, 163
  manufacture, 166
  speed ratio, 161